Experiment

II Uncertainty in Measurements

I. Preparing for Lab

The purpose of this lab is to show you how to estimate uncertainty in measurement, how to propagate uncertainties to calculated values, and why it is essential to track uncertainties in any measurements you make.

To prepare for this lab before your session starts, read through the **Appendix A** on **Error Analysis**; for further reference see Chapter 1 in your textbook.

If you wish to review it, a video walkthrough of the experimental setup is available here.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

digital calipers
mass scale (one per setup)
Excel Template for Experiment 2
Numbered sample box containing:
Three blocks made from different metals, one metal cylinder, one sample bolt

Safety Warning: *Be careful when handling the calipers. The ends of the jaws are sharp enough to easily cut you. If you accidentally drop the calipers, don't attempt to catch them - you are risking injury.*

II. Experiment

Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 2** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

Part B: Measurements and Uncertainty

The goal of this part is to learn how to determine the uncertainty in measurements. You will take repeated measurements of the same quantity and examine how much the results vary. A key concept is that you can use the **standard deviation** to find the uncertainty of a measurement when you have multiple measurements of the same quantity that are not all identical.

(1) Turn on the calipers and push the button for the "**mm**" setting. Next *gently* push the jaws together completely. The display should show 0.00 mm; if not, push the "**zero**" button.

(2) Use the calipers to measure the outer thread diameter of the sample bolt (this is called the **major diameter**), by inserting the bolt threads **lengthwise** into the caliper jaws; if you try to measure the threads transversely, the jaws will likely slip from the unstable equilibrium into the thread valley. Again, you need to push rather gently on the caliper jaws when making a measurement to get an accurate reading. Record the diameter in mm in your spreadsheet.[1]

(3) You need to take several measurements of the bolt to properly consider the uncertainty in the diameter. Trade off with your lab partner, rotate the bolt around its axis, and clasp different numbers of threads to maximize variation. Make a total of five measurements of the diameter of the bolt, and record the diameters in your spreadsheet.

(4) In the designated area in your spreadsheet, use the Excel function =AVERAGE (…) to find the average of your five diameter measurements.

(5) **If all five of your measurements are not identical, go to the next step**. If all the numbers are exactly the same, then you are probably doing something wrong—go back and try doing the measurements again. Make sure you aren't looking at the scale while you are adjusting the position of the jaws—it is a mistake to keep fiddling until you see the same number you saw for the first measurement.

If you still find that the five are identical, it means that your measurements are instead *limited by the* ***resolution*** *of the calipers digital display* (*i.e.* the number of digits in the readout); in any such a case, the *uncertainty is taken as 1/3 of the smallest increment*, known as the **least count** of the device, that can be displayed. For these calipers, the smallest increment displayed is 0.01 mm; thus the uncertainty in your measurements would be approximately 0.003 mm. **Appendix A** has a discussion of where this "1/3 rule" comes from. Enter this number into your spreadsheet and **skip step 6**.

(6) The variations in the five measurements are direct evidence that there is some uncertainty in the diameter value. In any such a case, you can find an estimate for the uncertainty in each measurement by taking the **standard deviation** of your results using the Excel function =STDEV(….). Enter the formula into the designated area of your spreadsheet using references to the five-cell range.

[1] We will usually work in SI units in this course sequence, but here, the presence of exclusively small objects, as well as the lack of any dynamic derived-unit calculations, makes grams and millimeters much more convenient.

Part C: Volume, Mass, and Density

The goal of this part is to find the uncertainty in a quantity calculated from other measured values. Such an uncertainty is found through a process typically referred to as uncertainty or "error" **propagation**.[2] You will study this method using both length and mass measurements and their corresponding uncertainties; in Section III you'll use those values to calculate the density of the materials, and propagate the measurement uncertainties to the density value.

(1) Find **block A** based on which letter is assigned to which object in the image provided in your spreadsheet. Place the block on the scale and wait for the display to settle to a fixed value. Record the value for this mass in the designated cell of your spreadsheet template.

Since the scale re-reads the value continually, here we have a case of "multiple measurements" giving an identical result. Hence the measurement is **limited by the resolution of the display**; see the rest of step 5 in part B, if you didn't encounter this earlier. So the uncertainty is 1/3 of the smallest increment displayed, which is in this case is about 0.03 g. Enter this value for "δM" into your spreadsheet.

(2) Repeat the measurement process for blocks B and C.

(3) Use the calipers to measure the lengths a, b, and c of the three sides of each of the three blocks. Let length *a* be the shortest length, *b* the next shortest, and *c* the longest; it is o.k. if two sides are very close in length. Again, push gently on the caliper jaws to obtain an accurate measurement. Record these lengths, nine values total in your spreadsheet.

(4) Measure the length *L* and diameter *D* of the small metal cylinder and record them in your spreadsheet.

(5) All the lengths above were measured using the same calipers, so it is reasonable to assume the uncertainties δa, δb, δc, δD, and δL of each is the same as the value you found for your bolt diameter uncertainty in part B, which may have been 1/3 the least count of the caliper display (0.003mm) or a larger value determined by the standard deviation. We will call this shared uncertainty δx. Copy that uncertainty from part B into your spreadsheet for δx.

III. Analysis

In this section you will use the data from part C of the previous section to find the volume and density of each object; but, the main goal here is the more complex problem of finding the uncertainties in volume and density. To do this you will need to **propagate** the measurement uncertainties.

[2] Physics teachers hate that the notion of uncertainty was ever called "error". These values are NOT due to mistakes or imperfect performance; *they are an **intrinsic and inevitable** part of the measurement process*. If you are thinking of it as the former, please, for the sake of our sanity, correct your thinking NOW.

Propagating Uncertainties (*aka* "Error" Propagation)

For a generic function $f(a,b,c)$ of three measured variables with uncertainties δa, δb, and δc, the general expression for the uncertainty δf is (see Appendix A):

$$\delta f = \sqrt{\left(\delta a \frac{\partial f}{\partial a}\right)^2 + \left(\delta b \frac{\partial f}{\partial b}\right)^2 + \left(\delta c \frac{\partial f}{\partial c}\right)^2}\ . \qquad (1)$$

For the volume of a rectangle $V = abc$ the relevant partial derivatives are

$$\frac{\partial V}{\partial a} = bc \qquad \frac{\partial V}{\partial b} = ac \qquad \frac{\partial V}{\partial c} = ab\ ,$$

so the formula for the uncertainty δV is

$$\delta V = \sqrt{(\delta a \cdot bc)^2 + (\delta b \cdot ac)^2 + (\delta c \cdot ab)^2}\ . \qquad (2)$$

Recall that here the three length uncertainties are all equal and dubbed δx. The above expression is valid and usable, but it can be put into a much simpler form that is easier to understand, and to enter into Excel. Notice that we could also write the derivatives as

$$\frac{\partial V}{\partial a} = bc = \frac{V}{a} \qquad \frac{\partial V}{\partial b} = ac = \frac{V}{b} \qquad \frac{\partial V}{\partial c} = ab = \frac{V}{c}\ ;$$

plugging these expressions into equation (1) as applied to volume gives instead

$$\delta V = V\sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta c}{c}\right)^2}\ . \qquad (3)$$

A quantity of the form *e.g.* $\delta a/a$ is the **relative** or **fractional uncertainty** in a, which is a dimensionless ratio.

For the cylinder volume $V = \pi D^2 L/4$ the relevant uncertainties are (there are only two here, for each of the two variables D and L)

$$\frac{\partial V}{\partial L} = \frac{\pi D^2}{4} = \frac{V}{L} \qquad \frac{\partial V}{\partial D} = \frac{2\pi DL}{4} = \frac{2V}{D}$$

so the uncertainty in the cylinder volume is

$$\delta V = V\sqrt{\left(\frac{\delta L}{L}\right)^2 + 4\left(\frac{\delta D}{D}\right)^2}\ . \qquad (4)$$

Below, you will use the above results in (3) and (4) to complete the valid scientific calculations of the densities of the various materials including their corresponding uncertainties.

Calculating Uncertain Densities for the Objects in II-C

(1) In the designated area in your spreadsheet, calculate the volume V of each block ($V = abc$) and the cylinder ($V = \pi D^2 L/4$). Use valid Excel formulas that reference the cells containing the lengths you measured; *if you are **retyping measured numbers** into the formula you are making a mistake*.

(2) Using equations (3) and (4), add Excel formulas for the uncertainties δV in the volume of each sample in the designated areas of your spreadsheet.

(3) Construct a valid Excel formula for the density $\rho = M/V$ for each object in the appropriate cells. Note the units for density here are g/mm^3. Because kg/m^3 or g/cm^3 are much more conventional choices, write a formula to convert each of your densities to g/cm^3 in the next column; note that 1 cm^3 = 1000 mm^3.

(4) Finally, you need to find the uncertainty $\delta\rho$ for each sample by propagating the uncertainties δM and δV. The relevant partial derivatives are

$$\frac{\partial \rho}{\partial M} = \frac{1}{V} = \frac{\rho}{M} \qquad \frac{\partial \rho}{\partial V} = -\frac{M}{V^2} = -\frac{\rho}{V},$$

so the uncertainty in density is

$$\Delta\rho = \rho\sqrt{\left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}\,. \tag{5}$$

Using eqn (5) and the cell references for ρ (in g/cm^3), M, V, δM and δV, enter formulas in the designated area for the uncertainty $\delta\rho$ of each sample. Note that by using fractional uncertainty for V, $\delta\rho$ will have the same units as ρ even though the V units were mm^3.

Final Question 1: *In your spreadsheet template Table 1 shows the major diameter of different bolts and their manufacturing tolerance. Using your measurements from Part B of the major diameter of the bolt and its uncertainty, identify all the possible bolt types that are consistent with your measurements.*

Final Question 2: *Also in your spreadsheet template, Table 2 shows the density of several materials. Using your measurements of the density and this table, identify the composition of each block sample and the cylinder. Include which alloys are statistically consistent with your data for your aluminum sample. To be consistent, the measured and accepted density should differ by no more than twice your uncertainty (see **Appendix A** for a discussion of the **two-sigma rule**).*

Final Question 3: *Suppose that your calipers are **systematically** reading **inaccurately** by consistently reporting lengths that are **too short** by 1%; so that the calipers displays 9.9 mm when the length is really 10.0 mm. (a) Will your calculated block densities be too high or too low compared to the real density? (b) By what percent will they be incorrect?*

IV. Finishing Up Before Leaving the Lab

(1) **Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.**

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is correct.

(3) Save the spreadsheet using the button provide near the bottom right of the spreadsheet workspace - the template will generate two files, one with your name in the title and one with your lab partner's name in the title.

(4) Before leaving the lab, Log into ELMS, go to the Physics 261 assignments, and submit your spreadsheet.

(5) Log out of ELMS and allow your lab partner to log in and submit their own assignment also.

(6) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab ... don't believe anyone who tells you otherwise!