

Experiment 8

Conservation of Linear Momentum

I. Purpose

The purpose of this experiment is to observe the conservation of linear momentum.

II. Preparing for the Lab

You must prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Next, watch the Youtube video of this experiment at <http://youtu.be/7wcC3l36C-Y>. Finally, don't forget to answer the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. Pre-lab Questions (*You must submit answers to Expert TA before your section meets*)

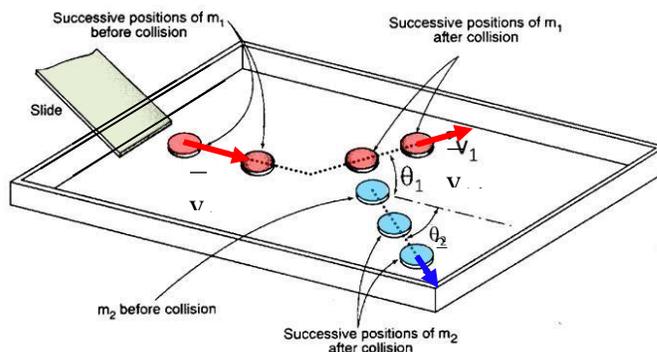
Questions and multiple-choice answers on Expert TA may vary from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

#1. (10 points) Consider the collision of a moving puck with a second, initially stationary puck, as shown in the figure. According to the law of conservation of momentum

$$m_1 \bar{v}_1 = m_1 \bar{v}_{1f} + m_2 \bar{v}_{2f}$$

where $m_1 = 0.50$ kg is the mass of the first puck with initial **velocity** \bar{v}_1 and final **velocity** \bar{v}_{1f} and $m_2 = 0.60$ kg is the mass of the second puck with outgoing velocity \bar{v}_{2f} . Assume the incoming velocity of the first puck is along the x -axis and the outgoing velocity of mass m_1 is at angle $\theta_1 = 50^\circ$. Suppose the **initial speed** of mass m_1 is $v_1 = 1.5$ m/s and the **final speed** of mass m_1 is $v_{1f} = 0.5$ m/s. Mass m_2 leaves at angle θ_2 with respect to the x -axis and you will need to find this angle as part of this problem.

- What is the x -component of the momentum of mass m_1 before the collision?
- What is the y -component of the momentum of mass m_1 before the collision?
- What is the x -component of the momentum of mass m_1 after the collision?
- What is the y -component of the momentum of mass m_1 after the collision?



- (e) What is the x-component of the momentum of mass m_2 after the collision?
- (f) What is the y-component of the momentum of mass m_2 after the collision?
- (g) What is the angle θ_2 in degrees? If the mass m_2 goes below the x-axis (as shown in the figure) assign a negative value to the angle θ_2 , while if the mass m_2 goes above the x-axis, assign a positive value to θ_2 .

IV. References

For a review of the law of conservation of momentum, see for example Chapter 9 in *College Physics a Strategic Approach* by Knight, Jones and Field. For two dimensional collisions, pay particular attention to section 9.6 in Knight. For tutorials on momentum and other topics in physics see for example the set of open-source tutorials by Andrew Elby *et al*, at:

<http://umdperg.pbworks.com/w/page/10511239/TutorialsinPhysicsSense-Making>

The exercise in Part VII below was adapted from these tutorials.

V. Equipment

Air table with blower, web camera, and camera mount

Red and green air pucks

Computer

Excel

Digital weight scale

PowerPoint

Excel Template for Exp. 8

VI. Introduction

The **linear momentum** p of an object having mass m and velocity v is

$$\vec{p} = m\vec{v} \quad (1)$$

The arrow over the \vec{v} and \vec{p} mean that the momentum and velocity are vectors, *i.e.* they have magnitude and direction or x and y components. The **law of conservation of linear momentum** states that, when the total external force acting on a collection of objects is zero, the **total** linear momentum of the objects will not change. If two objects collide, and only exert forces on each other, this principle requires that the total linear momentum before the collision must be equal to the total linear momentum after the collision:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}. \quad (2)$$

Here \vec{v}_1 is the velocity of mass m_1 before the collision, \vec{v}_2 is the velocity of mass m_2 before the collision, \vec{v}_{1f} is the velocity of mass m_1 after the collision, and \vec{v}_{2f} is the velocity of mass m_2 after the collision. The left-hand side of Equation (2) is just the total linear momentum of the two objects before the collision

$$\vec{p}_{total}^{before} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2. \quad (3)$$

and the right-hand side of Equation (2) is the total linear momentum of the two objects after the collision.

To make things a little simpler, in this experiment you will try to arrange things so that puck 2 is at rest before the collision, so $\vec{v}_2 = 0$. In this case Equation 3 simplifies to

$$m_1 \vec{v}_1 = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}. \quad (4)$$

This is a vector equation. In two dimensions (x and y) it is equivalent to two equations, one for the x-components and one for the y-components. From Equation (4), for the x-components we can write:

$$m_1 v_{1x} = m_1 v_{1fx} + m_2 v_{2fx}. \quad (4a)$$

where v_{1x} is the x-component of the velocity of mass m_1 before the collision, v_{1fx} is the x-component of the velocity of mass m_1 after the collision, and v_{2fx} is the x-component of the velocity of mass m_2 after the collision. Similarly, for the y-components we can write:

$$m_1 v_{1y} = m_1 v_{1fy} + m_2 v_{2fy}. \quad (4b)$$

where v_{1y} is the y-component of the velocity of mass m_1 before the collision, v_{1fy} is the y-component of the velocity of mass m_1 after the collision, *etc.*

In the prelab question, puck 1 is initially moving along the x-axis with speed v_1 so that

$$v_{1x} = v_1. \quad (5a)$$

$$v_{1y} = 0. \quad (5b)$$

If puck 1 leaves the collision at angle θ_1 and puck 2 leaves the collision at angle θ_2 we have:

$$v_{1fx} = v_{1f} \cos(\theta_1). \quad (6a)$$

$$v_{1fy} = v_{1f} \sin(\theta_1). \quad (6b)$$

$$v_{2fx} = v_{2f} \cos(\theta_2). \quad (6c)$$

$$v_{2fy} = v_{2f} \sin(\theta_2). \quad (6d)$$

where v_{1f} is the speed of puck 1 after the collision and v_{2f} is the speed of puck 2 after the collision. We can then rewrite Equations 4(a) and 4(b) as:

$$m_1 v_0 = m_1 v_{1f} \cos(\theta_1) + m_2 v_{2f} \cos(\theta_2). \quad (7a)$$

$$0 = m_1 v_{1f} \sin(\theta_1) + m_2 v_{2f} \sin(\theta_2). \quad (7b)$$

Equations 4(a)-4(b) look different than Equations 7(a) and 7(b), but they are really just different forms of the same equations and both express conservation of momentum. Which equations you should use depends on whether you have the x and y components of the velocity vectors, in which case use 4(a)-4(b), or the speeds and angles, in which case you should use Equations 7(a)-7(b).

VII. Warm-up 1-D tutorial exercise before taking data

1. Work through this warm-up tutorial on conservation of momentum with your lab partner.

2. A 100 g puck is sliding in the x-direction at 5 m/s with negligible friction. The first puck collides with a second 100 g puck that was not moving before the collision. Right after the collision the first puck has completely stopped. Using your best physics intuition, what is the velocity of the second puck? Be sure to give the **speed and direction** the second puck is moving and briefly explain your reasoning.
3. Consider again a 100 g puck that is sliding in the x-direction at 5 m/s with negligible friction. It collides with a second 100 g puck that was not moving before the collision, but this time the two pucks stick together and after the collision both pucks slide together as a single unit. If you are having trouble seeing how the two pucks could stick together, just imagine their sides are covered with sticky tape. Using your best physics intuition (don't use a formula at this point), what is the post-collision speed of the two pucks? Briefly explain your reasoning.
4. Next suppose the 100 g puck traveling at 5 m/s collides with a 300 g puck (see Figure 1). This time, instead of sticking together, there is a crunching sound as the 300 g cart gets knocked directly forward by the 100 g puck, and the slightly damaged but still intact 100 g puck comes to rest. Intuitively, what is the post-collision speed of the 300 g puck?

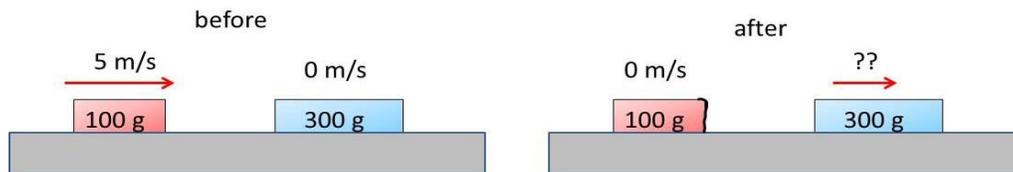


Figure 1. Before and after views of two pucks that collide.



Figure 2. Before and after views of two pucks with the same mass and opposite velocities that collide and stick together.

5. Next consider two identical pucks 100 g pucks covered with sticky tape that slide directly *toward* each other with equal speeds of 5 m/s (see Figure 2). The pucks collide and stick together. Intuitively, after the collision, how fast do the blocks move and in what direction?
6. Based on your answers to the above questions 2-5, does the total momentum of the two pucks go up, go down, or stay the same during each of the collisions? Explain why you think so.

VIII - Collecting 2D collision data

In this experiment, you will test whether the total linear momentum of two objects is conserved (does not change) when they collide on an air table. A diagram of a collision is shown in Figure 3 and a photograph of the setup is shown in Figure 4. After practicing a few “dry runs”, you will use a web camera to take a video of two pucks colliding. You will then use a custom Excel

template to process the video and extract a picture that shows the location of each puck at a series of equally spaced times. After your TA verifies that your picture looks OK, you will analyze the picture and determine if momentum was conserved in the collision.

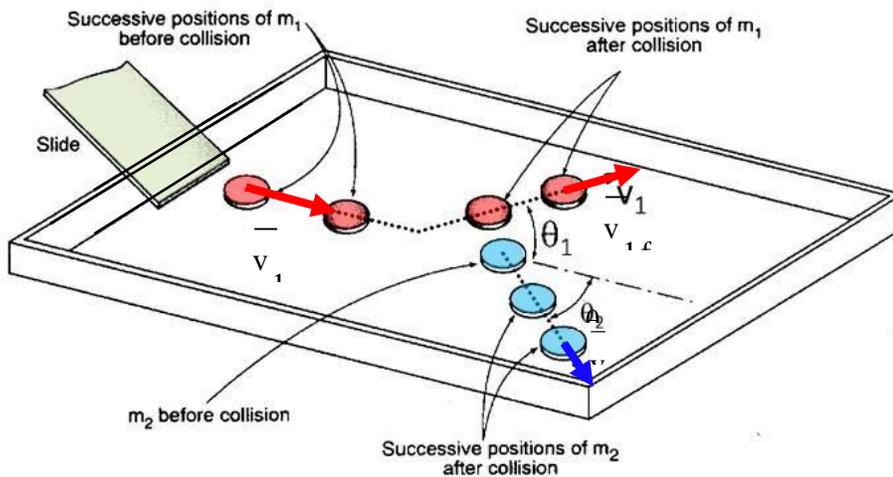


Figure 3. Diagram showing a collision between two pucks on the air table.

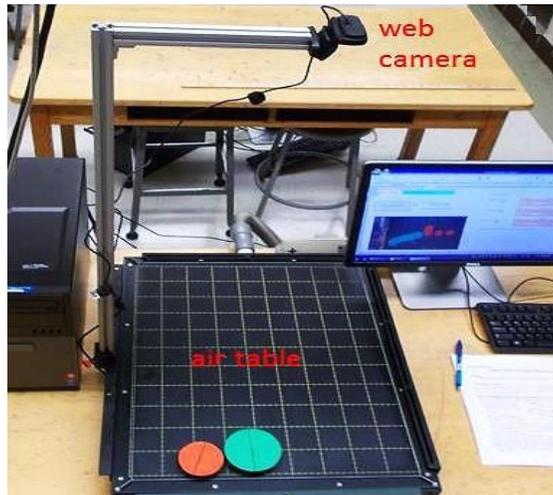


Figure 4: Air table, red and green pucks and web camera.

Part A - Take a Video of a Collision

1. For best results, the TA should have turned on the lights in the room and closed the window blinds. This arrangement produces lighting for the video that is compatible with the software you will be using to process the images.

2. Go to the **Templates** folder on the computer desktop and open the Excel template for **Experiment 8**.
3. You should find a red puck and a green puck at your setup. Choose which puck will be the projectile puck (Puck 1) and which puck will be the target puck (Puck 2). Record the color of Puck 1 and Puck 2 in your spreadsheet template in the designated cells.
4. Measure the mass of each puck and record in the designated cells in the spreadsheet template.
5. Save a copy of your template in the **My Documents** folder. Be sure to include your name and the name of your lab partner in the file name.
6. If you are working with a lab partner, decide who will handle the pucks (person#1). The other person (person #2) must start and stop the video. If you do not have a lab partner, you can ask the TA for assistance. You can also both launch the puck and start/stop the video, although this takes more practice to get a good video than if you are working with a partner.
7. The air table should have been leveled by the lab technicians, but you will need to check whether it is still reasonably level. To do this, turn on the blower, set the level to about 50% (just enough that the puck moves freely), place the target puck in the center of the table, try to gently release it without any velocity, and see if it remains at rest. If you see the puck pick up speed in some direction, then the table is tilted downward in that direction and you need get your TA to take a look and assist you. The air tables will never be perfectly level, but your TA should let you know if it is good enough. With your TA, figure out which foot needs to be turned which way and by how much, and then use the puck to again check if the air table is level.
8. Take a look at the setup and notice the spring-and-string bumpers that the pucks bounce off at the edges of the table (see Figure 5(a)). You will use one of the bumpers for launching the projectile puck with a good speed (not too fast or too slow for the video). Press the projectile puck against one of the bumpers and release it so that it collides with the target puck. Try to have the puck strike a somewhat glancing blow, rather than head-on, as this produces a more interesting video. Practice until you are confident that you can launch the projectile puck in the right direction so that it collides with the target puck.

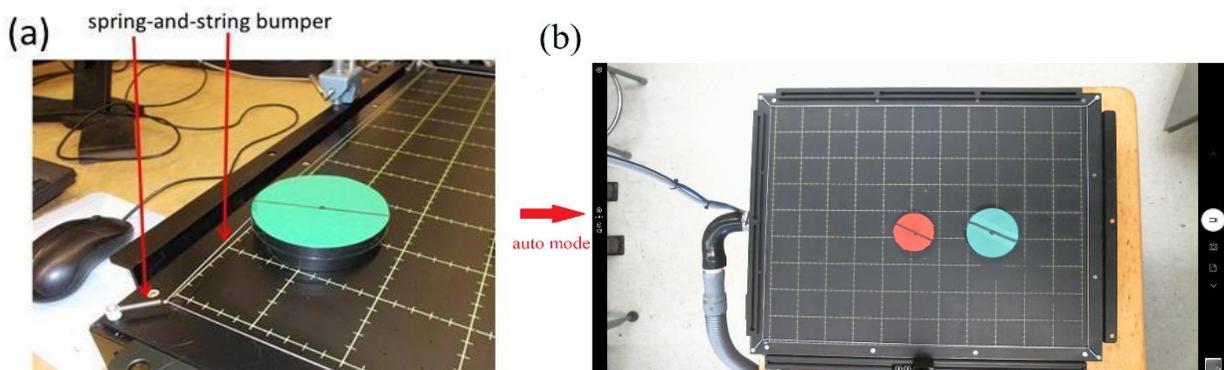


Figure 5. (a) Green puck pressed against one of the spring-and-string bumpers at the edge of the air table. (b) Start-up control panel for the camera.

9. On the computer desktop, click the Camera icon to start up the webcam software. The window should open with a live display. The picture should be in-focus and centered on the air-table. If it is not, verify the focus is in auto mode (see Figure 5(b)). Next click on the Take Video button (see Figure 6(a)). The camera will immediately start recording. End the video by clicking on the **red stop recording button** (see Figure 6(b)). The program automatically saves a copy of each video in the Camera Roll folder.

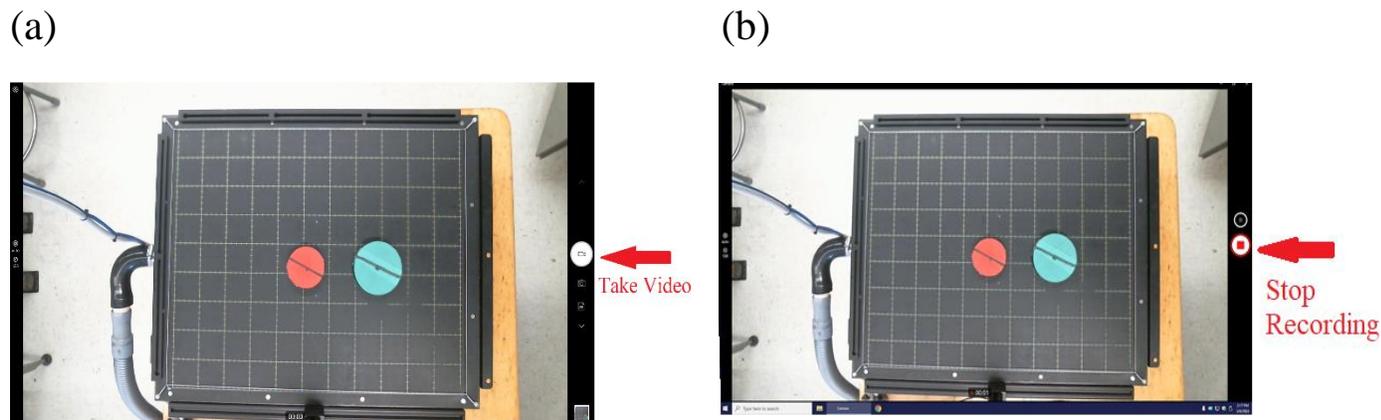


Figure 6. Camera panel showing (a) start recording button, (b) stop recording button.

10. Now that you and your partner know how to launch a puck and how to record a video, it is time to practice doing both at the proper time to create a short video that shows two pucks colliding. Practice the following:
 - Person 1 places the target puck (puck 2) stationary in the center of the table
 - Person 2 clicks on the start recording button
 - Person 2 launches the projectile puck (puck 1)
 - After the collision is over and either puck has reached the edge of the air table, click the stop recording button on the camera's control panel.
11. To view the video, go to the **bottom of the camera control panel** and click on the icon for your video. Consider the quality of the collision you just observed. If you are not satisfied with the video, hit the back arrow button and keep taking videos until you are satisfied with the result.
12. Once you have taken a video that looks good, have your TA verify that it looks OK. Then record the name of your video file in your Excel template so you can find it again. To see the name of the video file, click on the three dots labeled "More" in the right hand corner of the panel, click Open Folder and locate your video file.

Part B - Create a composite picture

1. Go to your Excel template for Experiment 8 (it should still be open from the previous part) and click on the light gray macro button that says "**Create Composite Image**" (see Figure

7). A window will open that shows you the videos that the webcam saved. Click on your video and then click OK. Excel then runs a macro that grabs your video and creates a composite or multiple-exposure picture that shows the puck's position at successive times (see Figure 8(a)). It typically takes about 10 seconds for the routine to run and output an image.

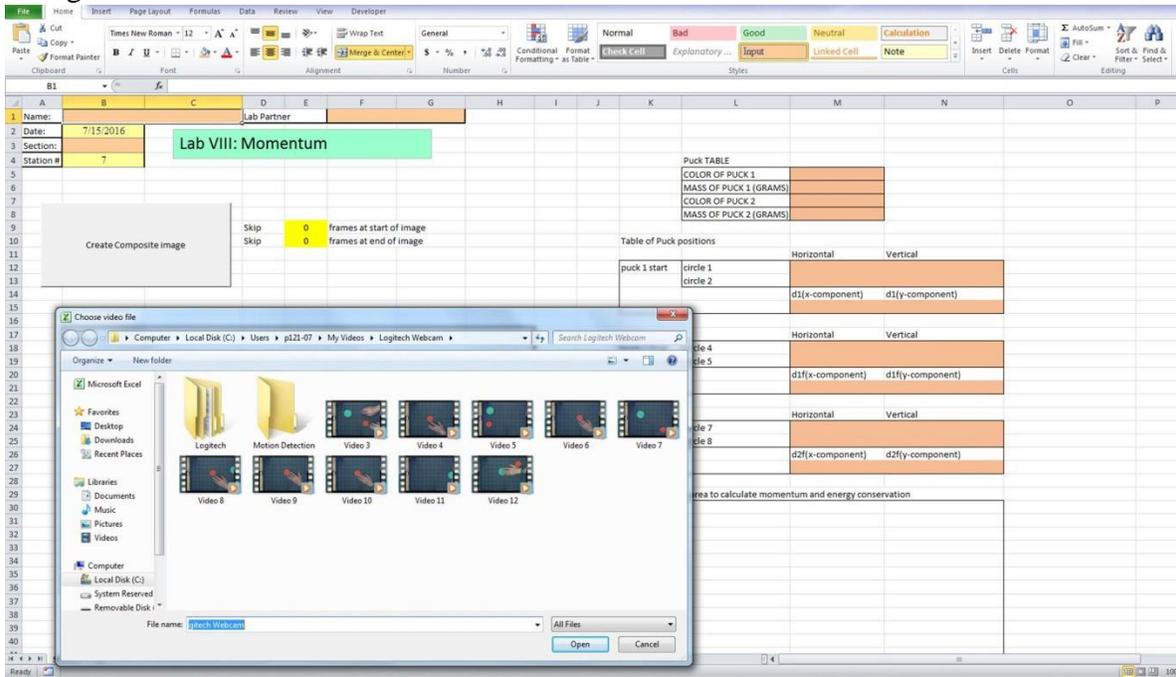


Figure 7. View of the Experiment 8 template after you click on the light gray button that says “Create Composite Image”. A window opens that shows you the videos saved by the webcam.

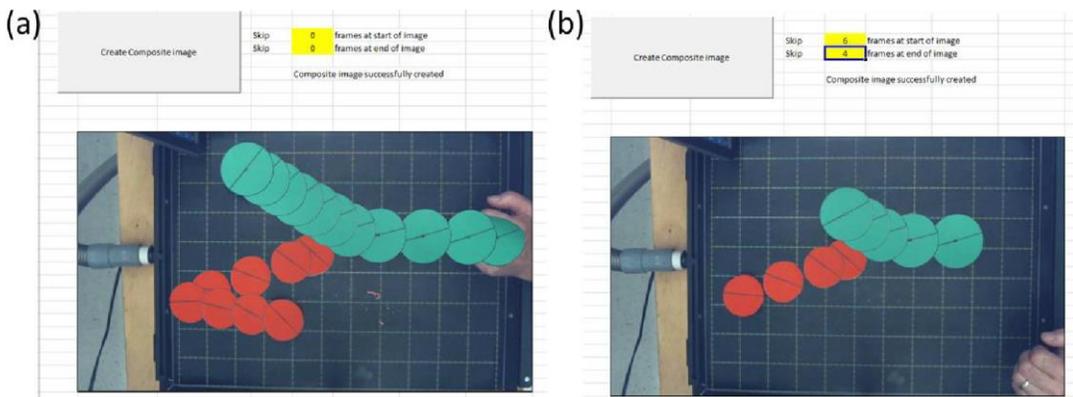


Figure 8. (a) Composite image in Excel template after the macro has run. (b) Composite image from same video, but with 6 frames removed from beginning and 4 frames removed from end.

2. Your Excel template should now display a picture that looks like a multiple exposure photograph of your collision. Take a careful look at your composite picture and compare with Figure 8(a). Of course, your picture will look different. However, you should be able to see both the red and green pucks and you should clearly see the projectile puck (red puck in this case) both before and after the collision. If the target puck (green puck in Figure 8(a)) was at rest before the collision, all the images of it before the collision will be in one spot and appear as a single puck. If it was moving slowly, you might be able to see a few closely overlapped circles. If the target puck was clearly moving, consult with your TA to decide if you need to retake the video.
3. In the example shown in Figure 8(a), the red puck bounced off the bumper at some point after the collision. Of course, this part of the picture cannot be used to determine if momentum is conserved. There is also one red puck in the middle of the picture in Figure 8(a) and it is difficult to tell for sure whether it was before or after the collision. It is actually just before the collision in this example, but it would be best not to use this puck position in an analysis since it is not clearly before or after the collision. **The key point:** *To determine if momentum is conserved, your picture should have two images of the projectile puck before the collision, two images of the projectile puck (and target puck) after the collision, and one image of the projectile puck that may be either just before or after the collision.* To do this, the Excel template has a simple tool for processing the video to remove images of the puck from the beginning and end. In the example shown in Figure 8(b), we removed 5 images from the start (the person held the puck for 5 frames at the beginning) and 3 images from the end.
4. Figure out how many puck images you want to remove from the beginning and the end of your composite picture and enter these numbers into cells E9 and E10 (highlighted yellow). Next click on the “**Create Composite Image**”, select your video again and then click OK. Excel reruns the macro and creates a new composite picture with the corresponding frames removed from the beginning and end of your video. If you removed too many pucks or need to remove more, just enter new numbers into cells E9 and E10 and repeat the process. *It is OK if you have an extra puck image or two at the end, but make sure you have the correct number (2) at the start of the video before the one image of the projectile puck that may be either just before or after the collision.*
5. Once you are satisfied with your final composite image, consult with your TA to decide if your picture is good enough to analyze. If your picture is not good, take another video and repeat the above steps. It only takes a few seconds to make a video so you can be choosy.

IX. Photo-Analysis of the Composite Image

1. In this part you will use simple software tools to analyze your composite digital image. Right-click on the composite image in your Excel template and select **Copy**.
2. Open **PowerPoint** and click on **Paste** (in the top left on the menu bar). A copy of your composite image should now be visible in PowerPoint.

3. Right-click on the PowerPoint slide at a point that is not on the picture. Select **Grid and Guides** from the pop-up menu, then uncheck **Snap to Grid**, and in the **Grid Settings** set the **Spacing** to 1/24" or 0.042" then click OK. These settings allow precise measurements.
4. You can expand the size of your picture if you want, but you must make sure that you do so by exactly the same amount in the x and y directions. To check whether your picture has been stretched in PowerPoint, right-click on the picture, then select **Size and Position** from the pop-up menu. Under **Scale**, check if percentages listed in the **Height** and **Width** boxes are different (if you see a number that is not a percentage you are looking at the wrong box - go to the box under **Scale**). If the percentages are different, just type the same number into both boxes to correct things. It is then a good idea to check-off the box for **Lock Aspect Ratio** and PowerPoint will then make sure that any more stretching you do in x will be the same as in y.
5. According to the law of conservation of momentum, when one puck collides with a puck that is initially not moving, you should find

$$m_1\vec{v}_1 = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}. \quad (8)$$

Of course, this assumes that the second puck was initially not moving. If you knew the velocities of the pucks, you could verify this equation by substitution. However, from the composite picture, what you can measure are the displacement vectors d corresponding to how far each puck moved between successive video frames. Since each puck moves with a constant velocity before and after the collision, each puck's displacement vectors d will be proportional to the time t between successive video frames. Multiplying equation (8) by the time interval t gives:

$$m_1\vec{d}_1 = m_1\vec{d}_{1f} + m_2\vec{d}_{2f}. \quad (9)$$

Where:

- $\vec{d}_1 = \vec{v}_1 t$ is the displacement of puck 1 between two successive images before the collision,
- $\vec{d}_{1f} = \vec{v}_{1f} t$ is the displacement of puck 1 between two successive images after the collision,
- $\vec{d}_{2f} = \vec{v}_{2f} t$ is the displacement of puck 2 between two successive images after the collision.

To test if momentum is conserved, you will find these three displacement vectors and check if Equation (9) is obeyed.

6. To find the displacement vectors, start by using PowerPoint to carefully draw a circle around the outer edge of one of the pucks (see Figure 9). For best results, we recommend:
 - Use an unfilled circle (right-click on the circle, select **Format shape**, then select **no fill**)

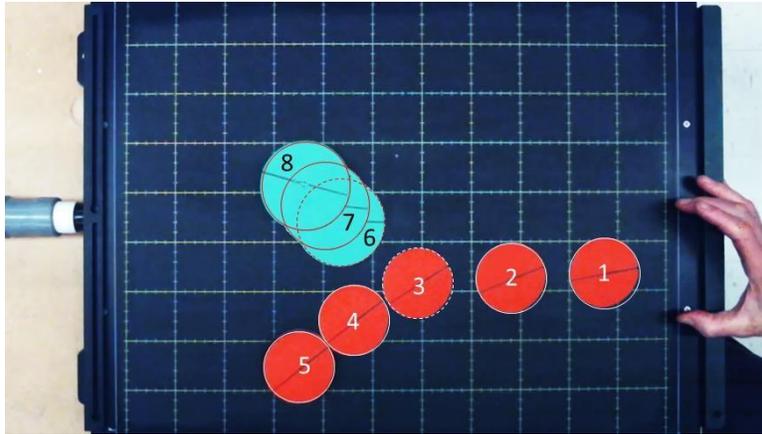


Figure 9. Composite picture with pucks circled and labelled using PowerPoint.

- Use a white line color to stand out against the dark background (right-click on the circle, select **Format shape**, then select **Line Style**, and choose white from the color pallet)
- Use a fine linewidth so that you can line up the circle with the puck edge (right-click on the circle, select **Format shape**, select **Line Style**, and set the **Width** to 0.5 pt)

After you have one circle drawn that fits exactly over the puck image, use **Copy** and **Paste** to make circles that are all exactly the same size for the rest of the images of that puck. If you make a mistake at any point, such as moving the image rather than a circle, just click on the **Undo** button on the top left in the Excel menu bar.

7. Repeat the previous step for the other puck - you will need to draw a circle with a different size since the two pucks are different sizes.
8. Check that each circle is precisely centered on the corresponding puck image. To make a fine adjustment up/down or left/right of a circle, left-click once on a circle to select it, then hold down the **Ctrl** button on your keyboard and press the corresponding **up/down left right arrows** on the keyboard. Also, use PowerPoints zoom control (lower left) to see details.
9. Add labels 1 to 8 your puck images so that your composite picture looks something like that shown in Figure 9.
10. In Figure 9, the circle labelled 3 shows puck 1 just before the collision, but you have to look very carefully to be sure. For this reason, you should not use your image of puck 1 at closest approach (labelled 3) in the following analysis. For a similar reason, you should not use the image of puck 2 at the closest approach (labelled 6) in your analysis.
11. You can now use PowerPoint to find the precise x and y location of each circle. To do this, **right click** on a circle, select **Size and Position** from the pop-up menu, and then select **Position** in the next window menu. You can now read off the **Horizontal** and **Vertical** position of the circle. To find the position of another circle, don't close this last window, but instead just click on another circle and its position will be given. Each time you click on a

circle, make sure the circle is highlighted (selected), and not the entire picture - otherwise you will get the location of the entire picture and not the circle!

Puck TABLE			
COLOR OF PUCK 1	GREEN		
MASS OF PUCK 1 (GRAMS)		124.1	
COLOR OF PUCK 2	RED		
MASS OF PUCK 2 (GRAMS)		167.2	

Table of Puck positions			
		Horizontal	Vertical
puck 1 start	circle 1	1.52	2.98
	circle 2	2	3
		d1(x-component)	d1(y-component)
			0.48

Table of Puck positions			
		Horizontal	Vertical
puck 1 final	circle 4	4.88	2.43
	circle 5	5.53	1.83
		d1f(x-component)	d1f(y-component)
			0.65

Table of Puck positions			
		Horizontal	Vertical
puck 2 final	circle 7	5.34	4.19
	circle 8	5.88	4.72
		d2f(x-component)	d2f(y-component)
			0.54

Figure 10. (a) Table in the Excel template with horizontal and vertical positions of Circles 1 and 2 entered. Cells change from beige (unfilled) to yellow (filled). (b) Table with positions of Circles 1, 2, 4, 5, 7 and 8 entered and x and y-components of vectors \vec{d}_1 , \vec{d}_{1f} and \vec{d}_{2f} calculated. Your numbers will be different.

12. Your Excel template contains a table (see Figure 10) in which you can record the Horizontal and Vertical position for the six circles labelled 1, 2, 4, 5, 7 and 8. Make sure you do not use the closest approach circles labelled 3 and 6. As you fill in the table, the cells change from beige to yellow.
13. Now fill in the rest of the table by using Excel to find the x and y components of the displacement vectors. To find the x-component of the vector \vec{d}_1 , for example, just use Excel to find the **Circle 2** Horizontal position minus the **Circle 1** Horizontal position. Similarly, to find the y-component of the vector \vec{d}_1 , just use Excel to find the **Circle 2** Vertical position minus the **Circle 1** Vertical position. Of course, your numbers will be different. Repeat this procedure and fill in the rest of the x and y components. When you are all done, your table should look something like in Figure 10(b).

14. Now that you have the x and y components of the displacement vectors \vec{d}_1 , \vec{d}_{1f} and \vec{d}_{2f} , use the designated area in the spreadsheet to check if the x-component of the vector equation:

$$m_1\vec{d}_1 = m_1\vec{d}_{1f} + m_2\vec{d}_{2f}. \quad (10)$$

is satisfied by your data (does the left side equal the right side?). Next check if the y-component of this equation is satisfied by your data. What does this result tell you about conservation of momentum in your collision?

15. Finally, if kinetic energy was conserved during the collision, then one would expect:

$$\frac{1}{2}m_1\vec{v}_1^2 = \frac{1}{2}m_1\vec{v}_{1f}^2 + \frac{1}{2}m_2\vec{v}_{2f}^2 \quad (11)$$

For this experiment this implies

$$m_1d_1^2 = m_1d_{1f}^2 + m_2d_{2f}^2, \quad (12)$$

Use your data to check if kinetic energy is conserved during the collision. Don't forget that d_1^2 is the square of the length of the vector \vec{d}_1 , which is equal to the square of its x-component plus the square of its y-component. Is Equation 12 satisfied? Discuss your result.

- 16.** To save a copy of your marked up image for your lab report, go into PowerPoint, use the mouse to select the photo and all of the circles and labels you made, then right-click on the selected image and click on **Save as Picture** and save to the **My Documents** folder. You can then save a copy of this file to a memory stick, or e-mail yourself a copy, and later paste this saved picture directly into an MS Word file for your lab report.

X. Final Questions

1. Suppose the pucks start spinning after the collision, whereas they were not before. Will this affect your momentum conservation results?
2. Momentum is conserved in either an elastic or inelastic collision, that is, whether or not kinetic energy is conserved. Why, then, is it important that the air table be frictionless?
3. Suppose a student finds that the total y-component of the outgoing momentum is consistently nonzero. Besides not correctly choosing the x-axis to be parallel to the direction of motion of the incoming puck, what else could be causing this discrepancy?

XI. Finishing Up

- Make sure you get your lab partner's name and his or her contact information.
- Save a copy of your PowerPoint file, spreadsheet and photo files on a memory stick or e-mail yourself a copy.
- In addition, don't forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus). Also, make sure to review the Guidelines for Writing a Lab Report, right after the main introduction in this Lab Manual.
- Finally, don't forget to prepare for the next lab and submit your answers to the prelab questions for the next lab before they are due.