Experiment 9

Conservation of Energy

I. Purpose

The purpose of this experiment is to examine conservation of energy and learn about kinetic energy, gravitational potential energy and the potential energy stored when a spring is compressed.

II. Preparing for the Lab

You must prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Next, don't forget to answer the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. Pre-lab Questions *(You must submit answers to Expert TA before your section meets)*

Questions and multiple choice answers on Expert TA may vary from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

#1. (5 points) Consider the potential energy U stored in a spring:

$$U_s = \frac{1}{2}k\ell^2$$

where $k= 10\ N/m$ is the spring constant and ℓ is the distance that the spring has been compressed from its equilibrium length. If the spring is compressed by distance ℓ_1 = 0.070 m it will have potential energy U_{s1} and if the same spring is compressed by distance ℓ_2=0.14 m, it will have potential energy U_{s2}.

(a) The ratio U_{s1}/U_{s2} does not depend on *k*. True or false?

(b) What is the ratio of the potential energies U_{s1}/U_{s2}? Give numerical answer to two significant figures.

#2. (5 points) Consider an object that is given an initial potential energy U_s by a compressed spring. The object is then "launched", converting all of the spring potential energy into kinetic energy. Conservation of energy tells us that the total mechanical (potential + kinetic) energy of a system is conserved for a frictionless process. Thus the total energy E_0 at the start of the process is equal to the total energy E_f at the end of the process. In this situation, $E_0 = U_s$ and E_f is equal to the kinetic energy K of the object and thus

$$\frac{1}{2}k\ell^2 = \frac{1}{2}mv^2$$

where k = 7 N/m is the spring constant and m = 0.6 kg is the mass of the object. Suppose the spring is compressed distance ℓ_1 = 0.070 m and this gives the object a speed v_1. Next suppose the spring is compressed by twice the distance, *i.e.* to ℓ_2 = 0.14 m and this gives a speed v_2.

(a) The ratio v_2/v_1 does not depend on *k*. True or False?

(b) The ratio v_2/v_1 depends on *m*. True or False?

(c) What is the ratio v_2/v_1? Give numerical answer to two significant figures.

IV. References

For a review of conservation of energy, see for example Chapters 10 and 11 in *College Physics a Strategic Approach* by Knight, Jones and Field.

V. Equipment

Air track with spring launcher
cart
timer
blower
ruler

VI. Experimental Procedure

Part A. Flat Launch

1. Examine your cart and air track. Find the unique identifier number written on each and record this for your lab report.

2. To level the air track, turn on the airflow so that the cart slides easily above the track. Place the glider near the middle of the air track and release it from rest; if the glider begins to move, turn the leveling knob (see Figures 1 and 2) and start again until the glider remains reasonably stationary when released. If the glider remains stationary, its acceleration must be zero, and by Newton's law, the net force acting on it must be zero. If you cannot get your air track to level before the screw is at the end of its range, let your TA know and a riser will be provided to be placed under the foot of the leveling knob.

   Note: The air-tracks are not perfectly flat, so you may want to check for zero acceleration in different places along the track. Try aiming for flatness on average or the middle of the track.

CAUTION: Do not drop the glider!

3. Move the glider until it just touches the steel spring launcher at the end of the track. Record the reading x_0 from the centimeter scale mounted along the air track (see Figure 1). **Please do not write on the air track**.

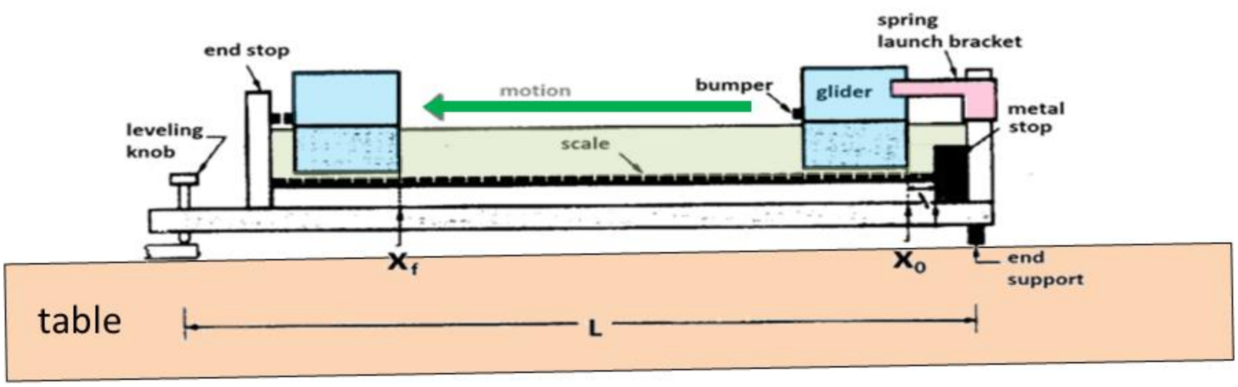

Figure 1. Schematic of the air-track.

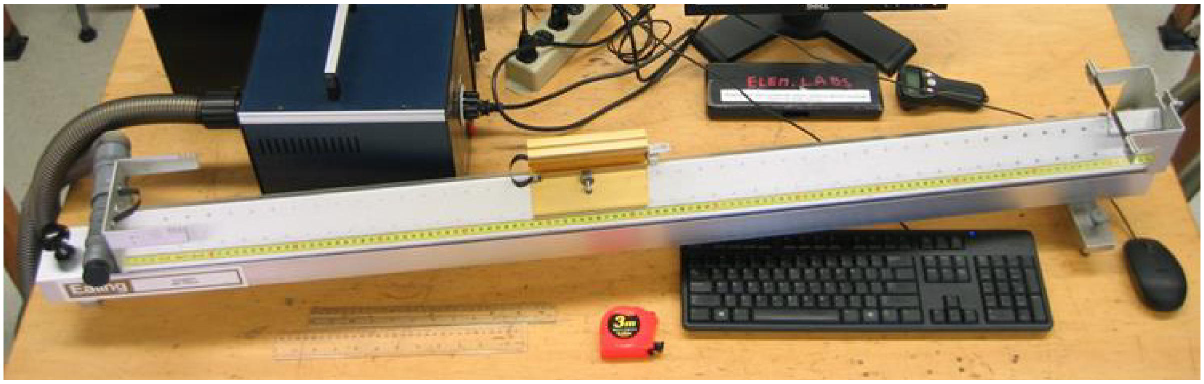

Figure 2. Photo of the air-track.

4. Find the two angled pieces of aluminum that can rest securely atop the track. You can use combinations of these pieces as stops against which the glider will rest when it is pulled back against the spring for launch. This will allow you to maintain consistency in the compression distance from one trial to the next. Find an aluminum piece that results in a compression distance of about 1 cm; measure the precise compression distance and record its value as l_1.

5. The uncertainty of l_1, as with the uncertainty in any ruler measurement, depends only on the limitations of the ruler itself; the ruler scale is marked off in millimeters, so using the "half the least-count" rule for analog scales (see Experiment 2), the uncertainty in each position δx can be taken as ½ mm. Since you need two positions to measure a distance, the uncertainty in the compression distance is

$$\delta l_1 = \sqrt{(\delta x)^2 + (\delta x)^2} = \sqrt{2}\,\delta x. \qquad (1)$$

Find a numerical value for δl_1 and record it in your spreadsheet with appropriate units along with the value for l_1.

6. Pull the glider back against the spring until it touches the stop; release the glider and start the timer simultaneously; stop the clock precisely when the glider reaches the other end of the track. Once you are comfortable with the timing process. Record the time t_1 elapsed for five trials.

7. Record the final position of the glider when it is in contact with the stop on the far side of the track. Be sure to use the position of the "back" of the glider, as shown in Fig. 9.1, for consistency with the initial position. Calculate the total distance traveled by the glider $d_1 = x_f - x_0$. Note that since the final position depends on the well-defined position at the end of the track, the uncertainty δd_1 will be equal to δl_1. In principle, this value is so small compared to the distance d_1 that we can ignore it (convince yourself this was not the case for l_1).

8. Repeat steps 3 and 7 for a compression distance l_2 of around 2 cm. Again, be sure to note the precise value of l_2.

Part B. Uphill Launch

1. Incline the track by turning the leveling knob clockwise an integer number of turns, *n*. The best inclination of the air track should result in the glider traveling about 1 m before coming to rest when released from a compression distance of $l_2 \sim 2$ cm. Measure and record the total length of the track *L*, between the foot of the leveling screw and the far end support, as indicated in see Figure 1. Again, we can ignore the uncertainty δL since $L >> 1$ mm.

2. As described in part A, launch the glider using the compression distance $l_1 \sim 1$ cm. Make sure you use the exact same compression for that you used l_1 in part A. This time, record the position x_f of the glider when it just comes to rest. Again do this for five trials and record the data in your spreadsheet. As the value of x_f may change from one trial to the next, be sure to note and record the value *for each trial*. Again, be sure to use the position of the back edge of the glider for consistency with the starting position measurement. The uncertainty for this position data will this time be defined in terms of standard deviation.

3. Repeat step 2 for the compression distance $l_2 \sim 2$ cm. Make sure you use the exact same compression for that you used l_2 in part A.

VII. Analysis and Discussion

Part A. Analysis of Flat Launch

1. Use a spreadsheet to compute the average t_1 and standard deviation σ of the five times you recorded for launch data using compression distance l_1. Next calculate the uncertainty $\delta t_1 = \sigma/\sqrt{N}$ in the average time and record its value as well.

2. Calculate the (constant) velocity v_1 of the glider using the measured full-track distance d_1 and the average time t_1 computed in step 1. Since we are taking d_1 to be exact, the uncertainty in the velocity depends only on the uncertainty δt_1 in time calculated in step 1. You can find the uncertainty $\delta \mathrm{v}_1$ in the velocity v_1 using:

$$\delta \mathrm{v}_1 = v_1 \frac{\delta t_1}{t_1} \tag{2}$$

3. Repeat steps 1 and 2 for your data for the compression distance l_2.

4. Calculate the ratio of the velocities v_2/v_1 and then calculate its uncertainty using,

$$\delta\left(\frac{\mathrm{v}_2}{\mathrm{v}_1}\right) = \frac{\mathrm{v}_2}{\mathrm{v}_1}\sqrt{\left(\frac{\delta \mathrm{v}_1}{\mathrm{v}_1}\right)^2 + \left(\frac{\delta \mathrm{v}_2}{\mathrm{v}_2}\right)^2} \tag{3}$$

Note: this expression can be used for finding the uncertainty in the ratio of any two quantities v_2 and v_1.

5. Right after you released the glider and it broke contact with the spring, *all* of the springs potential energy was converted to kinetic energy so $E_f = K = U_s$. Thus from conservation of energy you

should expect that

$$\frac{1}{2}k\ell_1^2 = \frac{1}{2}mv_1^2. \tag{4}$$

and also that

$$\frac{1}{2}k\ell_2^2 = \frac{1}{2}mv_2^2. \tag{5}$$

Taking the ratio of these two Equations and simplifying things gives:

$$\frac{\ell_2}{\ell_1} = \frac{v_2}{v_1} \tag{6}$$

Using your values for l_2 and l_1, calculate the ratio l_2 / l_1 and its uncertainty (see comment in step 4 for how to find the uncertainty in a ratio of two quantities. Using this value for l_2 / l_1 and your value for v_2/v_1 from step 4 above, do you find that Equation (6) is obeyed to within the uncertainties?

Part B. Analysis of Uphill Launch

1. Compute the average x_{f1} and standard deviation of the five final positions you recorded for launch data using compression distance l_1. Calculate the associated distance traveled $d_1 = x_{f1} - x_0$. Calculate the uncertainty $\delta x_{f1} = \sigma/\sqrt{N}$ and let $\delta d_1 = \delta x_{f1}$. Record its value as well.

2. Repeat step 1 for compression distance l_2 to find $d_2 = x_{f2} - x_0$.

3. Using the values you just found for for d_2 and d_1, calculate the ratio d_2 / d_1 and its uncertainty $\delta(d_2 / d_1)$. See the note in step 4 of the Part A analysis for how to find the uncertainty in a ratio of two quantities.

4. You should have found in part A that the potential energy in the spring is completely converted to kinetic energy as soon as the glider breaks contact with the spring. Conservation of energy also tells us that when the glider finally comes to rest after traveling uphill on the track, its (initial) kinetic energy has been completely converted into gravitational potential energy (final). Recall that the potential energy of an object in a gravitational field is

$$U_g = mgh \tag{7}$$

Therefore $E_0 = E_f$ can be written as

$$\frac{1}{2}mv^2 = mgh \tag{8}$$

Note that mass can be canceled out of both sides of this equation. The v here is the initial velocity of the glider, which is the same as the constant velocity produced by the launch from part A. But how can we determine h? As one can see in Figure 3, if the glider has traveled a distance d along the track, then trigonometry tells us that the corresponding change in height is:

$$h = d\sin\theta. \tag{9}$$

Plugging Equation (9) into Equation (8) and rearranging things, you can write

$$2gd\sin(\theta) = v^2 \tag{10}$$

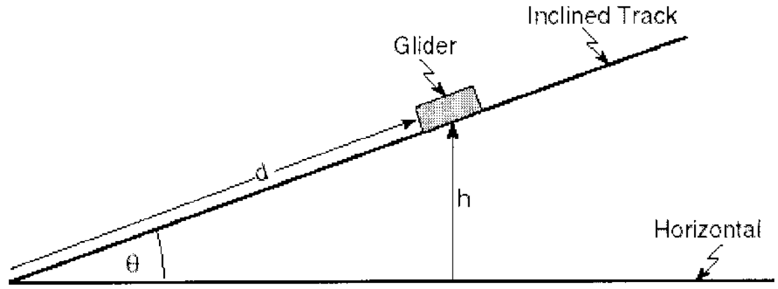

Figure 3: Glider moving up an inclined track

Thus, conservation of energy tells us that:

$$2gd_1\sin(\theta) = v_1^2 \tag{11}$$

and also

$$2gd_2\sin(\theta) = v_2^2 \tag{12}$$

Taking the ratio of these two Equations gives:

$$\frac{d_2}{d_1} = \left(\frac{v_2}{v_1}\right)^2 \tag{13}$$

Using your value for d_2/d_1 and your value for v_2/v_1 from Part A, is Equation (13) obeyed to within the uncertainty?

Part C. Determining the acceleration due to gravity g

1. From the number of turns n used in step 1 of part C, calculate the differential height H of the raised end of the sloped track using the relationship $H = (n / 20) \times 2.54$ cm (the screw has 20 threads per inch, so that $n / 20$ is the height in inches; 1 inch = 2.54 cm).

2. From this value for H and your recorded length L of the track, calculate

$$\sin\theta = H / L. \tag{14}$$

3. Starting from Equation (10) you can solve for g and get:

$$g = \frac{v^2}{2d\sin(\theta)} \tag{15}$$

   Calculate the acceleration due to gravity g using your values for $\sin\theta$, v_1 and d_1.

4. Repeat step 3 using v_2 and d_2 instead of v_1 and d_1.

5. Average your two values for g, and compute the standard deviation σ_g and corresponding uncertainty in the average:

$$\delta g = \frac{\sigma_g}{\sqrt{2}} \tag{16}$$

6. Does your average value for g agree with the accepted value of $g = 9.801\ m/s^2$ to within the uncertainty? If not, what are some factors that may have created unaccounted-for systematic errors in your measurements? Think back about some of the track characteristics discussed in this experiment and earlier experiments that used the tracks.

IX. Finishing Up

- Make sure you get your lab partner's name and his or her contact information.

- Save a copy of your spreadsheet or other data files on a memory stick or e-mail yourself a copy.

- In addition, don't forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus). Also, make sure to review the Guidelines for Writing a Lab Report, right after the main introduction in this Lab Manual.

- Finally, don't forget to prepare for the next lab and submit your answers to the prelab questions for the next lab before they are due.