Experiment

III Motion Due to a Constant Force

I. Preparing for Lab

The purpose of this lab is to model the velocity and acceleration of an object acted on by a constant force (gravity on an incline) using an air track and a sonic motion detector, as well as to explore the validity of a model based on whether it "looks right" versus what the statistical fit results tell you.

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see Chapter 2 in your textbook. You may also want to review **Appendix A** in this manual.

If you wish to review it, a video walkthrough of a similar experimental setup is available here.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

Air glider with sonar reflector and 2-50g masses
Air track with blower
Vernier GoMotion motion sensor
Excel spreadsheet template
Vernier calipers
3-step block
LabPro interface
Logger Pro software

II. Physical Theory

Newton's second law says that when a net force $\vec{F}_{net}$ is applied to an object of mass m, the object obeys

$$\vec{F}_{net} = m\vec{a}\,, \tag{1}$$

where $\vec{a}$ is the acceleration of the object. In this lab, you will use a tilted air track to study the motion of a mass which is acted on by a constant force. The accelerating mass will be a cart that rides on an **air track**; the track reduces friction and allows motion in only one direction. The constant force will be applied by tilting the track, so that gravity pulls the cart down the track.

If the track is tilted at an angle θ from the horizontal, and we choose the x-axis so that it runs parallel the track with the positive x-direction pointing down the incline, then the x-component of the total force on the cart is (treating track friction and air resistance as negligible)

$$F_x = mg\sin\theta \qquad (2)$$

where g = 9.801 m/s^2 is the acceleration due to gravity. The x-component of the acceleration is therefore

$$a_x = F_x / m = mg\sin\theta / m = g\sin\theta\,. \qquad (3)$$

From this result you can see that the acceleration of the cart only depends on the acceleration g due to gravity and the tilt angle θ of the track. In particular, the acceleration does not depend on time nor on the **velocity** / **direction** the cart is moving.

Since

$$a_x = \frac{dv}{dt},$$

where v is the velocity of the cart, eqn (3) is a separable differential equation; as you have likely seen in lecture, it can be integrated to give the following solution for velocity v as a function of time:

$$v(t) = (g\sin\theta)t + v_0\ , \qquad (4)$$

Here v_o is the cart's initial velocity, at time t = 0. Note that the cart's velocity is a linear function of time; if you plot v versus t, it will be a straight line.

Equation (4) can be integrated once more to yield the position x of the cart as a function of time:

$$x(t) = \frac{1}{2}(g\sin\theta)t^2 + v_0 t + x_0\,, \qquad (5)$$

where x_0 is the initial position of the cart. Note that x is a quadratic function of time; if you plot the position x versus the time t, it will be a parabola.

To measure the position of the cart, you will use a sound-based motion detector or "**sonic ranger**". a speaker emits a sequence of brief sound pulses that reflect off the cart and return to the speaker, where they are detected by microphone. The farther away the cart is, the longer it takes for the sound to make the trip. If the round trip requires a time interval Δt, then the distance to the cart must be Δx = ½ v_s Δt, where v_s = 343 m/s is the speed of sound at room temperature. The "½" follows from the fact that the distance is covered twice: there and back again.

An electrical circuit in the device measures the time interval Δt, and the computer then automatically computes the distance Δx to the object, which can be used with eqn (5) once a position for x = 0 is chosen.

The output from the sonic ranger will be recorded by the **LoggerPro software** as a series of points which will allow you to find the position of the cart x_1, x_2, ... x_j ... at times t_1, t_2, ... t_j … By fitting eqn (7) to your data, you will obtain best-fit estimates for x_0, v_0 and the acceleration a, from which you can calculate a prediction for the acceleration due to gravity constant via $a = g\sin\theta$.

III. Experiment

Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 2** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

(4) Make sure that the indicator light on the sonic ranger is glowing green. If it isn't, check that the cord for the ranger is plugged into the LabPro interface. If not, contact your TA.

(5) Start the LoggerPro software by opening the LoggerPro Templates folder on the desktop and clicking on the file labelled **Sonic Ranger**; if LoggerPro was left running from the previous lab section, close and restart it.

Part B. Setting up the Air-track and Sonic Ranger

(1) Check that the Sonic Ranger face is pointing parallel to the track.

(2) Make sure the rubber band bumpers on the cart are moderately tight so it will bounce smoothly off the air track end stops.

(3) Turn on the air supply and turn the power level knob to about the 1 o'clock position. Some blowers will need a higher setting to achieve adequate air flow; if needed fine tune the setting so that it is just enough to enable glider movement to appear frictionless.

(4) To level the air track, first check remove the 3 step block and ensure nothing else is under the feet of the track. Place the cart on the air track at the center and try to stop it from moving, then let go without imparting any impulse. If the cart moves on its own, rotate the adjustable feet of the track until it passes this leveling test.

(5) Test the collection some position data by first pressing the **COLLECT** button on the LoggerPro screen's toolbar; after a brief delay, the sonic ranger should start sending out clicks and you should see the results being plotted on your screen. To stop collecting data,

just click on the **STOP** button (it replaces the COLLECT button when data is being collected).

(6) Next send a cart down the track by giving it a small push and check that the ranger is faithfully recording its position. You may find that the sonic ranger loses the cart when it gets too far away or too close; if this happens, you need to fix the problem before going any farther:

- With the program running and the cart in motion, try tilting the ranger face slightly up/down and see if the range plot improves.
- Make sure that you and your lab partner, your hands, and other equipment are not blocking the path of the ranger.

If you still are having trouble, call your TA over to help.

(7) To zero the ranger, **turn off the air flow**, put the leading edge of the cart at about the 70 cm mark on the track, and then hit the **ZERO button** (next to the **COLLECT** button). This sets the ranger's origin (“0”) at the 0.7 m mark on the track.

(8) **To obtain data for calculating a reasonable estimate of the uncertainty in cart position on the track,** turn the **air supply back ON,** put the cart at the 70 cm position, release it from rest, and collect at least 3s worth of data (60 data points). It is ok and expected that the cart will drift slightly during this motion Copy and paste the position and time for **the most variable 20** of the data points into the designated area in your template. It does not matter which time range the points come from, just that the data contains the most significant fluctuations apparent from the entire range.

Part C. Measuring the Accelerating Cart

(1) Get the distance L of the track by measuring from the center of the air-track’s single foot on one end to the center of the two feet on the other end; record this value in your spreadsheet, in SI units. To estimate the uncertainty ΔL in this measurement, since you can measure to the nearest mm, then $\Delta L = 0.0003$ m. Record this value as well in your template.

(2) Use the vernier caliper to measure the height h of the first step on the 3-step block and record this in your template (labeled “step 1” in your template). We will ignore the uncertainty in this measurement since it is relatively small compared to other uncertainties in this experiment.

(3) Insert the first step under the single leg of the track, closest to the sonic ranger. To ensure usable data, try this method for recording the motion of the cart: place the cart near the top of the track, well to the uphill side of the 70-cm mark “zero; release it from rest or with a very small push, then click on COLLECT just a moment later, well before the cart reaches the bottom; let the cart bounce off the bumper, travel uphill, stop, and repeat the downhill motion, until the 15s of data collection completes.

(4) Examine the plot of your data on the screen and verify that it looks qualitatively similar to **Figure 1**. You will need to analyze the data between bounces, so make sure that at least one complete bounce is recorded in your graph, as is highlighted in red, between the turning points, in the figure; you will also need the turning points to be rather sharp and smooth. If it doesn't look good, just try again; if you are not sure your data is OK, ask your TA to take a look.

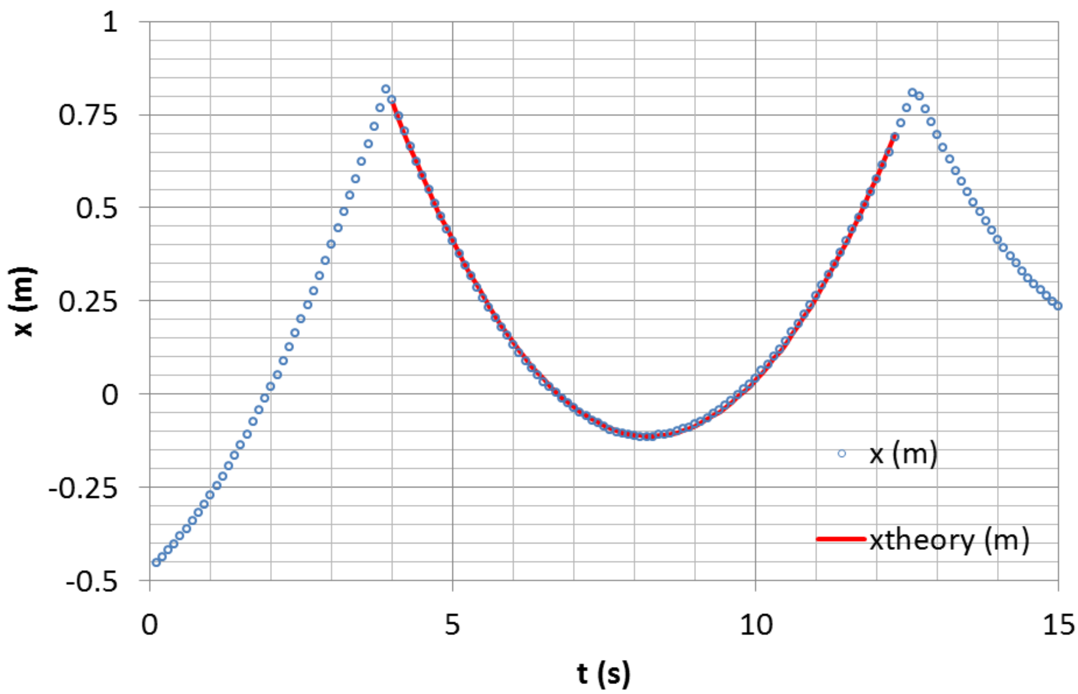

Figure 2. Sample x vs t data (blue points) and fit to theory (red curve).

(5) When you have obtained satisfactory data, copy and paste the time and position data columns from LoggerPro into the "time" and "x1" columns in your template in part C. Be sure to copy the entire set of measurements that Logger Pro recorded; you can click in the column heading to select the entire column, and drag to the second heading to select both columns.

(6) Click the "Check my data" button in the template to run a macro that will try to locate the approximate times for the turning points of your first full parabola. Visually check the start and end times it gives you with the plot to see if the numbers are near (and just inside) the turning points. If they seem OK, continue to step (7); if they seem incorrect, try retaking your data or notify your TA.

(7) Repeat steps (2)-(6) for the second and third steps of the step block. **Only copy and paste the position data into columns "x2" and "x3" for the second and third steps on the step block.** The times will be the same as for step 1 and are not repeated.

(8) Turn off the air supply when you are done taking data to reduce noise in the lab and extend the life of the blower.

IV. Analysis

(1) In the designated area of your template, take the standard deviation of the uncertainty data you recorded in step (8) of part B. This is the uncertainty Δx in each position measurement and you will use this for the error bars on your position vs time data points.

(2) Make a graph of your measured position x vs time t data for the first step of the step block. This plot should look the same as it did on the LoggerPro display. Plot data point markers with no connecting line. Add position error bars to your data and titles and units to the axes.

(3) Click the "Fit x vs t" button in the template to fit the theoretical function

$$x_{theory} = x_0 + v_0 t + \frac{1}{2} a_0 t^2$$

to your data. The macro minimizes χ^2 by varying the fitting parameters x_0, v_0 and a_0 and returns the corresponding points and best fit parameters with uncertainties. It only fits to the first **full** parabola in the data/ plot.

(4) Add the curve for your best-fit theory to the graph for the first step block. Use a smooth line without markers to distinguish it from the data points.

(5) Repeat steps (2) and (4) (but not (3)) for the second and third steps of the step block, **placing all three position vs time plots on the same set of axes.** You don't need to click the macro button again.

Final Question 1: *Given the χ^2 that you got in step (3), and the meaning of χ^2 in general, is the resulting parabola a **statistically** good fit to your position vs time data? Explain your answer in terms ONLY of the rigorous interpretation of χ^2 (see Appendix A for how to interpret χ^2).*

Final Question 2: *Now consider the validity of the fit from a simple ***visual*** inspection. Does the fit "look good"? What are some possible explanations for any discrepancy between the statistical validity and the apparent validity? Might any of your uncertainties be underestimated? Or, are there any systematic errors you can think of that may be present and unaccounted for?*

(6) In the **non-gray** sections of the "v1", "v2", and "v3" columns, compute the velocity at each position for each step on the step block. Do this by taking the difference between consecutive position measurements (the actual position data points, not theoretical fit values) and dividing by the time difference (always 0.05s):

$$v_i = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}.$$

Note you can't use this equation for the last position data point in your data set.

(7) Plot the velocity versus time in the provided box; only use the data from the starting time to the ending time. It should be a straight line. **Again plot all three v vs t plots on the same set of axes.**

(8) Using the best fit parameters found earlier, add the theoretical velocities for each step on the step block in the appropriate columns, again in the **non-gray** sections only,

$$v_{theory} = v_0 + a_0 t$$

(9) Add your theory data to the velocity graphs; again reformat the theory points to display as a line without markers.

(10) We know $a = g \sin\theta$, and for the small tilts used in this apparatus, we can use the approximation, $\sin\theta \approx h/L$. This then gives

$$a_0 = gh/L$$

which shows that the acceleration a_0 is a linear function of the step height h, as well as being proportional to g. In your template, click the "**Fit line to a0 vs h**" button to fit a line of the form $a_0 = (slope)h + int$ to your a_0 vs h values. The slope of this line is g/L, and the intercept should be consistent with a value of 0.

(11) Compute the value predicted by your model for $g = L*slope$, using the slope found in step and your value for L.

(12) Find the uncertainty Δg in your value of g using the formula

$$\Delta g = g\sqrt{\left(\frac{\Delta(slope)}{slope}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}.$$

Final Question 3: *Taking the uncertainty Δg into account, is your predicted value for g statistically consistent with the expected value g = 9.801 m/s^2? Explain how you know.*

V. Finishing Up Before Leaving the Lab

(1) Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is correct.

(3) Save the spreadsheet using the button provide near the bottom right of the spreadsheet workspace - the template will generate two files, one with your name in the title and one with your lab partner's name in the title.

(4) Before leaving the lab, Log into ELMS, go to the Physics 261 assignments, and submit your spreadsheet.

(5) Log out of ELMS and allow your lab partner to log in and submit their own assignment also.

(6) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab ... don't believe anyone who tells you otherwise!