Experiment

IX Ideal Gas Law and Absolute Zero

I. Preparing for Lab

The purpose of this lab is to examine the relationships among the pressure, volume, and temperature of air inside a closed chamber. To do this, you will test the Ideal Gas Law for two scenarios:

1. Changing volume with temperature held constant (Boyle's Law)
2. Changing temperature with volume held constant (Charles' Law).

In the latter, you will also attempt to extrapolate a value for **absolute zero**, the unreachable temperature at which all kinetic energy vanishes and the pressure of an ideal gas reaches zero.

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see chapter 13 in your textbook.

If you wish to review it, a video walkthrough of a similar experimental setup is available here, although the latter half on Charles' Law uses a different apparatus.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

Boyle's Law Apparatus	temperature sensor
Charles' Law apparatus	pressure sensor
Logger Pro	Excel spreadsheet template for Experiment 9

II. Physical Theory

The **Ideal Gas Law** says that the pressure p, volume V, and temperature T of an ideal gas are related by:

$$pV = nRT \tag{1}$$

where n is the number of moles of the gas and R is a constant of proportionality. The ideal gas law is obeyed fairly well by many real gases, including air, provided the temperature does not get so low or the pressure so high that the gas starts to condense into a liquid.

In a sealed but not thermally insulated container, both number of moles n and the temperature are constant, and so the right hand side of Equation (1) is a constant:

$$pV = C_{\mathrm{B}}; \tag{2}$$

we have defined the constant $C_B = nRT$. The resulting rule is known as **Boyle's law,** which implies that changes in pressure and volume at constant temperature will be ***inversely related*** to each other for a fixed amount of ideal gas.

If we instead fix the volume of our container, then one finds changes in temperature and pressure are ***directly proportional***:

$$p = C_c T \,, \qquad (3)$$

where we have defined the constant $C_c = nR/V$. Equation (3) is known as **Charles' law**.

From Equation (3) you can see that if the pressure could decrease to zero, then the temperature must also decrease to zero; this would be the lowest possible temperature **absolute zero**.

It is important to realize that equations (1-3) are only valid if you use an absolute temperature scale, such as the **Kelvin scale**, in which the lowest possible temperature is set to zero. On the more common Celsius scale, absolute zero is known to be just under -273ºC.

III. Experiment

Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 9** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

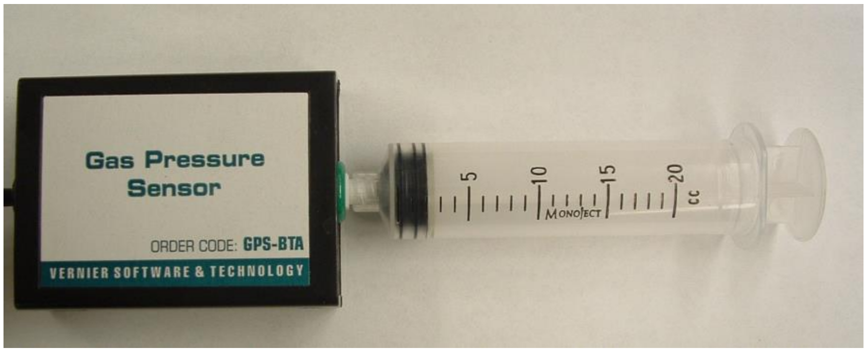

Figure 1. Boyle's Law pressure-volume apparatus

Part B: Testing Boyle's Law

(1) To calibrate the gas pressure sensor, first unscrew the plastic syringe from the gas pressure sensor box (see Figure 1). From the "LoggerPro Templates" desktop folder, open the "Boyle's Law" template. From the "Experiment" drop-down menu, click on "Calibrate" and choose "Gas Pressure Sensor".

(2) Go to the wall of the lab room with the windows, locate the barometer, and read the atmospheric pressure in kilo-Pascals (1kPa = 10^3Pa). In LoggerPro, click "Calibrate Now" and type the pressure reading from the barometer into the designated place in the open window. Click "Keep" and then "Done" to complete the process.

(3) Return to the syringe, and push the plunger in until the inside end of it is at the volume 12 cc mark on the barrel. Then screw the syringe back onto the pressure sensor without disturbing the plunger location.

(4) With the plunger still at the 12 cc mark on the barrel, start collecting data in LoggerPro. Keep the plunger in place for about 15-20s, then push the plunger in until the inside edge of the seal is at the 11 cc mark **and hold it there steadily as well as you can with a sturdy force**. Again wait about 15-20s with the plunger held firmly and steadily in place. Continue this process until you reach the 8 cc mark. As you progress, in the plot of your *p* vs *t* data you should see **nearly constant plateaus** between the jumps in volume, as seen in Figure 2.

(5) With the plunger still steady and data still collecting, start pulling the plunger out in 1-cc steps, still with pauses of 15-20s, until the 16cc mark is reached. Continue to monitor the validity of your data in the plot. Once you have data for 16cc, stop the data collection, and you can finally release the plunger.

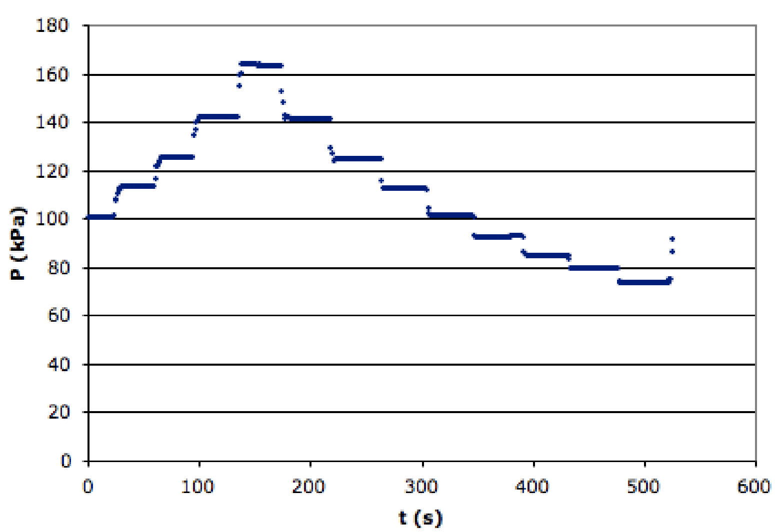

Figure 2. Sample plot showing pressure as a function of time during Part A of the experiment.

(6) If your pressure vs time data seems valid, select, copy, and paste it into your spreadsheet in the designated area of Part B. Note that the volume values you used, notated as V', are already present in the next column.

(7) Examine the scale on the plunger barrel and estimate how precisely you can read the location of the inside edge of the seal. From this determine an appropriate uncertainty ΔV' in your volume readings and record the value in the designated column (use the same uncertainty for every V' in the column).

Part C: Testing Charles' Law

Safety Warning

The apparatus used in Part C to test Charles' Law uses a low-voltage nichrome coil to heat a closed copper cylinder to **temperatures as high as 130^{o}C**.

The coil and copper cylinder are surrounded by a metal wire screen to prevent you from contacting the coil or other hot surfaces. **Do not remove the screen and do not push anything into or through the screen** as this could damage the coil or connections. Before using the apparatus, make sure that it is stable, not too near the edge of the bench, and the electrical cord is out of the way so that it cannot be snagged and cause the apparatus to be knocked over.

(1) Set the toggle switch on the base of the Charles Law apparatus to the center (OFF) position (see Figure 2).

(2) Carefully unscrew the plastic syringe from the pressure sensor box and reconnect the box to the long narrow plastic tube that goes to the copper cylinder (see Figure 2).

(3) Close the "Boyle's Law" template and open the LoggerPro template "Charles Law". Check the current pressure reading to see if it is still near the room pressure value. If it seems off, follow the same procedure as before to calibrate the pressure sensor.

(4) The apparatus uses a small temperature sensor fastened to the side of the copper cylinder and insulated from the ambient air with a small piece of foam rubber. Be sure that this sensor is plugged into the LabPro interface.

(5) In the LoggerPro template, check that the temperature reading is below 30^{o}C. If not, set the heater toggle switch on the apparatus to FAN to cool the cylinder; once the cylinder is at or below 30^{o}C, turn off the fan and allow the apparatus to equilibrate for about 2 minutes.

(6) Click the Collect button in LoggerPro and verify that data collection has started, then move the heater toggle switch to the HEAT position. This will energize the nichrome coil and begin heating the copper cylinder as well as the air that it contains. You should see the pressure and temperature rising gradually, but note that the data is P vs T and does not track the time.

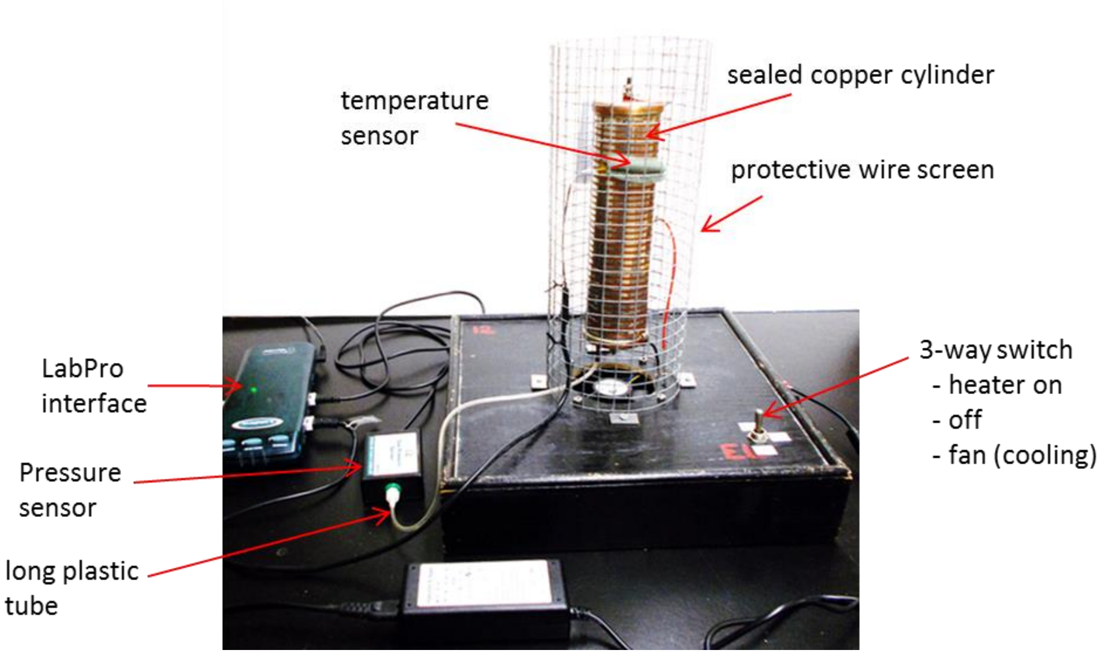

Figure 2. Photo of the Charles' Law pressure-temperature apparatus.

(7) The data collection process will take about 15 minutes; in the meantime you may want to begin the Analysis for the Boyle's Law part of the experiment

(8) Continue collecting data with the heater on until the temperature reaches about 120^{o}C or else stops rising significantly for several minutes. At that point, stop data collection and switch the heater back to FAN to help it cool off. Once the FAN has run for little while, return the toggle switch to the OFF position.

(8) Copy and Paste the temperature and pressure data from LoggerPro into the designated columns of Part C of your spreadsheet.

IV. Analysis

Part B: Testing Boyle's Law

(1) Use EXCEL to plot your pressure versus time data; be sure to add a chart title, axes titles and units.

(2) In your pressure vs time data columns, the template will do its best to find the flat constant-pressure sections and **highlight them in purple**; its choices may not be perfect, so you should examine the ranges and decide if you should make adjustments to the values you choose. In the designated column of the smaller p vs V' table, calculate the average pressures

p_{avg} associated with each volume (they will be in order) using Excel's AVERAGE function and the constant pressure ranges you have selected following the purple guidelines.

(3) The manufacturer of the pressure sensor says that the uncertainty in an average pressure measured by the sensor is $\Delta p = p_{\text{avg}} / 500$. Enter this formula for the uncertainties Δp into the column on your spreadsheet.

(3) Note that the volume readings V' correspond to the volumes of gas in the syringe, which **excludes a volume V_0** found inside the gas pressure sensor itself as well as the hose connecting it to the syringe. So the total volume V should be

$$V = V' + V_0 \tag{4}$$

Unfortunately we cannot measure V_0 and so will need to determine it from the data. Substituting this volume into Equation (2), we get

$$p_{\text{avg}}\left(V' + V_0\right) = C_{\text{B}} \tag{5}$$

Rearranging this to a "y = mx + b" form, we find

$$V' = C_{\text{B}}\left(\frac{1}{p_{\text{avg}}}\right) - V_0 \tag{6}$$

Hence a plot of V' vs $1/p_{\text{avg}}$, will be a straight line with a slope of C_{B} and a y-intercept of $-V_0$.

Calculate $1/p_{\text{avg}}$ for use in testing this model.

(4) To calculate the uncertainty $\Delta(1/p_{\text{avg}})$ you will need to propagate the uncertainty in the pressure Δp. (yes $1/p_{\text{avg}}$ is a function of p_{avg}). To do this, start from the general result

$$\Delta\left(\frac{1}{p_{avg}}\right) = \sqrt{\left(\frac{d\left(1/p_{avg}\right)}{dp_{avg}}\right)^2 \left(\Delta p_{avg}\right)^2}, \tag{7}$$

calculate the needed derivative, and plug it in to equation (7). Simplify the expression and input the formula into the designated column in your spreadsheet.

(5) After you have filled in the column for $\Delta(1/P_{\text{avg}})$ in your spreadsheet, run the "Fit line to V' vs 1/P" macro to perform a straight-line χ^2 fit to your $1/p_{\text{avg}}$ vs V' data and extract the best fit estimate for V_0 and its uncertainty ΔV_0.

(6) Now use EXCEL to calculate $V = V_0 + V'$ and its uncertainty ΔV using

$$\Delta V = \sqrt{\left(\Delta V'\right)^2 + \left(\Delta V_0\right)^2} \tag{8}$$

(7) After you have filled in the column for ΔV in your spreadsheet, run the "Fit power law to P vs V" macro to perform a χ^2 fit to your p_{avg} versus V results using a general equation of the form

$$p_{avg} = C_{\text{B}} V^n \tag{9}$$

If your data is consistent with Boyle's Law, we expect to find a good fit with a value for the exponent n that agrees with -1. Notice that the macro also gives the best fit estimate for C_{B}, the uncertainty Δn, and the reduced χ^2 value.

Final Question 1: *From Part B of the experiment, does your data statistically agree with Boyle's Law? Justify using the pertinent uncertainties and the reduced* χ^2 *value.*

Final Question 2: *The technique we had you use to analyze your Boyle's Law data was a bit strange because in order to find* V_0*, we had to use Boyle's Law itself (see Equation 6). For this potentially problematic trick to be physically valid, the correction* V_0 *that was added to all your V' readings had to be* ***the same value*** *and it should have been* ***relatively small****. What was the ratio* V_o/V' *for your smallest volume V'?*

Part C: Testing Charles' Law

(1) Use Excel to plot your pressure vs temperature data. Be sure to add a chart title, axes titles and units.

(2) In the designated areas in Part C of your spreadsheet, calculate the uncertainties ΔT and Δp. The manufacturer of the temperature sensor says that the uncertainty ΔT is $\Delta T = 0.003T + 0.2\,^{\circ}\mathrm{C}$, and recall that the uncertainty in the pressure is given by $\Delta p = p/500$.

(3) After you have the uncertainties, run the "Fit line to P vs T" macro to perform a χ^2 fit to your data using the function

$$p = C_C\left(T - T_0\right) \tag{10}$$

and extract the best fit estimate for T_0 and ΔT_0; the macro also gives the best fit estimate for C_C, and the reduced χ^2.

(4) Add the fit macro fit to your p vs T plot using equation (10) and the fitting parameters found in the previous step and format the data to be a line with no markers.

(5) Notice that Equation (10) is not exactly the same as the Charles' Law Equation (3) because it has the shift T_0 due to use of the Celsius temperature scale. Therefore if your experiment was valid, the y-intercept T_0 in Equation (10) and from your fit should correspond to **absolute zero on the Celsius scale.**

Final Question 3: *Given the uncertainty* ΔT_0*, is your value for* T_0 *in statistical agreement with absolute zero, -273.15 °C? Can you think of any reason why this apparatus might fail to test Charles' law at high precision? Consider the conditions required for Charles' law to apply, and whether the copper cylinder apparatus should adhere to those conditions during a 100°C temperature change. Could these discrepancies affect the fit intercept?*

Final Question 4: *From Part C of the experiment, does your data statistically agree with Charles' Law? Justify using the pertinent uncertainties and the reduced* χ^2 *value. Consider the point brought up in question 3 as well.*

V. Finishing Up Before Leaving the Lab

(1) Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is incorrect.

(3) Save your spreadsheet using the provided button and submit your spreadsheet on ELMS before you leave. Both partners should do this.

(4) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab.