Experiment 7

Centripetal Acceleration

I. Purpose

The purpose of this experiment is to observe centripetal acceleration in an object that is undergoing uniform circular motion.

II. Preparing for the Lab

You must prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Next, don't forget to answer the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. Pre-lab Questions *(You must submit answers to Expert TA before your section meets)*

Questions and multiple choice answers on Expert TA may vary from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

#1. This set of prelab questions are all about the experimental setup shown in the Lab Manual as Figure 1 below. Why are there 2 carriages in this experiment?

#2. If the fixed carriage and the sliding carriage are at a radius of 15 cm and you add a total mass of 300 g to the sliding carriage, what total mass should you have in the fixed carriage?

#3. If both carriages have a total mass of 200 g and the sliding carriage is at a radius of 20 cm, what radius should the fixed carriage be at?

#4. The direction of the carriage's velocity changes as it undergoes circular motion. Suppose there is also a change in magnitude of the velocity. Since the carriage is travelling in a circle, the net force on the carriage would still be "centripetal", i.e. towards center. True or false? Hint: try drawing a Free Body diagram.

#5. Due to friction in the axle, the rotating system slows down and eventually comes to rest. This means that the speed of the carriage is not constant! Why is it still justified to use the expression we have derived for centripetal force assuming uniform speed?

#6. Even though the carriage is slowing down, the tension force in the string attached to the carriage (which the force sensor measures) is still the centripetal force. True or False?

IV. References

For a review of uniform circular motion, see for example Chapter 6 in *College Physics a Strategic Approach* by Knight, Jones and Field.

V. Equipment

Vernier's Centrifugal Force Apparatus

computer — LoggerPro template

VI. Introduction

In uniform circular motion an object moves with constant (uniform) speed on a circular path. Although the speed v of an object remains constant while undergoing uniform circular motion, the velocity vector $\vec{v}$ continually changes direction (see Figure 1). Any change in the

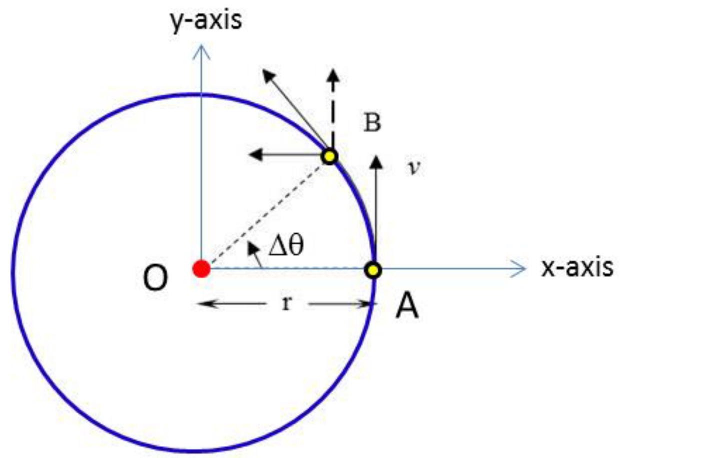

Figure 1. An object (yellow point) travels from point A to point B along the arc of a circle that is centered at point O (red point).

velocity of an object, either due to a change in speed or direction, means the object is accelerating. According to Newton's second law, an object will accelerate only if there is a non-zero total force acting on the object. For uniform circular motion, it turns out that this force must always be pointing towards the center of the circle that the object is travelling around and we say that there is a **centripetal force** present**.**

You can use Newton's second law and a little bit of geometry to understand why an object undergoing uniform circular motion must be experiencing a force that is pointing towards the center of the circle the object is travelling on. If the acceleration vector pointed above or below the plane of the circle, the velocity vector would also end up pointing out of the plane that the circle is on. Since the velocity vector always stays in the plane of the circle, it must be that the acceleration is also in the plane of the circle. Next recall that if a force acts along the direction of velocity it either increases or decreases the magnitude of velocity. Since the speed is constant in uniform circular motion, the force must be pointing perpendicular to the object's velocity vector. This only leaves two possible directions for the acceleration vector, either pointing towards the center of the circle or radially out from the circle's center.

To see which direction the acceleration is pointing, consider Figure 1 which shows an object of mass ***m***, moving along a circle of radius r centered about the red point O. The object starts at point A and travels with constant speed v. After a small time interval Δt it reaches point B covering an arc of length $\Delta s = v\Delta t$. The small angle covered in this time interval is:

$$\Delta\theta = \frac{\Delta s}{r} = \frac{v\Delta t}{r} \tag{1}$$

At point A, the velocity points in the positive y direction, as shown. At point B the magnitude of velocity is still the same but the direction has changed. By carefully examining the figure, you can see that the new y component of velocity at point B is $v\cos(\Delta\theta)$ and the new x component is - $v\sin(\Delta\theta)$. Note the minus sign. The change in velocity is thus

$$\Delta\vec{v} = \vec{v}_B - \vec{v}_A = -v(1-\cos(\Delta\theta))\hat{y} - v\sin(\Delta\theta)\hat{x} \tag{2}$$

For very small time intervals, the angle Δθ will be very small and you can write:

$$\cos(\Delta\theta) \approx 1 \tag{3}$$

and:

$$\sin(\Delta\theta) \approx \Delta\theta = \frac{v\Delta t}{r}. \tag{4}$$

Plugging this into Equation (2) gives:

$$\Delta\vec{v} = -v(\Delta\theta)\hat{x} = -\frac{v^2\Delta t}{r}\hat{x} \tag{5}$$

The acceleration is then the change in velocity divided by the change in time:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = -\frac{v^2}{r}\hat{x} \tag{6}$$

Using Newton's Law $\vec{F} = m\vec{a}$ we then find:

$$\vec{F} = -\frac{mv^2}{r}\hat{x} \tag{7}$$

Note that this is the force on the object when it is at point A (or an arbitrarily small time after it passes through point A) so the direction of the force is towards O i.e. towards the center of the circle. If we had chosen any other point on the circle to do this analysis, we would have found that the force was also always pointing to the center of the circle. It is important to note that this force has to be provided to the object. Otherwise the object would not change its direction and would continue to move along its initial direction. If you take the magnitude of the vectors on both side of Equation (7) one finds that the magnitude of the force on the object is:

$$F = \frac{mv^2}{r} \tag{8}$$

The rate at which the angle θ changes is called the **angular velocity ω**. Rearranging Equation (1) we can then write:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{r\Delta t} = \frac{v}{r} \tag{9}$$

or:

$$v = \omega r \tag{10}$$

Substituting Equation (10) into Equation (8) then gives another relationship for the magnitude of the centripetal force on an object undergoing circular motion

$$F = m r \omega^2 \tag{11}$$

Figure (2) shows the experimental setup. There are two carriages placed on a horizontal beam that can rotate about its center. Beneath the rotating beam is a photogate that measures the angular displacement of the beam as it rotates. One of the carriages is fixed on the beam and the other can slide on the beam. Each carriage can hold a mass and the two masses do not have to be the same. The sliding carriage is connected to the center of the rod by a string of negligible mass. The other end of the string is connected to a force sensor that measures the tension in the string. As you rotate the beam, the string becomes taut and it is this tension that provides the necessary centripetal force to the carriage.

With this arrangement you can measure the output from the force sensor (which gives the magnitude of the centripetal force F) and the "angular distance" the object moves as a function of time. From the radius r at which the mass is located and the angular distance versus time, you can obtain the speed of the object. Equation (8) says that a plot of the force versus the square of the speed will be a straight line. Similarly, Equation (11) says that a plot of the force F versus the square of the angular velocity ω will be a straight line.

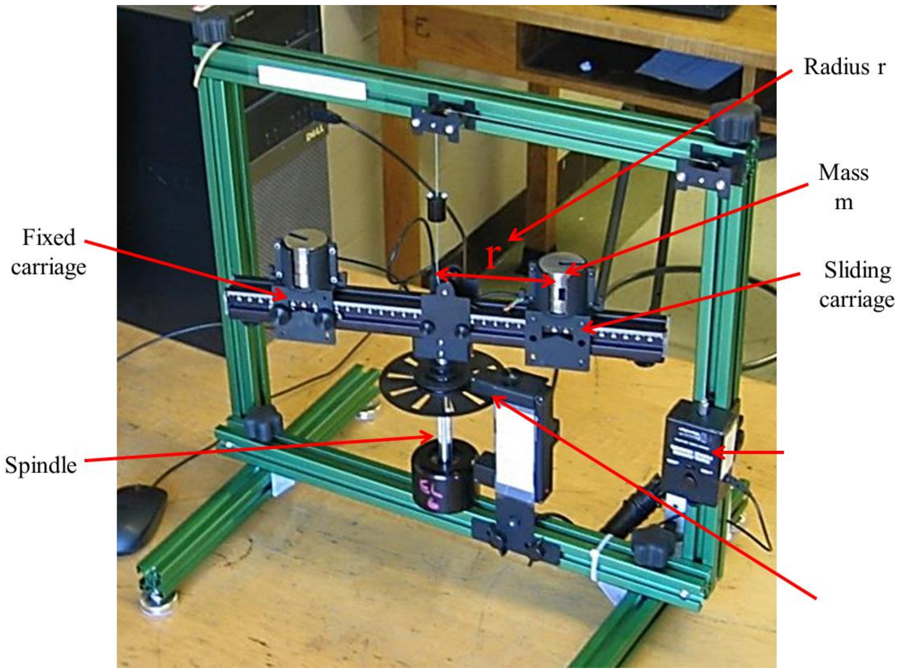

Figure 2. Photograph of Centripetal Force Apparatus (CFA) with some of the parts labelled.

VII. Experimental Procedure

Part A - Setting up the Apparatus

1. Check that the Dual-Range Force Sensor and Vernier Photogate are attached to the Centripetal Force Apparatus as shown in Figure 2.

2. To set up **automatic data collection**, find and open the file **Centripetal Acceleration.cmbl**.

3. Next verify that the photogate and the force sensor data are being correctly read by the software. To do this, place your hand in the photogate. The red LED on the top of the gate should illuminate and the software should display "Blocked" next to GateState above the data table, in the data collection area. If this does not happen, check all your connections. There is a small sliding switch on the lower side of the upper arm of the photogate. Try shifting its position so that the LED glows **only** when the photogate is blocked.

4. Go to "Experiment" → "Data Collection" or simply press Ctrl+D to go to the data collection set up. Verify (and modify if required) that under the collection Tab, the mode is set to "Time based" , "length" is set to 120 seconds and "Sampling rate" is set to 10 "samples/second". All other boxes should be left unchecked.

5. Data collection has been set up so that Logger Pro records the angular distance the carriage on the beam travels during its circular motion. You can examine the formula by double-clicking on the column header for **Angular Distance**. This **Angular Distance** is simply the total angle the cart has travelled in radians and the formula is:

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stepColumnBased("GateState", 0, 0.628, 1, 1)
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The factor of 0.628 comes in because 10 spokes go through the photogate in each full rotation. Therefore the Angular Distance moved from one spoke to the next is $2\pi/10 = 0.628$ radians.

7. Go to "File" → "Settings for Centripetal Acceleration". Check the number of points being used for calculating derivatives and smoothing functions. The number should be 97.

8. You can press the spacebar to stop collecting data.

Part B: Measure the centripetal force

1. Determine the mass of the empty sliding mass carriage. Add 100 g mass to both the sliding and fixed mass carriages as directed by your instructor. Record the total mass of the sliding carriage and additional mass.

2. Zero the force sensor. This can be done by using the "Set Zero" button on the Logger Pro software. This button is just next to the "Collect" button.

3. Position the fixed carriage so that its center is 10 cm from the axis of rotation. Adjust the position of the force sensor on the rail. Note the "Force" reading in the Logger Pro. Since the string stretches a little bit, adjust the length of the string so that it is exactly at 10 cm for the midway range of the force sensor. Typically we shall work in the 4-5 N force range. Therefore try to adjust the string length to 10 cm when the force reading is 2-2.5 N.

4. Spin the beam by twisting the knurled spindle of the CFA with your fingers. When the force reaches 4-5 N, begin collecting data. **Do not** exceed 5 N.

5. On the 1st page you can see a dynamic plotting of the force vs. velocity plot. Stop if you get spikes and redo. When you stop data-collection, use the friction between your hand and the knurled spindle to stop the beam.

Part C - Evaluation of data from Part B

1. Choose Next Page from the Page menu. Note that the vertical axis displays Force-interpolated; these are values that Logger Pro has interpolated from the values of force measured by the sensor.

2. Plot Force-Interpolated versus ω^2. According to Equation 11, this plot should be a straight line. From this Equation what can you say about the physical meaning of the slope of this straight line?

3. Use "Linear Fit" to find the equation of the line that best fits your linearized graph. Save your Logger *Pro* file.

4. Store the 1st run by going to "Experiment" → "Store Latest Run"

5. In case you get spikes in your data, retake the data set.

Part D - Investigating the effect of mass

1. Recall that Equation (11) says that the magnitude F of the centripetal force can be written as:
$$F = m r \omega^2 \tag{11}$$
From this result you should expect that doubling the mass will double the centripetal force, all other things being equal. In this part you will test this prediction by varying the mass.

2. Change the mass on both the fixed and sliding carriages by putting an additional 100 g and record the value of the total mass of the sliding carriage and any additional masses. Return to Page 1 of your experiment file.

3. Re-zero the force sensor, and then spin the beam as you did before. Once the force reaches 3–5 N, begin collecting data. DO NOT exceed 5 N. As the string attached to the carriage would stretch appreciably and the radius would change. When you stop data collection, stop the beam as you did in Part B-4. It's important to Store all your mostly "spike-free" runs (at least a couple of blips are very likely).

4. Change the system mass again and record the value of the total mass of the sliding carriage and any additional masses, then repeat Step 2.

Part E - Investigating the effect of varying radius

1. Equation 8 implies that the magnitude F of the centripetal force will be cut in half if the radius is doubled, provided all other things are equal. In this part you will test this prediction by varying the radius of the object's motion.

2. Keep the mass on the carriage fixed at 200 g for the rest of the measurements.

3. Decrease the radius of both the sliding and fixed mass carriages by 2–3 cm. Record the value of the radius. It's very important to change the radius parameter in the "distance" formula in Logger Pro. Otherwise your slope would be erroneous.

4. Re-zero the force sensor, and then spin the beam as you did before. Once the force reaches 4–5 N, begin collecting data. When you stop data collection, stop the beam. Store this run.

5. Now, change the radius so that it is 2–3 cm greater than your initial value. Record the value of the radius, and then repeat Step 3.

V. Data Analysis for Parts C and D

1. Go to Page 2 of your Logger *Pro* file.

2. Plot Force-interpolated (F_i) *vs.* ω^2 and select the interpolated force for the three runs in

which mass was varied. On the graph you should see a family of 3 curves.

3. Perform linear fits on all three sets of data. Be sure that the parameter “radius” has the correct value. Record the value of the slope of each of the equations of the lines. What relationship appears to exist between the value of the slope and the total mass of the sliding carriage? Is it expected based on the theoretical formula? For all the different values of m, compare the observed and the theoretical slopes and calculate the percentage difference between them.

4. To analyze the effect of changing the radius, go to “Page” and add another page. Select More on the vertical axis of the F_i *vs.* ω^2 graph. De-select Force-interpolated for the runs you examined in Step 3. Now, select it for one of the runs in which you changed the radius.

5. Because the velocity was calculated using a different value of the radius, you must set this parameter to the radius used for each run you wish to examine. Perform a linear fit on the data for the desired run. Compare the value of the slope for this run to that for your first run ($r = 0.10$ m).

6. **Question**: In this lab you investigated the dependence of centripetal force on the magnitude of the angular velocity, mass of the object and the radius of the circular path. Were your experimental results consistent with the theoretical expression?

7. **Question**: Using the slopes you found from the plots of F_i *vs.* ω^2, and the masses used in the experiment, can you verify the radius used for each run?

IX. Finishing Up

- Make sure you get your lab partner’s name and his or her contact information.

- Save a copy of your spreadsheet or other data files on a memory stick or e-mail yourself a copy.

- In addition, don’t forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus). Also, make sure to review the Guidelines for Writing a Lab Report, right after the main introduction in this Lab Manual.

- Finally, don’t forget to prepare for the next lab and submit your answers to the prelab questions for the next lab before they are due.