Experiment 6

Equilibrium of Forces

I. Purpose

The purpose of this experiment is to observe an object in static equilibrium, find the magnitude and direction of each force acting on the object and show that these forces add like **vectors**.

II. Preparing for the Lab

You need to prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Finally, don't forget to do the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. Pre-lab Questions *(You must submit answers to Expert TA before your section meets)*

Questions and multiple choice answers on Expert TA may vary from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

#1. In this lab, the use of a bold symbol such as $\mathbf{F}_1$ means that this quantity is a vector. True or false?

#2. Although a vector has both magnitude and direction, like everything else in physics it can be quantified by a single number. True or False?

#3. Three forces are pulling on the same object such that the system is in equilibrium. Their magnitudes are $F_1 = 2.83$ N, $F_2 = 3.35$ N, and $F_3 = 3.64$ N, and they make angles of $\theta_1 = 45.0^o$, $\theta_2 = -63.43^o$ and $\theta_3 = 164.05^o$ with respect to the x-axis, respectively.

(a) What is the x-component of the force vector $\mathbf{F}_1$? Give answer in Newtons N to three significant figures.

(b) What is the x-component of the force vector $\mathbf{F}_2$? Give answer in Newtons N to three significant figures.

(c) What is the x-component of the force vector $\mathbf{F}_3$? Give answer in Newtons N to three significant figures.

(d) What is the x-component of the force formed from the resulting sum of $\overline{F}_1$, $\overline{F}_2$ and $\overline{F}_3$? Hint: the system is in static equilibrium.

#4. Three forces are pulling on the same object such that the system is in equilibrium. Their magnitudes are $F_1 = 1.00$ N, $F_2 = 1.50$ N, and $F_3 = 2.18$ N, and they make angles of $\theta_1 = 30.00^o$, $\theta_2 = -30.00^o$ and $\theta_3 = 173.41^o$ with respect to the x-axis, respectively.

(a) What is the y-component of the force vector $\overline{F}_1$?

(b) What is the y-component of the force vector $\overline{F}_2$?

(c) What is the y-component of the force vector $\overline{F}_3$?

(d) What is the y-component of the resulting force formed by the sum of $\overline{F}_1$, $\overline{F}_2$ and $\overline{F}_3$?

IV. References

For a review of forces, see for example Chapter 4 from *College Physics a Strategic Approach* by Knight, Jones and Field.

V. Equipment

Force table	4 pulleys	4 strings
4 weight hangers	level	protractor
weights	ruler	mirror

VI. Introduction

According to Newton's first law, if a body has zero acceleration (*i.e.* its velocity neither changes magnitude nor direction), then the total force acting on the body must vanish. For example, if there are just three forces $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ that are acting on an object that is not accelerating, then it must be that:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0. \quad (1)$$

Note that we have used bold symbols $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ for the forces to indicate that these are **vectors**. In other words, each force cannot be quantified by just a number but rather each force has both a magnitude and a direction, *i.e.* it is a vector. Thus, Equation 1 says that the **vector sum** of these forces must be zero.

Static **equilibrium** is a special case of no acceleration. In static equilibrium, an object not only has no acceleration but it remains at rest. It follows that Equation 1 is also a requirement that an object must obey if it is in for equilibrium.

The fact that forces are vectors means that you need to keep track of both the length and the angle the vector makes with a chosen direction. When you have several forces acting on an object the easiest way to add the vectors mathematically is to set up a Cartesian coordinate system by specifying directions x, y, and z. The force vector $\mathbf{F}_1$ for example can then be described by giving its component in the x, y, and z direction. We denote these components by the symbols F_{1x}, F_{1y}, and F_{1z}, respectively and the vector force $\mathbf{F}_1$ can be written as simply:

$$\mathbf{F}_1 = (F_{1x}, F_{1y}, F_{1z}) \quad (2)$$

Similarly:

$$\mathbf{F}_2 = (F_{2x}, F_{2y}, F_{2z}) \quad (3)$$

$$\mathbf{F}_3 = (F_{3x}, F_{3y}, F_{3z}) \quad (4)$$

With this notation, the vector Equation 1 is equivalent to the three component equations

$$F_{1x} + F_{2x} + F_{3x} = 0 \quad (5)$$

$$F_{1y} + F_{2y} + F_{3y} = 0 \quad (6)$$

$$F_{1z} + F_{2z} + F_{3z} = 0 \quad (7)$$

Equation (5) means that for an object in equilibrium the sum of the x components of all the forces acting on the object must vanish. Similarly, Equations (6) says that the sum of the y components must also vanish and Equations (7) says that the sum of the z components must also vanish.

In this experiment you will see that forces not only have magnitude and direction, but also that they do indeed **add** like vectors. In order to do this you will use a **force table** (see Figure 1). The force table has a collection strings, pulleys and weights hanging on weight holders. The strings are attached

Figure 1. Photograph of the force table, weights and level.

to a central ring, which we will call R. The ring R can be put into static equilibrium (at rest) at the center of the table by adding just the right amount of weight to each weight holder so that the ring is not touching the center post (no additional force from the post) and not moving. You can then measure the magnitude and direction of each force acting on the ring, and see if the vector sum of all the forces is zero.

One potentially confusing thing about this apparatus is that you will only be checking two components for each force. Let's choose the x, y, z coordinate system so that the z axis is vertical. The ring R has a small mass, and consequently experiences a small force F_{gz} in the negative z direction (downward) due to gravity. This force is balanced by a slight upward force provided by the strings. The strings are not quite horizontal as they lie above the force table, and consequently the forces they apply to the ring have small vertical components. Altogether, the downward force of gravity and the upward forces of the strings act in such a way that in static equilibrium

$$F_{1z} + F_{2z} + F_{3z} + F_{gz} = 0 \qquad (8)$$

If the mass of the ring is small enough, the strings will be almost perfectly horizontal as they lie above the force table and we can neglect this small z-component of the force from each string. In any case since there is no component of gravity in the x or y-direction, for static equilibrium we will still have:

$$F_{1x} + F_{2x} + F_{3x} = 0 \qquad (9)$$

$$F_{1y} + F_{2y} + F_{3y} = 0 \qquad (10)$$

It is these two relations that you will check in this experiment.

VII. Experiment

1. Place the level on top of the force table and then adjust the feet on the force table until the table is level side-to-side and front-to-back.

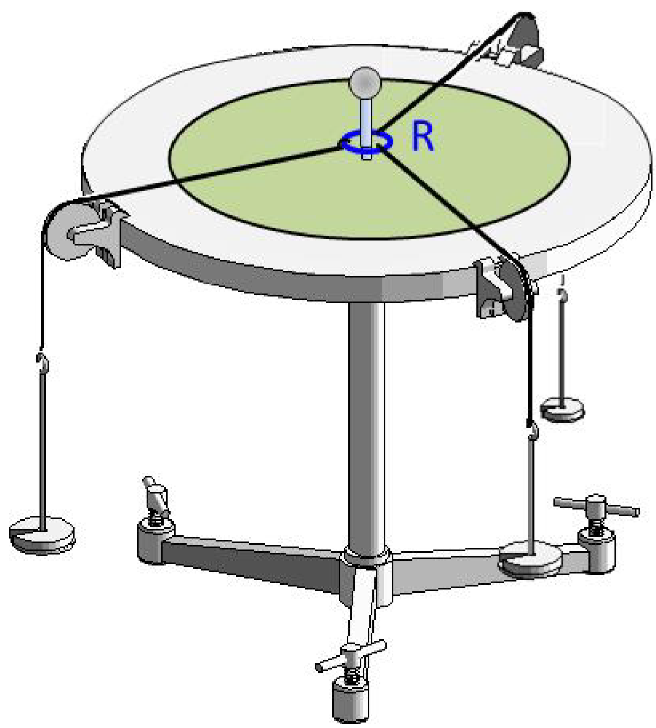

Figure 2. Force Table with three weights and pulleys pulling on the center ring.

2. Arrange three pulleys, with their associated strings and weight holders and weights, at arbitrary angles on the force table. **Hold the pulley securely when its clamp is loosened. This keeps it from falling and possibly breaking**.

3. Place enough weights on each of the three weight holders so that the ring *R* is centered on, but not touching, the post at the center of the force table. Make sure that all three strings are positioned on the ring in such a way that they point directly at the center post. Tap on the force table to determine whether you have found the desired equilibrium condition. If the ring displaced from the center when you tapped, your result was affected by pulley friction. Continue to adjust weights by small amounts until the ring stays centered on the post when you gently tap.

4. Record the total mass hanging on each string. Be sure to add the mass of each holder to the mass on each holder. Also record the corresponding angle of each string. Use the mirror to eliminate parallax in the measurement of angles. Ask the TA how to do this if you cannot figure it out.

5. For one of the strings determine δm, the largest change in the mass m hanging on the string that you can make without appreciably moving the ring from its position centered on the post. Do this by alternately adding and removing small weights from the holder. The quantity δm gives an estimate for the uncertainty in your determination of the mass m required to achieve equilibrium. Each mass m produces a force of magnitude $F = mg$ on the ring where g =9.801 m/s^2. Consequently, the uncertainty δm in m leads to an uncertainty δF in F. The quantity $\delta F/F$ is a measure of the relative error associated with the determination of F. To estimate this error, should you use the string with the smallest or largest mass? Decide which string you want to use, and record m and δm for this string.

6. Check to see whether having the weights swinging a small amount influences the determination of the masses required for equilibrium.

VIII. Data Analysis

1. For your measurements with three strings, calculate the magnitude of each of the horizontal forces acting on the ring in units of Newtons.

2. Make a drawing showing the forces acting on the ring. Select *x* and *y* coordinates axes with the center post at the origin. Draw the three force vectors as arrows. The lengths of the arrows should be proportional to the magnitudes of the forces, and the directions of the arrows should be along the strings. (Recall that a string under tension exerts a force that lies along the direction of the string.) Indicate the scale used to relate the magnitude of the force to the length of the vector. Label each force with the angle it makes with the positive *x*-axis and the magnitude of the force. Your picture should look something like the one shown below, but with angles and magnitudes labeled.

3. You can find the total force acting on the ring by adding the force vectors **graphically**. In Figure 3(a) we show three force vectors drawn with their tails at the origin, and is **not** what you should do to add them graphically. Instead, start by drawing one of the force vectors with its tail at the origin of the coordinate system (see $\mathbf{F}_1$ in Figure 3(b)). The next vector is drawn with its tail at the head of the first one (see $\mathbf{F}_2$ in Figure 3(b)). The next vector is drawn with its tail at the head of the second one (see $\mathbf{F}_3$ in Figure 3(b)) until you have added all of the forces. Be sure to label each force in your diagram. After you have added all the forces, draw a vector from the origin to the tip of the last vector. Call this vector the "error vector". From the equations describing the condition for equilibrium,

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0 . \tag{11}$$

this vector should actually be zero. That is, your 3 vectors should add to zero. In the example shown in Figure 3(b) the sum of vectors is located at the head of $\mathbf{F}_3$ and this is at the origin. For your actual data, you should not expect the forces to add exactly to zero because of the experimental errors involved even if you do the graphical addition correctly! Determine, as accurately as you can, the magnitude of the error vector and the direction it makes with the x-axis.

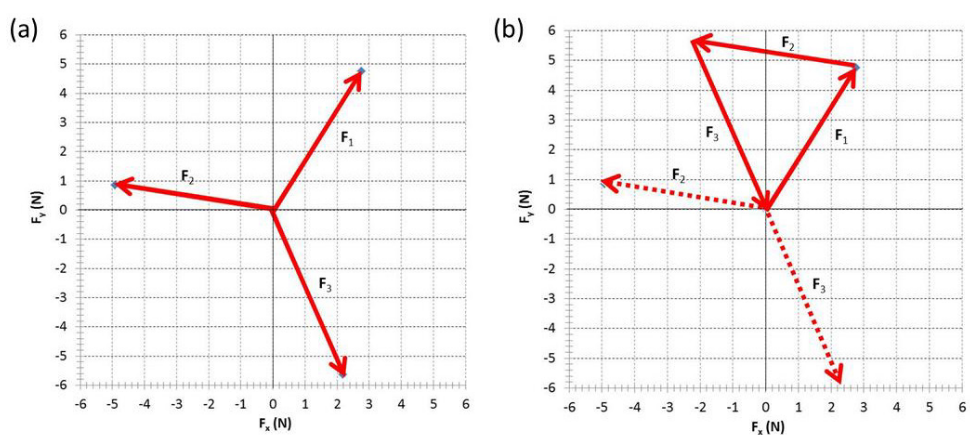

Figure 3. (a) Diagram showing three force vectors. (b) Graphical addition of three forces.

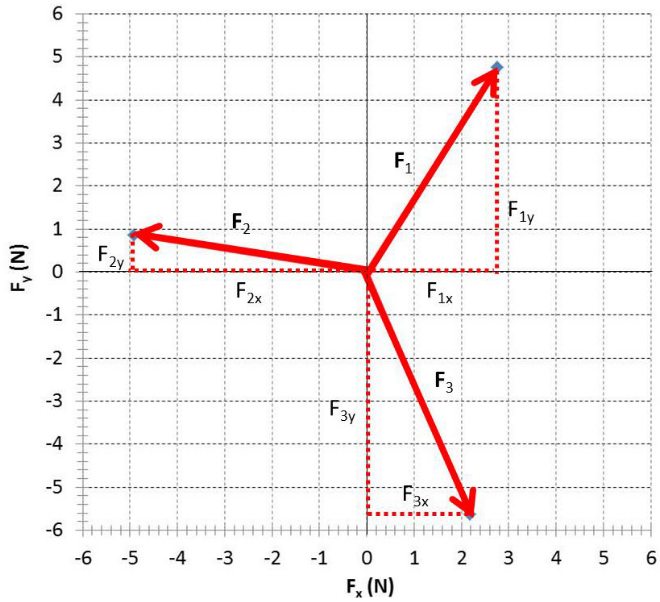

Figure 4. Diagram showing how to find the x and y components of each force vector. Note that in this example F_{2x} and F_{3y} are negative while all of the other components are positive.

4. Next add the three force vectors using **algebraic** addition of their components. Algebraic addition requires that you find th*e x* and *y* components of the forces. Figure 4 shows how to do this using a force vector diagram with each force vector drawn with its tail at the origin. Make sure you understand and keep track of whether a component is negative (lies along the negative x or y-axis). After you find the x-and y component of each force find the *x* component of the error vector by calculating

$$F_{1x} + F_{2x} + F_{3x} \tag{12}$$

Next find the *y* component of the error vector by calculating

$$F_{1y} + F_{2y} + F_{3y}\,. \tag{13}$$

Using these components find the magnitude of the error vector, the direction it makes with the *x*-and then check whether:

$$F_{1x} + F_{2x} + F_{3x} = 0 \tag{14}$$

$$F_{1y} + F_{2y} + F_{3y} = 0 \tag{15}$$

5. Compare the results you obtained using algebraic addition of the components with those obtained graphically. They should agree within the practical inaccuracy associated with graphical addition. If they do not, you have made a mistake in graphical or algebraic vector addition, or both! Try again.

6. Estimate the errors in your measurements of angles and in the forces acting on the ring. For the case of angles, estimate how accurately you were able to measure them. For the case of forces, estimate δF from the δm you measured in step 5 in the Experiment. Estimate the size of the error vector that could result from these errors. This can be done, for example, by repeating your graphical addition using vectors differing slightly from your measured ones by the amount of the estimated errors in both magnitude as well as direction. Alternatively, you could repeat your algebraic vector addition

using the slightly different vectors. Is your error vector consistent with your expected experimental errors?

IX. Finishing Up

- Make sure you get your lab partner's name and his or her contact information.

- Make sure you save a copy of your spreadsheets on a memory stick or e-mail yourself a copy.

- In addition, don't forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus). Also, make sure to review the Guidelines for Writing a Lab Report, right after the main introduction in this Lab Manual.

- Finally, don't forget to prepare for the next lab and submit your answers to the prelab questions for the next lab before they are due.