Experiment

IV

Momentum and Drag

I. Preparing for Lab

The purpose of this experiment is to test for conservation of linear momentum in the presence of resistive forces. For a simple collision between carts on an air track, small but non-negligible friction and drag forces should lead to small losses of momentum; can we use an algorithm based on a realistic model of the resistive forces (provided!) to restore momentum conservation?

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see 5.3 (drag) and Chapter 8 (momentum) in your textbook.

If you wish to review it, a video walkthrough of a similar experimental setup is available here.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

Two air track carts, each with 2-50g masses, optical "picket fence", and an attached cup

Air track with blower	Two soft rubber band bumper sets
Two optical gate sensors	LabPro interface box
Mass scale	LoggerPro template
Excel spreadsheet template	

II. Physical Theory

The Principle of Conservation of Momentum says that "**In a closed system with no net external force acting on it, the total momentum is constant**". The **total momentum** is the vector sum of the momenta of all parts of the system. The fact that momentum is a vector means that you will need to keep track of the direction as well as the magnitude of the momentum of each object in.

Formally, a **closed system** is a collection of objects in which the objects cannot enter or exit the system. In this lab, you should think of the closed system as being just the two carts. The key phrase "**no net external force**" means that you will need to prevent (or account for) any unbalanced interaction with entities outside of the system. That will firstly mean that you need to

level the air-track to ensure that the force of gravity on each cart is cancelled out by the normal force from the track.

Let cart A have mass m_A, initial velocity $\vec{v}_A{}^i$ and final velocity $\vec{v}_A{}^f$ and cart B have mass m_B, initial velocity $\vec{v}_B{}^i$ and final velocity $\vec{v}_B{}^f$. Three different types of collisions can occur:

(1) Inelastic Collision - the objects bounce off each other, but some kinetic energy is lost in the collision (to friction, deformation, or sound). Although the total kinetic energy will change, the total momentum will not change, so we can write

$$m_A\vec{v}_A{}^i + m_B\vec{v}_B{}^i = m_A\vec{v}_A{}^f + m_B\vec{v}_B{}^f \tag{1}$$

(2) Elastic Collision - the objects bounce off each other without loss of kinetic energy, so K will be the same before the collision as after the collision. For such a collision the total momentum will again not change, so we have both of the following relationships:

$$\begin{aligned} &m_A\vec{v}_A{}^i + m_B\vec{v}_B{}^i = m_A\vec{v}_A{}^f + m_B\vec{v}_B{}^f \\ &\frac{1}{2}m_A\left(v_A{}^i\right)^2 + \frac{1}{2}m_B\left(v_B{}^i\right)^2 = \frac{1}{2}m_A\left(v_A{}^f\right)^2 + \frac{1}{2}m_B\left(v_B{}^f\right)^2 \end{aligned} \tag{2,3}$$

(3) Perfectly Inelastic Collision - the objects collide and stick together, resulting in a maximum loss of kinetic energy. After the collision, the objects move together as one, so one can simplify eqn (1) to

$$m_A\vec{v}_A{}^i + m_B\vec{v}_B{}^i = \left(m_A + m_B\right)\vec{v}_f \tag{4}$$

Notice the A/B subscript is absent from the *shared* final velocity.

In this experiment, a perfectly inelastic collision will be simulated by dropping a small object (straight downward) into the cup attached to the moving cart. In that case, the vertical momentum of the falling object will be absorbed by the cart and track, so that conservation of momentum does not hold in the vertical direction; however it will still hold in the horizontal direction.

The common property of all three types of collisions is that the total momentum of the objects is unchanged by the collision, provided there is no net external force acting on the objects. Unfortunately, in the setup you will be using, there is some **net external force** that cannot be completely eliminated.

- The air-track allows the carts to float on a pillow of air, but a small **drag** force will still remain, causing a small slowing of the cart velocities.
- It is impossible to exactly level the track, and even a **small tilt** of the air track will cause the cart to measurably accelerate.

Although you can't completely eliminate tilt and drag from the setup, we will attempt to compensate for their presence in the analysis. This will require that you first make some measurements of the motion of a single cart and use the results to find the **residual tilt** in the track and determine the **drag force** acting on the cart. These calculations are doable but messy and time-consuming, so they will be done for you by a macro. With the drag force and tilt known, and additional macro will apply corrections to the collision measurements, allowing us to test for momentum and energy conservation during the collision. See **Lab 4 Appendix A** at the end of the manual for a description of the tilt-drag calculations.

To understand a bit about how the tilt and drag correction works, suppose there is a small drag force on the cart $F_d = -mv_x/\tau_d$, where τ_d is the **damping time**, and a small downhill force $F_g = -mg\sin(\theta)$ due to gravity and a small tilt angle θ of the track. According to Newton's second law, the net force will cause the cart to accelerate along the track:

$$\begin{aligned} -F_{gx} - F_d = ma_x \quad &\Rightarrow \quad -mg\sin(\theta) - \frac{mv_x}{\tau_d} = ma_x \\ \Rightarrow \quad a_x &= -\frac{v_x}{\tau_d} - g\sin(\theta) \end{aligned} \tag{5}$$

Here we have assumed that the tilt direction accelerates the cart in the same (negative) direction as the drag force, but note that for motion in the opposite direction on the track, this tilt would speed the cart up, while the drag force will still slow the cart down. So there are two different scenarios to model. This acceleration a_x will cause the cart's velocity to change, even if there is no collision.

III. Experiment

Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 4** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

(4) Make sure that the indicator light on the sonic ranger is glowing green. If it isn't, check that the cord for the ranger is plugged into the LabPro interface. If not, contact your TA.

(5) Now check the LabPro box to make sure that that the **left** optical gate is plugged into the **DIG/SONIC 1** input port and the **right** optical gate is plugged into the **DIG/SONIC 2** input port as shown in Figure 1.

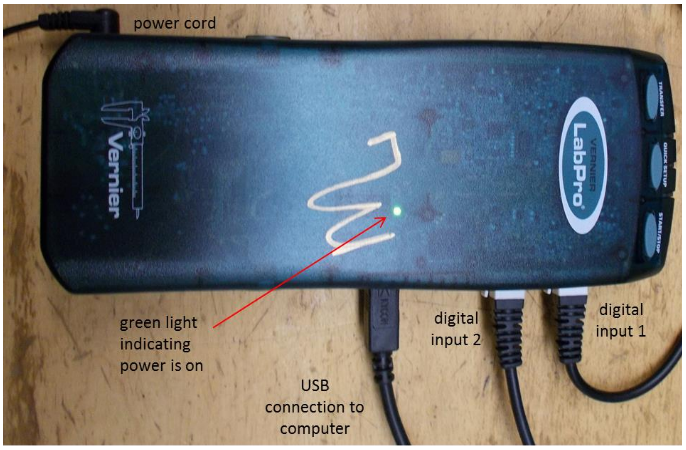

Figure 1. LabPro interface box showing optical gates connected to digital input ports 1 and 2.

(6) Start the LoggerPro software by opening the LoggerPro Templates folder on the desktop and clicking on the file labelled **Two Optical Gates**; if LoggerPro was left running from the previous lab section, close and restart it.

Part B. Setting up the Air track

(1) Check that your carts are both set up as seen in Figure 2. The rubber band bumpers should be relatively tight to ensure smooth collisions off each other and the end stops, and the bumper fixtures will need to be rotated to allow cart-cart collisions without the plastic brackets hitting together.

(2) The carts are labelled “cart A” and “cart B” to help you keep them straight. Measure the mass of each cart in kg and record in the designated area in your spreadsheet.

(3) Check that the optical “picket fence” on each cart is parallel to the track (See Figure 2) and the optical gates are perpendicular to the track.

(4) **CAREFULLY** check the height of the gates so that when the cart passes through a gate, the black bands on the picket fence each pass cleanly through the reader on the gate. **You should see the red light on the gate blink on and off rapidly 5x as the cart goes through, indicating each band successfully blocks the sensor.**

(5) Separate the optical gates by about 40 cm and ensure that there is at least 25 cm between a gate and the end of the track. Measure the exact distance between the centers of the optical gates in meters and record it in the cell labeled “d”. ***Do not move the gates for the rest of the lab.***

(6) Turn on the air supply and turn the power level knob to about the 1 o’clock position. Some blowers will need a higher setting to achieve adequate air flow; if needed fine tune the setting so that it is just enough to enable glider movement to appear frictionless.

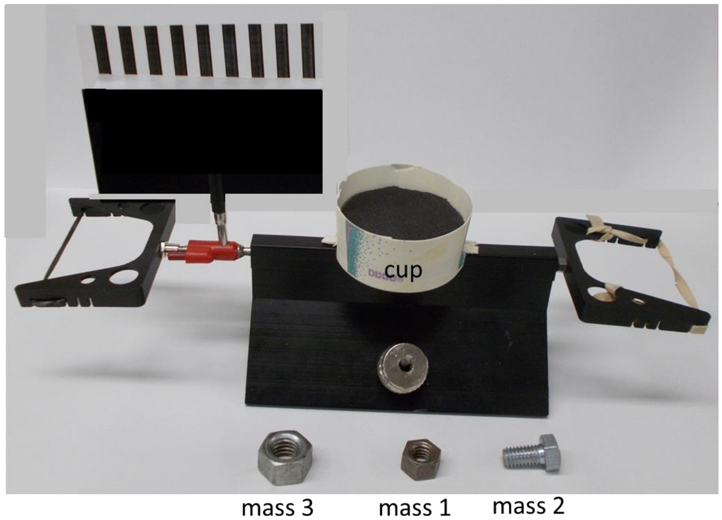

Figure 2. Early prototype of cart, and drop-weight masses for the inelastic collisions.

(7) To level the air track, place a cart at the **center** of the air track and try to stop it from moving, then carefully let go without pushing. If the cart moves on its own, rotate the adjustable feet of the track until it passes this leveling test.

Part C: Measuring the residual tilt and drag force

In this part, you will collect simple cart motion data that will be used to measure any tilt in the track as well as the drag force on a cart; the data will be used to ***recalibrate*** *the data in parts D and E to allow for "ideal" modeling of an* ***isolated system****.*

(1) Place cart A on the track, well outside of the area between the optical gates. In particular, the optical picket fence should not intrude into a gate.

(2) Click "Collect" in Logger Pro. It will wait for a trigger, so don't worry if nothing happens on the screen.

(3) Send the cart through the gates, not too fast or too slow, making sure to release it well before it enters the first gate. Allow the cart to bounce off the end of the track and pass back through both gates in the opposite direction before clicking "Stop" in Logger Pro.

(4) Select all of the data LoggerPro is displaying in its columns (Ctrl-A on the keyboard), copy the data (Ctrl-C or edit menu) and paste it (Ctrl-V or right click) into the table labeled "Trial 1" in your Excel spreadsheet for Part C. Inspect the data to ensure that the final line containing any data (about half the cells will be blank by design) is in the final line of the table (20 lines total). **If your data fills NO cells on the bottom row of the table, then either your gates are not set at the right height (see part B, step 4), or you picket fence was not clear of the gates before launch or at the point of collision with the track's end bumper (see part B, step 5 or part C, step 1). Correct the problem and repeat steps 2-4.**

(5) Repeat steps 1-4 with **small variations in the initial velocity** of the cart. We found that speeds in the 0.1 m/s to 0.5 m/s range worked well. If the cart's initial velocity is too similar to a previous trial (within 0.05 m/s or so), then don't record that trial. Repeat the process until you have 5 good trials with different velocities pasted into your spreadsheet.

Part D: An inelastic collision

In this part, you will observe a "normal" (inelastic) collision between the two carts and record before and after velocity data to test conservation of momentum.

(1) Place cart A on the left side of the track and cart B on the right side so that both picket fences are nearer to the center of the track, but also still well clear of the two optical gates in between them. Each lab partner should control one cart. Make sure the rubber bumpers are oriented so that the carts can collide with clean bumper-to-bumper contact and no plastic contact.

(2) Click "Collect" in Logger Pro and send the carts through the gates so that they collide when both picket fences are between the optical gates. The collision will send the carts back through the gates in the reverse direction. Allow the carts to completely exit the gates, then click "Stop" in Logger Pro (make sure to stop Logger Pro before the carts re-enter the gates).

It may take some trial and error to get a clean collision to occur in the appropriate location. (Don't use too much force in the push…if you hear a clank at launch, that is the cart hitting the track and you need to try again.)

(3) Once you have data for a good collision, select and copy all of the data LoggerPro is displaying in its columns and paste this data into the table labeled "Trial 1" in Part D of your Excel spreadsheet.

(4) Repeat the collision data collection process two more times, varying somewhat the initial velocities of the carts. If a trial is too similar in speed to a previous trial, try again.

Part E: A perfectly inelastic collision

In this part, you will drop small masses into the moving cart to observe some perfectly inelastic collisions; the vertical momentum of the objects needs to be small to avoid putting enough pressure on the cart to cause it to "bottom out" on the track, which would ruin the low-friction motion enabled by the air track.

(1) Measure the mass m_1 of the small drop-weight labelled mass 1 (see Figure 2) and record it in the designated cell.

(2) Place cart A on the track to the left of both optical gates, and orient the cart so that the cup is to the right of the picket fence. Recheck that the optical picket fence on the cart is straight and parallel to the track.

(3) Click "Collect" in Logger Pro and send cart A through the first gate, then ***before the cart enters the second gate,*** **drop mass 1 into the cup** from no more than 5cm above; the

collision must occur while the picket fence is *completely* in between the two gates. Click "Stop" after the cart completely exits the second gate.

(4) If you are satisfied with your data, select and copy all of the data LoggerPro is displaying in its columns and paste this data into the designated area labeled "mass 1" in Part E of your spreadsheet.

(5) Repeat steps 1-4 for small masses 2 and 3.

(6) Turn off the air supply when you are done taking data. This will reduce noise in the lab, extend the life of the blower and reduce the build-up of dirt inside the air track.

IV. Analysis

Part C: Tilt and Drag

(1) Notice that the template automatically extracted the average velocities and their uncertainties, through both gates in both directions, after you pasted in the data. v_1^i and v_2^i are the initial velocities of the cart through gates 1 and 2, before the collision with the end bumper, while v_1^f and v_2^f are the final velocities of the cart through gates 1 and 2, after the collision with the end bumper. Note that the cart will pass through the gates in **reverse order after bouncing off the end of the track**. $dv_{1,2}^{i,f}$ are the uncertainties of the average velocities. Note this is a break in convention for this lab manual in an attempt to minimize confusion in the calculations that follow.

(2) Calculate the difference in velocity** between gate 1 and gate 2 before the collision with the end bumper, $\Delta v^i = v_2^i - v_1^i$, and after the collision, $\Delta v^f = v_1^f - v_2^f$, as well as the uncertainties, $d\left(\Delta v^i\right) = \sqrt{(dv_1^i)^2 + (dv_2^i)^2}$ and $d\left(\Delta v^f\right) = \sqrt{(dv_1^f)^2 + (dv_2^f)^2}$. Clearly these calculations are meticulous and the notation is a bit difficult, so take your time and be careful!

****Notice the ORDER of $v_{1,2}$ is FLIPPED in the after case (indicated in red above) to maintain the order of passage through the gates.*

(3) Plot Δv^i vs v_1^i and Δv^f vs v_1^f on the same chart. These are the specific relationships utilized by the macro to correct for tilt and drag. Label the x-axis "v" and the y-axis "Δv" and add error bars to your plot.

(4) Click the "Fit theory" macro button to fill the theory column, Δv_{th} for a large range of v_{th} values; the macro also determines best fit values for the damping time τ_d and tilt angle of the track θ, as seen in the acceleration eqn 5 in section II. Add the theory to your plot and change the theory points to a smooth line without markers.

Final Question 1: *Briefly explain the physical meaning of the change of sign in $v_2 - v_1$ (initial vs final cases) in your data and plot above.*

Part D: An inelastic collision

(1) Notice that again the template has automatically extracted the average velocities v and their uncertainties dv. The subscripts A and B label the carts and the superscripts i and f indicate whether a velocity is before (initial) or after (final) the collision.

(2) Click the "Account for air resistance and tilt" button in the Part D Analysis section of the template. This macro will apply corrections due to air resistance and tilt based on your data in Part C and return the corrected velocities in the designated column.

(3) Using the corrected velocities, calculate the momenta of the carts before ($p_A^i = m_A v_A^i$ and $p_B^i = m_B v_B^i$) and after ($p_A^f = m_A v_A^f$ and $p_B^f = m_B v_B^f$) the collision and their uncertainties:

i. $dp_A^i = m_A\, dv_A^i$ $\qquad dp_A^f = m_A\, dv_A^f$

ii. $dp_B^i = m_B\, dv_B^i$ $\qquad dp_B^f = m_B\, dv_B^f$

for each trial. The uncertainty in the mass is negligible.

(4) For each trial, calculate the total momentum before and after the collision,

iii. $p_T^i = p_A^i + p_B^i$ and $p_T^f = p_A^f + p_B^f$

and their uncertainties,

iv. $dp_T^{\,i} = \sqrt{(dp_A^{\,i})^2 + (dp_B^{\,i})^2}$ $\qquad dp_T^{\,f} = \sqrt{(dp_A^{\,f})^2 + (dp_B^{\,f})^2}$

(5) Calculate $\chi^2 = \dfrac{(p_T^{\,f} - p_T^{\,i})^2}{(dp_T^{\,i})^2 + (dp_T^{\,f})^2}$ and the χ-dist value in the designated columns ("chidist" is the function).

Final Question #2: *Recall that a χ-dist value between 0.05 and 0.95 is indicative of statistical agreement or a good fit; above 0.95 is too high and can indicate an overestimation of the uncertainties, while below 0.05 indicates a disagreement between values. After accounting for air resistance and tilt, did you find that total momentum was conserved in the collision modeled in Part D? Briefly explain how you know, and offer a potential explanation for why not if applicable.*

Part E: A perfectly inelastic collision

(1) Notice that again the template has extracted the average velocities $v^{i,f}$ of the cart before the mass drop and after the mass drop, as well as the uncertainties for each trial.

(2) For each trial, calculate the total momenta before and after the drop and their uncertainties using methods similar to those for Part D. Recall that after a perfectly inelastic collision (the drop), objects move together at the same speed, with the total combined mass but shared velocity; for example, total momentum after the drop in trial 1 is $p_T^f = p_A^f + p_1^f = (m_A + m_1)\, v_f$

(3) Click the "Account for air resistance and tilt" button in the Part E analysis section. The macro will apply a correction based on your data in Part C and return the corrected momenta.

(4) Using the corrected momenta, calculate $\chi^2 = \frac{(p^f - p^i)^2}{(dp^i)^2 + (dp^f)^2}$ and the χ-dist value in the designated cells.

Final Question #3: *After accounting for air resistance and tilt, did you find that total momentum was conserved in the perfectly inelastic collision modeled in Part E? Briefly explain how you know, and offer a potential explanation for why not if applicable.*

V. Finishing Up Before Leaving the Lab

(1) **Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.**

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is correct.

(3) Save the spreadsheet using the button provide near the bottom right of the spreadsheet workspace - the template will generate two files, one with your name in the title and one with your lab partner's name in the title.

(4) Before leaving the lab, Log into ELMS, go to the Physics 261 assignments, and submit your spreadsheet.

(5) Log out of ELMS and allow your lab partner to log in and submit their own assignment also.

(6) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab.

VI. Appendix: How the macro accounts for tilt and drag

Suppose you now send the cart down the track so that it passes through two optical gates that are a distance L apart. The first gate will measure velocity v_1 at time t_1 and the second gate will measure velocity v_2 at time t_2. Given that the cart was accelerating for time $t_2 - t_1$ and that the external forces are very small, v_1 and v_2 will be nearly identical, and the acceleration will be nearly constant, so we can write:

$$v_2 - v_1 \approx a_x\left(t_2 - t_1\right) \approx \frac{v_1\left(t_2 - t_1\right)}{\tau_d} - g\sin\theta\left(t_2 - t_1\right) \tag{6}$$

For a cart moving in the positive x-direction we have $v_1 > 0$ and we know that $v_1(t_2 - t_1) \approx L$, since the gates are a distance L apart and the velocity of the cart changes very little. On the other hand, for a cart moving in the negative x-direction $v_1 < 0$ and we have $v_1(t_2 - t_1) \approx -L$. Thus the change in velocity on passing through the two gates can be written as:

$$v_2 - v_1 \approx \begin{cases} -\dfrac{L}{\tau_d} - g\sin\theta\dfrac{L}{|v_1|} & \textit{for } v_1 > 0 \\ \dfrac{L}{\tau_d} - g\sin\theta\dfrac{L}{|v_1|} & \textit{for } v_1 < 0. \end{cases} \tag{7}$$

Figure 3 shows a plot of the velocity difference $v_2 - v_1$ as a function of the cart's velocity v_1. The dashed curve is a fit to Equation (7) for L = 0.437 m, $\theta = 0.0013^{\circ}$ and τ_d = 94 s. The points are actual measured data from the apparatus. Notice that the velocity change is very small (just a few mm/s). Also the acceleration depends on the direction of the cart's motion and its speed $|v_1|$, so it is not so simple to think about or work with.

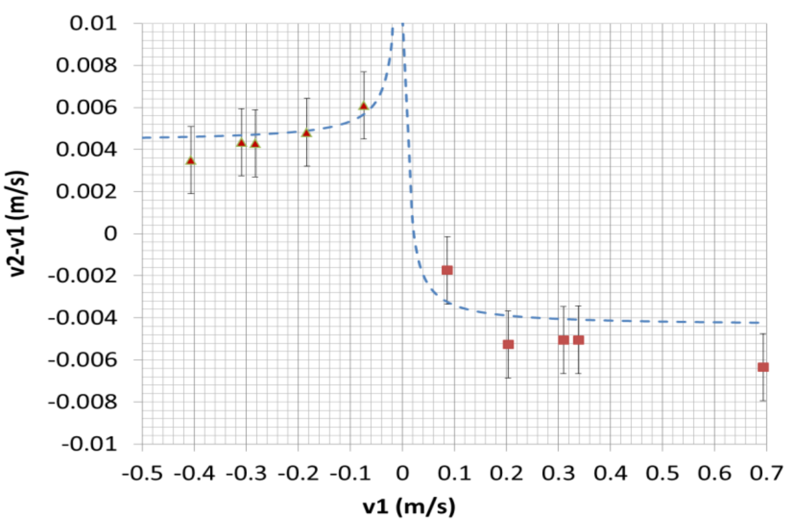

Figure 3. Change in velocity as a function of velocity for a cart that is going between two gates. Points are actual measured data, and the dashed curve is fit to a theory for the velocity change due to a small tilt and drag (Equation (7)).

To understand how you will be **correcting for drag and small tilts**, suppose that cart 1 passes through optical gate 1 at time t_1 where its velocity $v_1(t_1)$ is precisely determined, while cart 2 passes through optical gate 2 where its velocity is precisely determined. The carts then move towards each other and collide together at time t_c. Since drag and tilt are present, the velocity $v_1(t_c - \varepsilon)$ of cart 1 immediately before the collision (think of ε as being a very small time increment) will not be quite the same as the velocity $v_1(t_1)$ measured at gate 1. Similarly, if the cart has velocity $v_1(t_c + \varepsilon)$ immediately after the collision, then by the time it moves back to gate 1 and again has its velocity measured at time t_2 it will be moving at a slightly different velocity $v_1(t_2)$ because of drag and tilt. Thus the velocities you can measure at the gates, before and after the collision, $v_1(t_1)$ and $v_1(t_2)$, are not exactly the same as the velocities immediately before and after the collision $v_1(t_c - \varepsilon)$ and $v_1(t_c + \varepsilon)$. In the lab, you will determine the residual tilt and drag forces and then use a macro that takes these forces into account to find the velocities immediately before and after the collision to test for momentum conservation during the collision.