Experiment 5

Projectile Motion in Two Dimensions

I. <u>Purpose</u>

The purpose of this experiment is to observe and analyze projectile motion in two-dimensions.

II. Preparing for the Lab

You must prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Finally, don't forget to answer the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. <u>Pre-lab Questions (You must submit answers to Expert TA before your section meets)</u>

The Questions on Expert TA may vary somewhat from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

- 1. Figure 1 shows the motion of a puck on a tilted blue coordinate system, which is slightly rotated and displaced with respect to the un-tilted black coordinate system. Both coordinate systems have a 5 cm by 5 cm grid. According to the black coordinate system, what is the x-coordinate of the origin (blue point) of the blue coordinate system?
- 2. In Figure 1, a puck starts at the red point labeled 1 and follows the trajectory shown by the blue curve connecting points 1 to 15. Using the blue coordinate system, what is the x- coordinate of red point 1?
- **3.** In Figure 1, what is the y-coordinate of red point 1? Use the blue coordinate system?



- 4. In Figure 1, at red point 1 is the x-component of the velocity of the puck positive or negative? Use the blue coordinate system.
- 5. In Figure 1, it takes 1/6 of a second for the puck to go from point 1 to point 2, and 1/6 of a second to go from point 2 to point 3, etc. What is the x-component of the velocity of the puck at red point 1? Use the blue coordinate system and don't forget Question 4.

IV. <u>References</u>

For a review of projectile motion in two dimensions, see for example Chapter 3, section 3.6 in *"College Physics: A Strategic Approach"* by Knight, Jones and Field (Addison Wesley). To better understand what is going on in Part C of this experiment, we also recommend reading section 3.3 on *Coordinate Systems and Vector Components*.

V. <u>Equipment</u>

-	Air table with blower web camera, and camera mount	(2.54 cm tilt block)
-	Red air puck	
-	Computer with Excel	(Excel Template for Exp. 5)

VI. Introduction

Here we will treat a **projectile** as an object that is moving in two dimensions and falling freely, subject only to the force of gravity. Of course an object moving in air also experiences a drag force, but we will ignore that to simplify the discussion.

You will use an air-table to make sure that there is no friction and very low air drag. Tilting the table by angle θ will cause the puck to accelerate down the slope of the table (see Figure 2). We use a relatively small tilt angle θ to get a small acceleration. This lets us slow things down so that it is easy to take a good video of the resulting motion. To analyze the motion, you will use a coordinate system with the +y direction in the plane of the table and pointing up the slope (see Figure 2). The x-direction will be in the plane of the table and pointing horizontally (see Figure 2). With this choice of coordinate system the x-component of the acceleration is zero and the y- component of acceleration is:

$$a_o = g \sin(\theta)$$

where $g = -9.80 \text{ m/s}^2 = -980 \text{ cm/s}^2$ is the y-component of the acceleration due to gravity in free-fall. For example, for a tilt angle of $\theta = 3.0^\circ$ the puck will accelerate down the slope of the table with acceleration.

$$a = g \sin(\theta) = (-9.8 \ m \ / \ s^2) * \sin(3.0^\circ) = -0.51 \ m \ / \ s^2 = -51 \ cm \ / \ s^2$$
[2]



[1]

Figure 2. Diagram showing a red puck moving on a tilted air table. The initial position of the puck (labeled 1) is at position (xo,yo). The table is tilted by angle θ . The x-axis is horizontal and the y-axis is pointing up the slope of the table. The puck starts with initial velocity (vox, voy) at point 1 and accelerates in the negative y-direction during the motion.

This tilt angle and acceleration is similar to what is used in the apparatus.

With this choice of coordinate system, the object has no acceleration in the x-direction, so its x-component of velocity will be constant in time. We can thus write:

 $v_x = v_{xo} \tag{3}$

where v_x is the x-component of the object's velocity at time t and v_{xo} is the x-component of the object's velocity at t=0. Equation [3] says that the x-component of the velocity is constant.

Since the object has a constant x-component of velocity, its x-coordinate of position will obey

$$x = x_o + v_{xo}t \tag{4}$$

where x is the x-coordinate of the object's position at time t and x_0 is the x-coordinate of the position at time t=0. This equation just says that the x-coordinate increase linearly in time.

In the y-direction, the object undergoes a constant acceleration a₀. The **y-component of the object's velocity** will then obey:

$$v_v = v_{vo} + a_o t \tag{5}$$

where v_y is the y-component of the object's velocity at time t, while v_{yo} is the **y-component of the object's velocity** at t=0 and a_0 is the y-component of the object's acceleration. The y-coordinate of the object's position then obeys

$$y = = y_0 + v t_{y_0} + \frac{1}{2} a t_0^2$$
[6]

where y is the y-coordinate of the object's position at time t, and y_0 is the y-coordinate of the object's position at time t=0.

In this experiment you will be observing the path of a projectile. This path is called the **trajectory** and it is a curve that shows how y depends on x. Notice that Equation [4] gives x versus time t and Equation [6] gives y versus time t. So neither of these functions is the trajectory y versus x. However, with a little bit of algebra you can get an equation for the trajectory. Start by rearranging Equation [4] to get:

$$= \underbrace{(x - x_0)}_{v_{xo}}$$
[7]

Plugging this into Equation [4], multiplying things out and rearranging then gives:

t

$$y = \frac{a_o}{2v_{xo}^2} x^2 + \left(\frac{v_{yo}}{v_{xo}} - \frac{x_o a_o}{v_{xo}^2}\right) x + \left(y_o - \frac{v_{yo} x_o}{v_{xo}} + \frac{a_o x_o^2}{2v_{xo}^2}\right)$$
[8]

Notice that Equation [8] is of the form $y = Ax^2 + Bx + C$, where A, B and C are constants. This

is the equation of a **parabola** and thus the trajectory of a projectile should look like a parabola.

The blue curve in Figure 3 shows an example of a **parabolic trajectory.** The red points show the calculate location of a projectile at specific times, starting at time t=0 with the point labeled 1. Point 2 is the projectile's location at time t = 1/6 second, point 3 is the projectile's location at 2/6 of a second, etc. This 1/6 second time interval between points is the same interval used by the video-processing software in the experiment. Notice that point 1 is located at x = 50 cm and y = 5 cm, while point 2 is at about x = 47 cm and about y = 13.5 cm. Since the object changed its x-coordinate by about -3 cm in 1/6 of a second, its x-component of velocity is approximately $v_{0x} = -3$ cm/(1/6 s) = -18 cm/s. Our estimate of the position of point 2 is a bit rough.



Figure 3. Blue curve shows plot of trajectory curve, y versus x, of a projectile that is undergoing constant acceleration in the negative y-direction. The red points show the location of an object at successive times, starting at the point labeled 1. The blue curve is a parabola. The blue grid is 5 cm x 5 cm, the same as used in the experiment.

If you examine a few more points along the curve, you can see that the x-component of velocity is constant and you can get a better estimate of vox = -20 cm/s. Since the object changed its y-coordinate by about +8.5 cm in 1/6 second, the y-component of velocity at t=0 is voy \approx 8.5 cm/(1/6 s) = 51 cm/s. However, this estimate ignores the fact that the object is accelerating in the -y direction and is thus slowing down as it goes from point 1 to point 2. The actual value is voy =55 cm/s.

Overview of the Experiment

Figure 4 shows a photo of the set-up, which has a tilted air-table, a red puck and a web camera. The air-table allows the puck to float just above the table, eliminating friction and leaving just a little bit of air drag. Since the air-track is tilted, the puck will accelerate down the slope of the table and behave as a projectile moving on the 2-D surface of the air-table. The tilt of the air-table is small so the puck will have an acceleration ao that is a small fraction of the acceleration g = -980 cm/s² due to gravity. With a small acceleration, things can proceed slowly enough that the cheap web cameras we are using can follow the resulting motion.

In the experiment you will first practice a few "dry runs" and then use the web camera to take a

video of a puck moving on the tilted air table. You will then use a custom Excel template to process the video and extract a picture that shows the location of the puck at a series of equally spaced times. Connecting these points gives the puck's trajectory. Next you will find accurate values for the trajectory's parameters xo, yo, vox, voy and ao by manually fitting a parabola to the puck's trajectory. Finally, using a rearranged version of Equation [1].

 $g = a_o / \sin(\theta)$

[9]

and your value for the acceleration a_0 , you will determine a value for g, the acceleration due to gravity. Of course you will also need to know the tilt angle θ of the table.



Figure 4: Photograph of tilted air table, red puck and web camera. The 5 cm x 5 cm grid is visible on the table surface.

VII. Collecting 2D Projectile Data

Part A - Practice Launching the Puck and then Take a Video

- 1. For best results, the TA should have turned on the lights in the room and closed the window blinds. This arrangement produces lighting for the video that is compatible with the software you will be using to process the images.
- 2. Go to the **Templates** folder on the computer desktop and open the Excel template named:

Phys 121 Experiment 5 Projectile

- 3. Enter your name, your lab partners name and your section number into the cells B1, F1 and B3. Save a copy of your template in the **My Documents** folder. Be sure to include your name and the name of your lab partner in the file name. We recommend saving often so that you do not lose data.
- **4.** If you find a red puck and a green puck at your setup, use the **red puck** for taking data. It will be easier to locate the center of the red puck as it moves because it has a smaller diameter.
- 5. The air table should have been tilted by the lab technicians and it should not need any adjustments.
- **6.** Turn on the blower, set the power level to about 50% (just enough that the puck moves freely).

- 7. If you are working with a lab partner, decide who will handle the puck (person#1). The other person (person #2) will start and stop the video. If you do not have a lab partner, you can ask the TA for assistance. You can also do it solo, both launching the puck and starting/stopping the video, although this takes more practice than if you are working with a partner.
- 8. Look at the spring-and-string bumpers at the edges of the air-table (see Figure 5(a)). You will use the x and y bumpers at the corner for launching the projectile puck with a good speed (not too fast or too slow). Gently press the projectile puck against the two bumpers at the bottom corner of the table and release it. Try to have the puck get almost to the top of the table, without actually bumping into the bumper at the top. To get a really nice looking trajectory you also need to launch it with a bit of velocity in the x-direction (horizontally), rather than just heading straight up and down the slope of the table. Do this by pulling the puck against both the x and y bumpers at the corner. Practice until you are confident that you can launch the projectile puck in the right direction with a good speed so that it ends up moving along a nice parabolic trajectory, something like in Figure 1, 2 or 3.
- **9.** On the computer desktop, double click the Camera icon to start up the webcam software. The window should open with a live display. The picture should be in-focus and centered on the air-table. If it is not, verify the focus is in auto mode (see Figure 5 (b)). Next click on the Take Video button (see Figure 6(a)). End the video by clicking on the **red stop recording button** (see Figure 6(b)). The program automatically saves a copy of each video in the Camera Roll folder.



Figure 5. (a) Puck pressed against one of the spring-and-string bumpers at the edge of the air table. You will want to launch the red puck from the corner, touching both x and y bumpers so that it picks up x and y components of initial velocity. (b) Start-up control panel for the camera.



Figure 6. Camera control panel showing (a) start recording button, and (b) stop recording button.

- **10.** Now that you and your partner know how to launch a puck and how to record a video, it is time to practice doing both at the proper time to create a short video that shows a puck undergoing projectile motion. Practice the following:
 - Person 1 places the puck on the corner of the table and pulls back against the bumpers
 - Person 2 clicks on the start recording button
 - Person 1 launches the puck
 - After the puck returns to the bottom of the table, Person 2 clicks the stop recording button on the camera's control panel.
- **11.** To view your last recorded video, go to the bottom of the camera control panel and click on the icon for your video. Press the play button on the bottom of the control panel to view your video. Consider the quality of the motion you just observed. Keep taking videos until you are satisfied with the result. Also examine if the grid of the air-table appears to be aligned with the edges of the picture frame. Let your TA know if it appears that you have a big rotation if it is bigger than 5° or 10°, you can try rotating the camera, and reshooting the video. If it is less than this, don't worry about it. You will correct this small rotation in Part C.
- 12. Once you have taken a video that looks good, have your TA verify that it looks OK.
- **13.** Record the name of your video file in the specific cell provided in your Excel template (to see the name of the video file, click on the three dots labeled "More" in the right hand corner of the panel, click Open Folder and locate your video file. Notice that you can only enter data or text into the cells that are highlighted in tan or pale yellow.

Part B - Create a Composite Picture

1. In your Excel template, click on the light gray macro button that says "Create Composite Image". A window will open that shows you the videos that the camera saved. Click on your video and then click on Ok. Excel then runs a macro that grabs your video and creates a composite or multiple-exposure picture that shows the puck's position at successive times (see Figure 7(a)). It typically takes about 10-20 seconds for the routine to generate a composite.



Figure 7. (a) Composite image of puck trajectory made using the macro in the Excel template. The puck was launched on the right, followed a nice parabolic trajectory, bounced off the bumper at the bottom of the table and made a second parabola. (b) Composite image from same video, but with 9 frames removed from beginning and 12 from the end so that only first parabola is visible and none of the puck images are touching the bumpers.

- 2. Your Excel template should now display a picture that looks like a multiple exposure photograph of the puck's motion. Take a careful look at your composite picture and compare with Figure 7(a). Of course the details in your picture will look different. However, you should be able to see your hand holding the red puck at the start and a nice parabolic path as it undergoes projectile motion.
- **3.** In the example shown in Figure 7(a), the red puck started against the lower bumper and also bounced off the bottom bumper after completing its first parabola. Of course if the red puck is touching the bumper, it is feeling a force from the bumper and we cannot use that part of the picture for analyzing the projectile motion. <u>The key point</u>: *You need to remove all of the images of the puck that occur before the puck is initially launched and all the images of the puck that occur after it completes the first parabola.* To do this, the Excel template has a simple tool for removing images from the beginning and end of the video. In the example shown in Figure 7(b), 9 puck images were removed from the beginning (during which the person was holding the puck at the beginning) and 12 images were removed from the end.
- 4. Figure out how many puck images you want to remove from the beginning and the end of your composite picture and enter these numbers into cells E9 and E10 (highlighted yellow) in the spreadsheet. Next click on the "Create Composite Image", and select your video again and then click OK. Excel reruns the macro and creates a new composite picture with the corresponding frames removed from the beginning and end of the video. If you remove too many pucks or need to remove more, just enter new numbers into cells E9 and E10 and repeat the process. *Make sure that you have removed all the pucks at the start and end of the parabola where the puck is touching the bumper*.
- **5.** Once you have removed all the extra puck images, consult with your TA to verify that your picture is good. If your picture is not good, take another video and repeat the above steps. It only takes a few seconds to make a video so you can be somewhat choosy.
- 6. If you, your lab partner and the TA are all satisfied, right-click on the image, copy it and

then paste a copy into the box in your spreadsheet that says "**place your cropped image here**".

7. Now Right-click on the copy of your composite image and then Click on the **image cropping tool** (see Figure 8(a)). Use the tool to crop away the top, bottom, left and right sides of the image so that only the complete surface of the air table is showing, enclosed by the grid and bumpers. Your cropped image should look something like Figure 8(b). You can also take this opportunity to sharpen the image or adjust the brightness and contrast.



Figure 8. (a) The image cropping tool. (b) This is the same image as in Figure 7, after cropping. The image was also sharpened and the brightness lowered and contrast increased.

- 8. Verify that your cropped image has 11 squares going from side to side, and 9 squares from top to bottom. If it has fewer than that, you cropped away too much and need to re-crop the image.
- 9. Save your spreadsheet.

Part C. Aligning the Table's Grid with the Rotated Grid

- 1. Before you can analyze the trajectory, you will need to add an accurate scaled analysis grid to the cropped image. This Part walks you through how to do that.
- 2. Examine your cropped image and look closely at the table's grid. You will probably notice that the grid appears to be slightly rotated with respect to the edges of the image. The example in Figure 7 has the grid rotated by about -0.4° (the negative sign means it was a counter-clockwise rotation). To see this, notice the bottom side of the grid is closer to the bottom edge of the picture on the right side of the picture. This misalignment happened because the camera was slightly rotated with respect to the grid on the table. The first step in the analysis is to deal with this small rotation. In principle you could correct this problem by rotating the camera a little bit and taking another video, but this is difficult to do accurately. Instead in this part, you will use Excel to create a rotated analysis grid that lines up with the grid on the cropped image.
- 3. Right-click on the cropped composite image that you made in Part B and select Copy.

4. In your spreadsheet template, find the Excel plot labeled "Projectile Motion". It has a starting parabola on it and a tilted blue grid. Right-click on the chart background for this plot, by right-clicking at an empty location in the middle of the chart (any point that does not have a grid line, data point or curve going through it). Select "Format Plot Area" from the pop-up menu, select "Fill" from the new menu that appears on the right, then select "Picture or Texture Fill" and finally select "Clipboard". You should now see that your cropped image has been inserted into the background of the plot, with a parabola and tilted blue grid on top. The result should look something like the example shown in Figure 9.



Figure 9. A cropped image has been inserted as background into the projectile plot. The tilted blue analysis grid is still rotated and displaced with respect to the air-table's white grid. The calculated trajectory (blue curve) still needs to be fit to the measured trajectory (red pucks).

- 5. Examine the plot. You should notice that the tilted blue analysis grid is not exactly on top of the air-tables 5cm x 5cm white grid. Find the origin of the white grid and the origin of the blue grid (they should be on the lower left corner of the image). Next to the plot, find the light- yellow cells labelled for x₀₀ and y₀₀ (don't confuse these with x₀ and y₀). The grid parameters x₀₀ and y₀₀ are the coordinates of the origin of the titled blue grid (according to the untiled plotting grid). Adjust the values in the x₀₀ and y₀₀ cells until the origin of the blue grid is exactly on top of the origin of the air-tables white grid.
- 6. After the origins are aligned, find the light-yellow cells labelled for θ_{00} (deg). This sets the angle that the blue grid is tilted by. Adjust the values in the θ_{00} cell until the blue grid is tilted by exactly the same amount as the air-tables white grid. Positive angles cause a counter- clockwise rotation of the blue grid and negative angles are clock-wise.
- 7. You should now have the origins of the blue and white grid aligned and the blue grid rotated by the same amount as the air-table's white grid. However, you will probably see there is still a problem the blue grid appears to be stretched slightly with respect to the white grid. The amount of stretching depends on how much you cropped the image. To fix the stretching, right-click on the chart background again, select "Format Plot Area". In the Fill part of the menu, adjust the Offset Right and Offset Top values until the blue grid is the same size as the white grid in your image. You can type in decimal numbers

(such as 1.3) to get finer control of the stretching than you can get using the sliders.

- 8. If it looks like the origins are not aligned, the rotation is off or the blue grid is bigger or smaller than the white grid, repeat steps 5-7 to get things lined up. You will probably also see some distortion (bending of the white grid lines), particularly near the edges of the image. The distortion is due to the camera's lens and depends on details such as how centered your image is. The distortion should be pretty small, and you can't correct it, so ignore it.
- 9. After everything is lined up, show your TA and Save your spreadsheet.

Part D. Fitting a Parabolic Curve to the trajectory in the Composite Image

- 1. Now that the blue analysis grid is lined up with the air-table's grid, you can analyze the trajectory.
- 2. The plot in your template already has a calculated trajectory (the blue curve with large white circles) that is essentially just a plot of Equation [8] at specific times. Notice that the white circles are labeled, 1, 2, etc. The white circle labeled 1 is where the **calculated trajectory** starts at t = 0. The circle labeled 2 is the location of the calculated trajectory at t = 1/6 s, etc. This is the same time spacing used to acquire the puck images. Of course the white circles are probably not on the red pucks because we didn't know where your starting point was going to be, or how fast you were going to launch the puck.
- 3. Next to the plot, find the light-yellow cells labeled x_0 and y_0 (do not confuse these with the coordinates x_{00} and y_{00} for the origin of the blue gird). The x_0 and y_0 parameters are the coordinates of the calculated trajectory at t = 0 (according to the tilted blue grid). Adjust the x_0 and y_0 values until the white circle labeled 1 is exactly on the first red puck in your image. You now have the starting position of the puck.
- 4. Next adjust v_{0x} (the x-component of the initial velocity of the puck) until the white circle labeled 2 is at the same x-coordinate as the second puck image. Of course circle 2 is the calculated location of the puck at t = 1/6 second. It should be easy to set v_{0x} because the puck images are spaced at regular intervals along the tilted blue x-axis. If v_{0x} is set correctly, each white circle will be at the same x-value as a red puck and there should not be any extra red pucks that lack a white circle. You might see an extra white circle at the end of the trajectory that corresponds to a puck image you deleted from your composite image.
- 5. Next adjust v_{oy} (the x-component of the initial velocity of the puck) until the white circle labeled 2 is at the same y-coordinate (on the blue gird) as the second puck image.
- 6. If you did everything correctly so far, the first and second white circles should be centered exactly on top of the first and second red puck images. If not, you need to go back and repeat steps 3-5.
- 7. Now comes the tricky part. To get the rest of the trajectory to fit, you need to make small simultaneous adjustments in a₀ and v₀ until each white circle sits exactly on top of a red puck. Of course there should not be any red pucks that lack a white circle or vice versa. If the calculated trajectory shoots up higher than your red puck went, try making

the acceleration more negative while increasing the velocity somewhat. If you have more white circles than red pucks, you probably have set the acceleration to low and/or the v_{0x} and v_{0y} too low.

- **8.** Continue adjusting x₀, y₀, v_{0x}, v_{0y} and a₀ until every white circle is exactly on top of its corresponding puck. If you get stuck, consult with your TA.
- 9. When you think you have got it right, show your TA and save your spreadsheet.

Part E. Determining the Acceleration Due to Gravity

1. According to Equation [9], the acceleration g due to gravity is related to the acceleration ao that the puck experiences by the expression:

 $g = a_o / \sin(\theta)$

[9]

[10]

You found ao in part D, so you just need to know the tilt of the air-table to find g.

2. The table was tilted using a block with a height of h = 1" = 2.54 cm and the distance from the high foot to the center-line between the two low feet is d = 15.5" = 39.37 cm. This gives a tilt angle of

$$\theta = \arctan(h / d)$$

In the Part E section of your spreadsheet fill in the values for d and h, and Excel will automatically calculate θ .

- 3. Check with your TA if the digital inclinometer is available. If it is, have your TA measure the tilt angle θ of your apparatus. It should be close to the value you calculated in the previous step. If you get a measurement, enter it into your spreadsheet in the cell for the tilt angle θ (don't confuse this with the blue grid tilt angle θ oo). Otherwise just use the value Excel calculated based on h and d values.
- 4. Notice in the Part E section of the template that Excel also finds your measured value for g using Equation [9] and your ao and θ values. The template also displays the accepted value gacc = -980 cm/s² and the fractional error in your g value, which is (g-gacc)/gacc.
- 5. Verify that your value for g is negative and close to the accepted value. If you did an OK job fitting the parabola to your data, the fractional error should be less than 0.1, which less than 10%. If the fractional error is something like 0.2, 1 or 2, which means that your g disagrees by 20 %, 100% or 200% from the accepted value, then you have made a major mistake and need to figure out what you did wrong and no it is not an "equipment problem".
- 6. Show your TA you final value for g and save your spreadsheet.

Part F. Second trajectory (Check if your instructor requires this part)

1. Only work on this part if your instructor requires it and you have already completed all of Parts A-E. If you have not finished Parts A-E yet, go back and do so now. You will not be able to "work on it later" because completing parts A-E requires custom software that is only available in the lab.

- 2. Save your spreadsheet and then open a **new spreadsheet** template so that you do not overwrite all the work you did for the first video. Make sure you name your new spreadsheet with your names and indicate it is video 2.
- 3. Repeat parts A-E, but this time make the following changes:
 - (a) Switch person $1 \leftrightarrow$ person 2, so that the other person launches the projectile, etc.
 - (b) Launch the projectile from the bumper on the side of the table near the top, rather than from the bottom corner. The resulting trajectory should have the puck starting out by moving horizontally and falling towards the bottom of the track.
- 4. Save your second spreadsheet.

VIII. Final Question

1. State the value you obtained for g in Part E and compare to the accepted result g = -980 cm/s². By what percentage did your result differ from the accepted value? If you completed Part F and thus now have two values for g, compare your two values to each other (state each value and the percentage they differ from each other).

IX. Finishing Up

- Make sure you get your lab partner's name and his or her contact information.
- Save a copy of your spreadsheets and photo files on a memory stick or e-mail yourself copies.
- Don't forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus).
- Finally, don't forget to prepare for the next lab and answer the prelab questions before they are due.