Experiment 4

Motion with Constant Acceleration

I. Purpose

The purpose of this experiment is to study uniformly accelerated motion as caused by gravity.

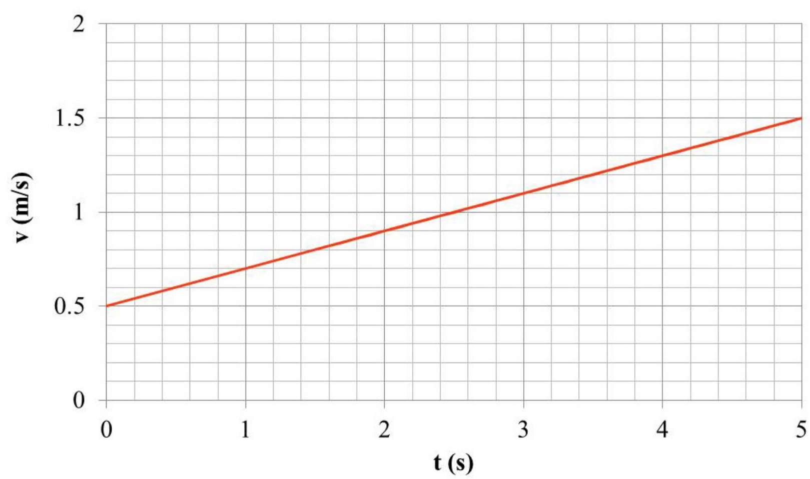
II. Preparing for the Lab

You need to prepare before going to the lab and trying to do this experiment. Start by reading through this lab write-up before you get to the lab. Finally, don't forget to do the Pre-Lab questions and turn them in to Expert TA before your lab starts.

III. Pre-lab Questions *(You must submit answers to Expert TA before your section meets)*

Questions and multiple choice answers on Expert TA may vary from those given below. Be sure to read questions and choices carefully before submitting your answers on Expert TA.

#1. The plot below shows the velocity v versus time t for an object that is moving.



(a) This object has constant acceleration during the time shown. True or False?
(b) What is the velocity of the object at time t = 1.0 second? Give answer in units of m/s and give it two significant figures.
(c) What is the acceleration of the object at t = 2 seconds? Give answer in m/s^2 to two significant figures.
(d) Suppose this object is at position x = 1.0 m at time t=0 seconds, at what position x will it be at time t = 2 seconds? Give answer in m to two significant figures.

#2. The position x of an object that is undergoing constant acceleration obeys:

$$x = v_0 t + \tfrac{1}{2} a_o t^2$$

Suppose v_0 = 4.3 m/s and a_o = -9.8 m/s^2.

(a) What is the position of the object at t = 2 s? Give answer in units of meters to two significant figures. Hint: the easiest ways to answer the following questions is to set up a spreadsheet to calculate the answers.

(b) What is the velocity of the object at t = 0.0 s? Give answer in units of m/s to two significant figures.

(c) What is the velocity of the object at t = 0.45 s? Give answer in units of m/s to two significant figures.

#3. When a ball is thrown straight up into the air, the acceleration of the ball is zero at the moment it reaches its maximum height. True or false?

#4. When a ball is thrown straight up into the air, the velocity of the ball is zero at the moment it reaches its maximum height. True or false?

#5. When you throw a ball straight up into the air, after it leaves your hand the acceleration is negative when the ball is going up and positive when it is falling down. True or false?

#6. Ignoring air resistance, when you throw a ball straight up into the air, after it leaves your hand the acceleration of the ball is always constant and downward. True or false?

#7. Last week in Experiment #3, you were told to record the unique identifier number on the track you used, the unique identifier number on the cart used and the best estimate for the velocity v_o that the launcher produced for that setup. (a) What is the unique identifier number on the track you used? (b) What is the unique identifier number on the cart you used? (c) What is the best estimate for the velocity v_o that the launcher produced for that setup?

IV. Equipment

Air track with blower	7-cm angle aluminum	digital timer
Air track glider	stepped support block	2-meter stick
Track-mounted springs	1-cm support block	
Excel Linear Regression Analysis template		

V. References

For a review of motion with constant acceleration, see for example Chapter 2 from *College Physics a Strategic Approach* by Knight, Jones and Field.

VI. Introduction

The air track provides a means for studying motion without having to worry about friction. It is a track on which carts or "gliders" slide on a cushion of air (see Figure 1).

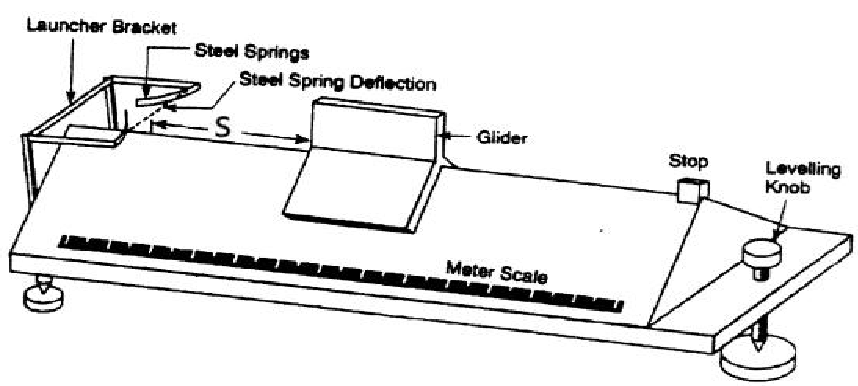


Figure 1. Glider on an air track.

If an object starts at $t = 0$ with an initial velocity v_0 and has a constant acceleration a, the distance s traveled in time t is given by

$$s = v_0 t + \tfrac{1}{2} a t^2 . \tag{1}$$

In this experiment, you will check if Equation 1 is obeyed for straight line motion along a level and a tilted air track.

All objects near the earth have a uniform downward acceleration due to gravity. One could study the free fall of objects, but in practice it is not so easy to measure the short time intervals involved in traversing the short distances available in our laboratory. By tilting the air track by a small amount, you effectively "reduce" the acceleration due to gravity. This idea was first employed by Galileo, who used an inclined plane rather than tilted air track.

Suppose g is the (downward) acceleration due to gravity. When an object of mass m is placed on an inclined plane, the downward force mg on the object due to gravity may be resolved into two components (see Figure 2). One component is **normal** (perpendicular) to the plane, and one is **tangent** to (along) the plane. The component of the force of gravity that is normal to the plane is balanced by the reaction force of the plane on the object (in this case the force provided by the air blowing out of the holes in the air track). Thus, the total normal force on the object is zero.

In contrast, the air track produces no tangential force on the object and the net tangential force on the object is just that due to gravity. Figure 2 shows that the tangential component of the force on the object is

$$F_{\tan} = \text{tangent force} = mg \sin\theta . \tag{2}$$

According to Newton's law of motion, the acceleration a (here the tangential acceleration) is related to the tangential force by the equation

$$a = F_{\tan}/m = (mg\sin\theta)/m = g\sin\theta . \tag{3}$$

Consequently we expect the acceleration a appearing in Equation 1 to be given by Equation 3. Finally note from Figure 2 that if the **length** of the air track (inclined plane) is L, and the **elevation** of the high end is H, then $\sin\theta$ is given by the geometric relation

$$\sin\theta = H/L . \tag{4}$$

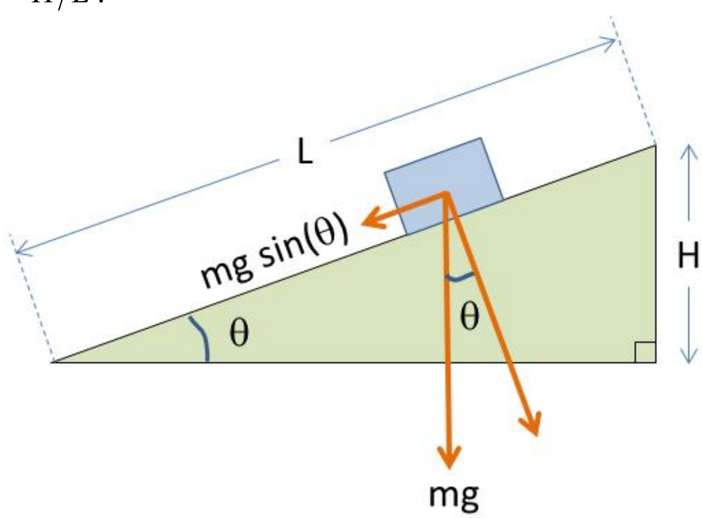


Figure 2. Diagram of a mass m on an inclined plane. The tangential component of the force of gravity accelerates the mass down the incline and is equal to *mgsin(θ)*.

VI. Experimental Procedure

Part A. Motion with No Acceleration

1. Examine the air track and find the identifier number on the air track and on the cart. Record the identifier numbers for both the track and the cart.

2. Last week you should have done ***Experiment #3 - Motion with Constant Velocity*** and found the initial velocity v_o that the launcher gives to a cart. Check the number of the track and the cart that you used last week and verify that you are working with the same airtrack and cart this week. If you are working with the same air track and cart as last week, retrieve from your records the initial velocity v_o and the uncertainty δv_o in the initial velocity for this setup.

3. If you were able to retrieve a value of v_o for this exact same setup from last week's experiment, skip the rest of Part A and just do Part B. On the other hand if you did not do ***Experiment #3- Motion with Constant Velocity*** or you do not have a value for v_o for the specific apparatus you are using this week, then you must complete the rest of this part (Part A) to find v_o before you do Part B.

4. To level the air track, turn on the airflow so that the cart slides easily above the track. Place the glider near the middle of the air track and release it from rest; if the glider begins to move, turn the leveling knob (see Figures 3 and 4) and start again until the glider remains reasonably stationary when released. If the glider remains stationary, its acceleration must be zero, and by Newton's law, the net force acting on it must be zero. If you cannot get your air track to level before the screw is at the end of its range, let your TA know and a riser will be provided to be placed under the foot of the leveling knob.

   Note: The air-tracks are not perfectly flat, so you may want to check for zero acceleration in different places along the track. Try aiming for flatness on average or the middle of the track.

CAUTION: Do not drop the glider!

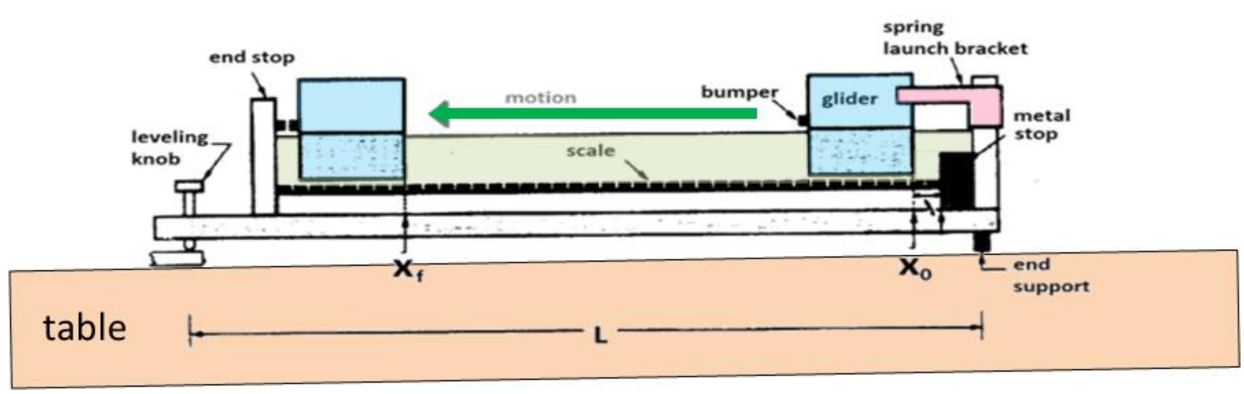


Figure 3. Schematic of the air-track.

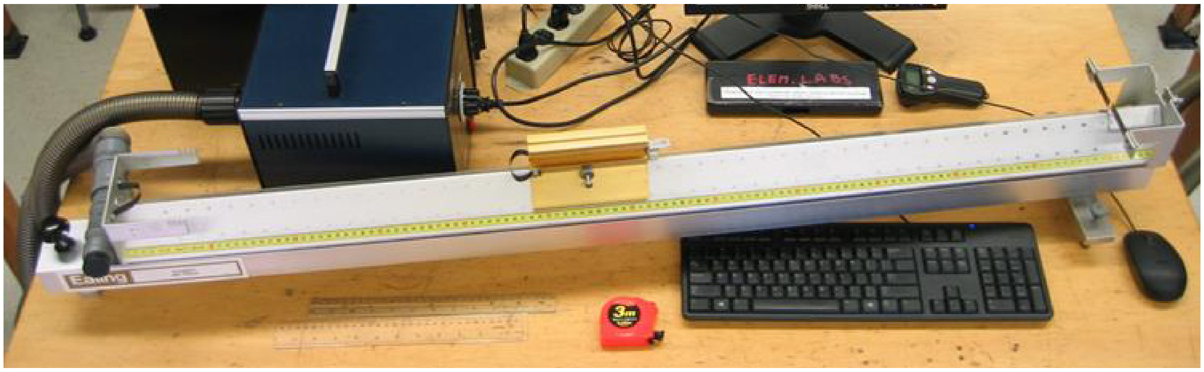


Figure 4. Photo of the air-track.

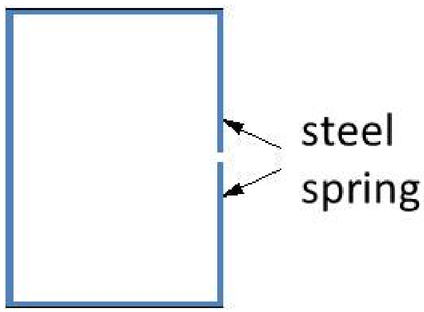


Figure 5. Top view of Launcher Bracket

5. Examine the track and note that there is a bracket at one end with two thin pieces of steel mounted, as shown in Fig. 5. These pieces act as a spring that you will use to launch the glider. Move the glider until it just touches the spring. Record the reading x_0, measured from the back edge of the glider from the centimeter scale mounted along the air track. **Do not write on the air track.**

6. Determine the location x_f on the scale that is 25 cm from the point x_o. That is, such that

$$d = |x_f - x_0| = 25 \text{ cm} \tag{5}$$

Note that x_0 and x_f must both be measured from the same end of the track! **Do not write on the air track.**

7. Your setup comes with an angled piece of aluminum that will sit securely on the track behind the spring and act as a stop for the glider when it is pulled back, compressing the spring. The length of this stop is such that the spring is compressed about 1.5 cm when the glider is brought back against the stop. This will keep the launch compression distances consistent from trial to trial.

8. Compress the spring about 1.5 cm by bringing the glider up against the stop. Release the glider, and use the stopwatch to measure the time t that it takes the glider to travel the 25 cm to the predetermined location x_f. It takes practice to be precise and consistent in your method for measuring the time. Once you are comfortable with the process, take 5 time measurements

and record your data in the "**Additional Data**" area of the **Excel Linear Regression Analysis template**.

9. Assign an uncertainty value δd to the distance by estimating how accurately you were able to stop the watch as the glider reached x_f. One contribution to the uncertainty is how precisely you can use the ruler to determine location x_f, as discussed in Experiment 2. However, in this case, that contribution should be small compared to ability to judge the location of the moving glider. If you aren't sure what δd should be, try observing while your partner launches the cart and uses the stopwatch. Did you agree with his/her timing based on the location of the cart at that moment? Work together to choose a reasonable value. Record the values d and δd in the "**dep variable**" and "**dep var uncert**" columns, respectively, in the Excel template.

9. Repeat steps 6 and 8 for two more values of x_f up to about d = 90 cm. You may use the same uncertainty δd for all of your values of d.

Part B. Motion with Constant Acceleration and no Initial Velocity

This part of the lab involves motion of an object with constant acceleration. You will time how long a glider takes to travel various distances when it is launched with zero initial velocity.

1. Measure and record the length *L* of the air track. See Figure 3.

2. Raise the leveling knob end of the air track 1 cm by inserting a 1 cm thick block between the end of the leveling screw and the bottom step of the supporting block (see Figure 3). You have set *H* = 1 cm.

3. Release the glider at rest (zero initial velocity) from the end of the air track nearest the leveling screw, and time how long it takes the glider to travel a distance of 5 cm. You will now have to measure distances from the opposite end of the air track. Take and record five timings.

4. Repeat step 3 for a travel distance of 20 cm.

5. Repeat step 3 for a travel distance of 45 cm.

6. Repeat step 3 for a travel distance of 80 cm.

Part C. Motion with Constant Acceleration and an Initial Velocity

In this part of the lab you will observe how long a glider takes to travel various distances when it is launched with a nonzero initial velocity uphill.

1. Launch the glider "uphill". Do this by using the launcher as before with the spring compressed about 1.5 cm when the glider is brought back against the stop. The glider should initially go up hill, slow down due to the force of gravity, eventually stop, "turn around", and finally slide back downhill with ever increasing downward velocity. The glider should not hit or touch the far end of the track. Launch the glider uphill 5 times. Record for each launch the time it takes from launch for the glider to just stop. Call the point where the glider stops the "turning point". Also measure the distance the

glider travels from the starting point to the turning point for each launch. Again be sure to always use the same end of the glider when measuring travel distances.

2. Pick a convenient intermediate point roughly half way between the starting point and the turning point. Measure carefully and record the distance from the starting point to the intermediate point.

3. Make several more uphill launches:
   a. Time how long it takes the glider (from the moment of launch) to reach the intermediate point.
   b. Note that after the glider is launched, it passes through the intermediate point **twice**. It passes through the intermediate point once on its way uphill, and a second time on its way downhill. Time how long it takes (from the moment of launch) to reach the intermediate point the second time.
   c. Time how long it takes the glider to make a full round trip. That is, time how long it takes (from the moment of launch) for the glider to come back to its starting point. Make 5 measurements of each timing.

To conserve batteries, please turn off digital timer when you are done using it.

VII. Data Analysis

Data Analysis for Part A. Motion with No Acceleration.

Only do this part of the analysis if you had to do part A above to find v_o.

1. For the data you recorded in Part A in the Excel Linear Regression Analysis template, for each distance d calculate the average time t_{avg} and the standard deviation σ_t for the corresponding set of times you measured for that distance. Record the values for t in the "**indep variable**" column in your template.

2. In your Excel spreadsheet, for each of your average times, find the corresponding uncertainty δt_{avg} in the average time using:
$$\delta t_{avg} = \frac{\sigma_t}{\sqrt{N}} \tag{6}$$
   where N is your number of data points. Record the values in the "**indep var uncert**" column. Make sure each value is in the appropriate row with its corresponding t_{avg} value.

3. Plot your data with d on the vertical axis and t on the horizontal axis. Put horizontal and vertical error bars on each data point using the values for d and t (you may use a median or other representative value for t since they are all different).

4. Add a fit line to your plot by right-clicking your data and choosing "**Add Trendline**". Make sure you choose the "**linear**" option, and also choose to "**display equation on chart**".

5. Click on the **macro button** in the template to run the linear regression fitting on your d and t_{avg} data, and note the resulting slope and intercept values, and their uncertainties. Note that the slope and intercept values should closely match the values in the equation on your plot.

6. Since you used the same launcher and same compression distance for every trial, the resulting velocity v_o should be the same for every trial. The lack of friction implies the speed should remain nearly constant and the distance d should be related to the corresponding time t_{avg} by:

$$d=v_o t_{avg}.$$

As a result, the slope of your plot should be equal to the (constant) velocity v_o of the glider, the uncertainty in the slope should be the uncertainty δv_o in the velocity, and the y-intercept should be zero. Examine the output from the spreadsheet's fit and determine whether the intercept from your fit is in fact statistically consistent with zero. That is, does the origin lie within your uncertainty? Include your answer and reasoning in your report.

Analysis of Part B. Motion with Constant Acceleration

1. You made timing measurements for travel distances *s* having the nominal values 5, 20, 45, and 80 cm. Assign an error value δs to these distances by estimating how accurately you were able to measure these distances.

2. Find the average time $\bar{t}$ that it took to travel 5 cm by averaging the five travel times *t* you measured for a travel distance of 5 cm. Also find the standard deviation σ_t of these five times.

3. Repeat step 2 above for the travel distances of 20, 45, and 80 cm. Record and present all your results in tabular form.

4. According to Equation 1,

$$s = v_0 t + \tfrac{1}{2} a t^2 . \qquad (1)$$

which means that a plot of *s* versus $\bar{t}$ should look like a parabola. You can convert "parabolic" data into a plot that looks "linear" data by plotting your data with *s* values along the vertical axis, and $\bar{t}^2$ values along the horizontal axis.

5. Next you will need to put error bars on each point. Each vertical error bar should extend from $s-\delta_s$ to $s+\delta_s$. Each horizontal error bar should extend from $\left(\bar{t}-\sigma_t\right)^2 \approx \bar{t}^2 - 2\bar{t}\sigma_t$ to $\left(\bar{t}+\sigma_t\right)^2 \approx \bar{t}^2 + 2\bar{t}\sigma_t$, where σ_t is the standard deviation associated with that particular $\bar{t}$. To get these error bars, which are the uncertainty in your values of $\bar{t}^2$ and will depend on t, use Excel to fill in a column with $2\bar{t}\sigma_t$. Note: when Excel first draws error bars on a plot, it automatically adds "Standard" error bars in x and y. These are totally wrong or fake error bars and you will need to change them to the correct error bars. To do this, click on an x-error bar, select Custom (in the pop-up menu on the right), then click on Specify Value, and then fill in the boxes for the left and right error bars to select the cells you just made with $2\bar{t}\sigma_t$. Don't forget to specify the y-error bars also.

6. According to Equation 1 with v_0=0, when you plot your data as s versus t^2 it should (ideally) lie on a straight line through the origin. Draw the **best** straight line that goes through the origin and also passes closest to your data points. Your plot should look something like Figure 6. Do your points indeed seem to lie on a straight line within the accuracy expected from the size of the error bars? Draw two other straight lines through the origin that are consistent with the error bars on your data points. One line should have a **larger** slope than the slope of the best straight line, and one should have a **smaller** slope.

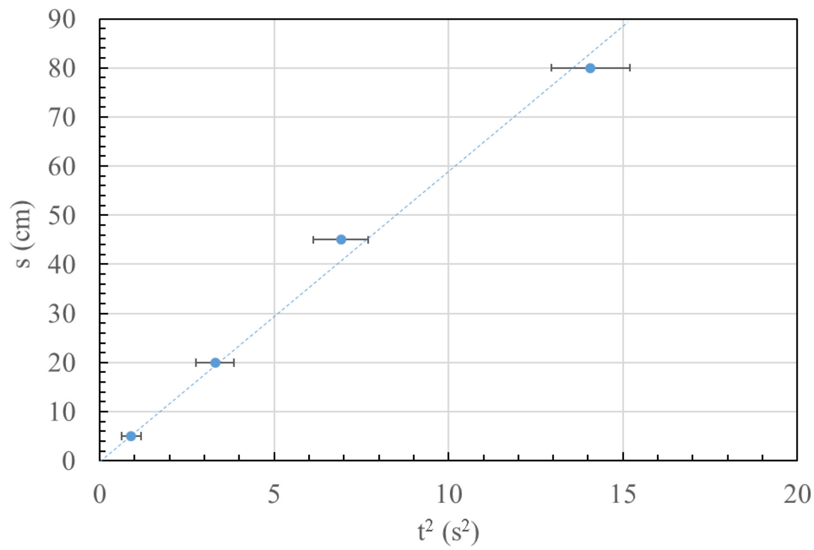


Figure 6. Plot of s (along the y-axis) versus t^2 (along the x-axis). The dashed line is a fit line through the data points.

7. Find the slope of each straight line. From each slope determine from Equation 1, with v_0=0, an associated value of a. Express your experimental result in the general form

$$a_{large} = a_{best} + u \quad (7)$$

and

$$a_{small} = a_{best} - w \quad (8)$$

Use these relations to determine u and w.

7. Using Equations 3, 4, and 7, find the value of g implied by your experimental result. Determine your experimental uncertainty in g by finding u and w so that

$$g_{large} = g_{best} + u \quad (9)$$

and

$$g_{small} = g_{best} - w\,. \quad (10)$$

Also express your estimated experimental uncertainty in percent by calculating u/g_{best} and w/g_{best} in percent.

8. Compare your result for g with the "correct" value of g.

Analysis of Part C- Motion with Constant Acceleration and an Initial Velocity

1. Consider the data you collected in Part C.
   a. Average the turning point travel distances to get an average turning point travel distance. Find the standard deviation in the turning point travel distance.
   b. Find the average turning point travel time, and its standard deviation.
   c. Consider the starting point. For this point you know $s = 0$ at $t = 0$ without any error.

d. Consider the intermediate point. Assign an error value δs to its distance by estimating how accurately you were able to measure its distance.

e. Consider the travel times associated with the intermediate point. You should have two sets of travel times (with 5 values each) for this point: one set of values from Part C step 3a and one set from Part C step 3b. Find the average and standard deviation for each set.

f. Consider the full round trip data you took in Part C step 3c. In this case we again have $s = 0$ without any error. From your data (Part C step 3c) find the average round trip travel time and its standard deviation. Put your results in tabular form.

2. Consider the relation

$$s = (v_{best})t + \tfrac{1}{2}(-a_{best})t^2, \tag{11}$$

where v_{best} is given by your value for v_o (either from doing Experiment 3 last week or from doing Part A this week) and a_{best} is given by your results from the Part B analysis above. Use Excel to make a graph of s versus t for this relation.

3. Add to the theory plot you just made a plot of your data from the analysis you just did in Part C step 1, with appropriate error bars, on the same graph. Consistent with the error bars, do your data fall on the curve Equation 11? Discuss briefly possible sources of error. Your plot should look something like what is shown below in Figure 7. Note that the data need not fall exactly on the curve. Be sure to use a smooth curve for the theory and points for your data. Also be sure to include scales, labels and units on both axes.

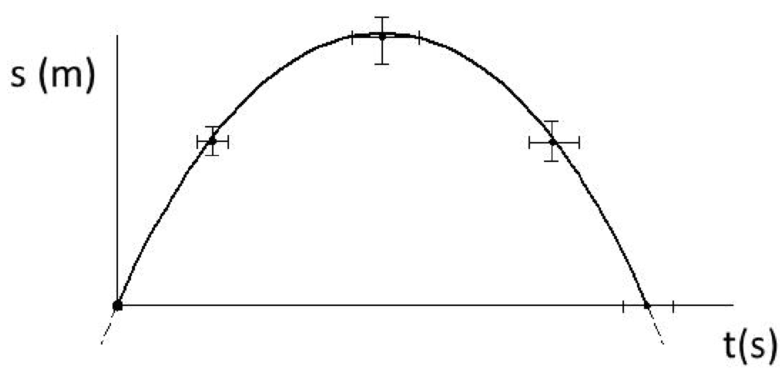


Figure 7: Plot of Up-Hill Motion Showing Expected Motion and Experimental Data. The Expected Motion Curve (a parabola) is shown as dashed for $s < 0$ since the Air Track ends at $s = 0$. This plot is missing scales on the x and y axis. Make sure that your plot has scales, labels and units on each axis.

IX. Finishing Up

- Make sure you get your lab partner's name and his or her contact information.
- Make sure you save a copy of your spreadsheet on a memory stick or e-mail yourself a copy.
- Don't forget that you need to prepare a Lab Report for this Experiment and it must be submitted before it is due (see the syllabus). Also, make sure to review the Guidelines for Writing a Lab Report, right after the main introduction in this Lab Manual.
- Finally, don't forget to prepare for the next lab and submit your answers to the prelab questions for the next lab before they are due.