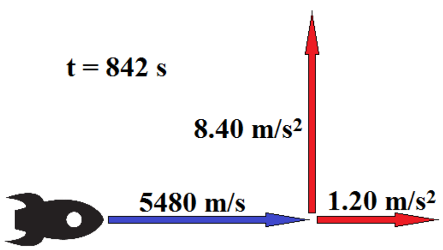
Lecture 9: Two-Dimensional Kinematics and Dynamics

Physics for Engineers & Scientists (Giancoli): Chapters 3 & 4
University Physics V1 (Openstax): Chapters 4 & 5

Example: A spacecraft is travelling with a velocity of v_{0x} = 5480 m/s along the +x direction. Two engines are turned on for a time of 842 s. One engine gives the spacecraft an acceleration in the +x direction of a_x = 1.20 m/s², while the other gives it an acceleration in the +y direction of 8.40 m/s². At the end of the firing, find (a) v_x and (b) v_y.



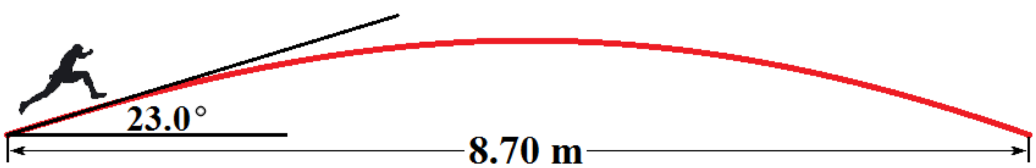
$t = 842\text{s} \quad v_{ox} = 5480\text{m/s} \quad v_{oy} = 0 \quad a_x = 1.20\text{m/s}^2 \quad a_y = 8.40\text{m/s}^2$

$$v_x = v_{0x} + a_x t = \left(5480\frac{m}{s}\right) + \left(1.20\frac{m}{s^2}\right)(842\ s) = 6490\frac{m}{s}$$

$$v_y = v_{0y} + a_y t = \left(0\frac{m}{s}\right) + \left(8.40\frac{m}{s^2}\right)(842\ s) = 7070\frac{m}{s}$$

If the quadratic equation gives you trouble, you can find your answer in another way!

Example: An Olympic jumper leaves the ground at an angle of 23.0° and travels through the air for a horizontal distance of 8.70 m before landing. What is the takeoff speed of the jumper?



$x_o = 0 \qquad x = 8.70\text{m} \qquad a_x=0 \qquad v_{ox} = v_x = v_o\text{Cos}(23.0°)$

$y_o = 0 \qquad y = 0 \qquad v_{oy} = v_o\text{Sin}(23.0°) \qquad v_y = -v_{oy} = -v_o\text{Sin}(23.0°) \qquad a_y=-g$

The x and y equations are linked via t (so we need equations with t), but we have 2 unknowns (v_0 and t). Need 2 equations (x-comp and y-comp) that have both v_0 and t.

x-components: $\quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t = v_0\text{Cos}(23.0°)t \qquad t = \frac{x}{v_0\text{Cos}(23.0°)}$

y-components: $\quad v_y = v_{0y} + a_y t \qquad -v_0\text{Sin}(23.0°) = v_0\text{Sin}(23.0°) - gt$

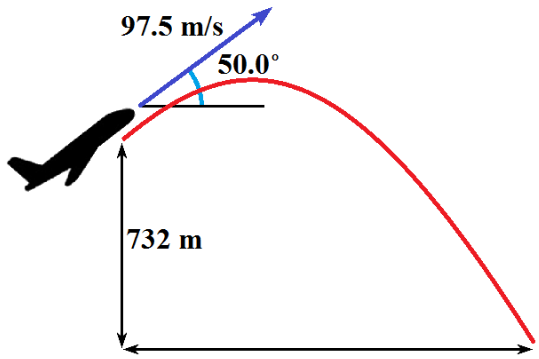
$$-2v_0\text{Sin}(23.0°) = -gt \qquad 2v_0\text{Sin}(23.0°) = gt$$

Plug in t (from x-comp): $\quad 2v_0\text{Sin}(23.0°) = g\left(\frac{x}{v_0\text{Cos}(23.0°)}\right) \qquad 2v_0\text{Sin}(23.0°) = \frac{gx}{v_0\text{Cos}(23.0°)}$

$$v_0 = \frac{gx}{v_0 2\text{Sin}(23.0°)\text{Cos}(23.0°)} \qquad v_0^2 = \frac{gx}{2\text{Sin}(23.0°)\text{Cos}(23.0°)}$$

$$v_0 = \sqrt{\frac{gx}{2\text{Sin}(23.0°)\text{Cos}(23.0°)}} = \sqrt{\frac{\left(9.80\frac{m}{s^2}\right)(8.70\ m)}{2\text{Sin}(23.0°)\text{Cos}(23.0°)}} = 10.887\frac{m}{s} \Rightarrow 10.9\frac{m}{s}$$

Example: An airplane with a speed of 97.5m/s is climbing upwards at an angle of 50.0° with respect to the horizontal. When the planes altitude is 732m the pilot releases a package. (a) Calculate the distance along the ground measured from a point directly beneath the point of release, to where the package hits the earth. (b) Relative to the ground, determine the angle of the velocity vector just before impact.



$x_0 = 0 \qquad x = ??? \qquad a_x = 0$

$v_{0x} = (97.5\text{ m/s})\text{Cos}(50.0°) = 62.6718\text{ m/s}$

$v_x = v_{0x} = 62.6718\text{ m/s}$

$y_0 = 732\text{ m} \qquad y = 0$

$v_{0y} = (97.5\text{ m/s})\text{Sin}(50.0°) = 74.6893\text{ m/s}$

$v_y = \qquad a_y = -g \qquad t =$

x-components (find x): $x = x_0 + v_x t = v_x t$ *{Need t.}*

y-components (find t, no v): $y = y_0 + v_{oy} t + \frac{1}{2} a_y t^2$ *{Solve with quadratic equation}*

$$0 = 732\text{ m} + \left(74.6893\frac{\text{m}}{\text{s}}\right)t - \frac{1}{2}\left(9.80\frac{\text{m}}{\text{s}^2}\right)t^2 \qquad 0 = 732\text{ m} + \left(74.6893\frac{\text{m}}{\text{s}}\right)t - \left(4.90\frac{\text{m}}{\text{s}^2}\right)t^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-74.6893\frac{\text{m}}{\text{s}} \pm \sqrt{\left(74.6893\frac{\text{m}}{\text{s}}\right)^2 - 4\left(-4.90\frac{\text{m}}{\text{s}^2}\right)(732\text{ m})}}{2\left(-4.90\frac{\text{m}}{\text{s}^2}\right)}$$

$$t = \frac{-74.6893\frac{\text{m}}{\text{s}} \pm \sqrt{5578.4915\frac{\text{m}^2}{\text{s}^2} + 14347.2\frac{\text{m}^2}{\text{s}^2}}}{-9.80\frac{\text{m}}{\text{s}^2}} \qquad t = \frac{-74.6893\frac{\text{m}}{\text{s}} \pm \sqrt{19925.6915\frac{\text{m}^2}{\text{s}^2}}}{-9.80\frac{\text{m}}{\text{s}^2}}$$

$$t = \frac{-74.6893\frac{\text{m}}{\text{s}} \pm 141.1584\frac{\text{m}}{\text{s}}}{-9.80\frac{\text{m}}{\text{s}^2}} \qquad t_1 = \frac{-74.6893\frac{\text{m}}{\text{s}} + 141.1584\frac{\text{m}}{\text{s}}}{-9.80\frac{\text{m}}{\text{s}^2}} = \frac{66.4691\frac{\text{m}}{\text{s}}}{-9.80\frac{\text{m}}{\text{s}^2}} = -6.78256\text{ s}$$

$$t_2 = \frac{-74.6893\frac{\text{m}}{\text{s}} - 141.1584\frac{\text{m}}{\text{s}}}{-9.80\frac{\text{m}}{\text{s}^2}} = \frac{-215.8477\frac{\text{m}}{\text{s}}}{-9.80\frac{\text{m}}{\text{s}^2}} = 22.0253\text{ s}$$

Now plug the value of t into the original equation to get x:

$$x = v_x t = \left(62.6718\frac{\text{m}}{\text{s}}\right)(22.0253\text{ s}) = 1380.36 \Rightarrow 1380\text{ m}$$

The value of t can also be used to get v_y, which is needed to produce the velocities angle (part b).

$$v_y = v_{0y} + a_y t = \left(74.6893\frac{\text{m}}{\text{s}}\right) + \left(-9.80\frac{\text{m}}{\text{s}^2}\right)(22.0253\text{ s}) = -141.159\frac{\text{m}}{\text{s}}$$

$$\theta = \text{Tan}^{-1}\left(\frac{v_y}{v_x}\right) = \text{Tan}^{-1}\left(\frac{-141.159\frac{\text{m}}{\text{s}}}{62.6718\frac{\text{m}}{\text{s}}}\right) = -66.0597° \Rightarrow -66.1°$$

Alternatively (instead of using quadratic equation) find v first, then t.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = \left(74.6893\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + 2\left(-9.80\,\frac{m}{s^2}\right)(0 - 732\,m) = 19925.69\,\frac{m^2}{s^2}$$

$$v_y = -\sqrt{19925.69\,\frac{m^2}{s^2}} = -141.158\,\frac{m}{s} \qquad \mathrm{v_y = v_{0y} + a_y t} \qquad \mathrm{v_y - v_{0y} = a_y t = -gt}$$

$$\mathrm{t} = \frac{\mathrm{v_y - v_{0y}}}{-\mathrm{g}} = \frac{-141.158\,\frac{\mathrm{m}}{\mathrm{s}} - 74.6893\,\frac{\mathrm{m}}{\mathrm{s}}}{-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 22.0252\ \mathrm{s}$$

$$\mathrm{x = v_x t} = \left(62.6718\,\frac{\mathrm{m}}{\mathrm{s}}\right)(22.0252\ \mathrm{s}) = 1380.36 \;\Rightarrow\; 1380\ \mathrm{m}$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{\mathrm{v_y}}{\mathrm{v_x}}\right) = \mathrm{Tan}^{-1}\left(\frac{-141.158\,\frac{\mathrm{m}}{\mathrm{s}}}{62.6718\,\frac{\mathrm{m}}{\mathrm{s}}}\right) = -66.0596° \;\Rightarrow\; -66.1°$$

If the quadratic equation gives you trouble, you may be able to find your answer in another way!

Forces

- Intuitively, a force is a push or a pull.
- Forces are vectors with both magnitude and direction.
- Contact forces are created when two objects are in physical contact.
- Non-contact forces are felt between objects that are not in contact.

Mass is a measure of the amount of matter in an object.

Newton's Laws of Motion

- **Newton's 1st Law of Motion**: **"The Law of Inertia"**

  Every object continues in its state of rest, or of uniform speed in a straight line, as long as no net force acts on it.

  - The tendency for an object to move at a constant speed in a straight line is called inertia.
  - Mass is a qualitative measure of the inertia of an object.
  - Velocity is a vector. A change in the speed or direction of motion is a change in inertia.
  - If something is speeding up, slowing down or changing direction Newton's First Law says there is an external FORCE acting on it.
  - An inertial reference frame is a coordinate system that obeys Newton's First Law. Any reference frame moving with constant velocity (or stationary) with respect to an inertia reference frame is also an inertia reference frame

- **<u>Newton's 2nd Law of Motion</u>: "The Law of Force and Acceleration."**

  The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportionally to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

  - The standard SI unit of mass is the kilogram (kg)
  - The standard SI unit of force is the Newton (N): $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$.
  - This is a vector equation.
    - The acceleration points in the same direction as the sum of the forces.
    - The magnitude of the net force equals the mass multiplied by the magnitude of the acceleration.
  - There are two cases
    1) In <u>static equilibrium</u> the acceleration (and the sum of the forces) is zero.
    2) In <u>non-equilibrium</u> the acceleration is not zero (and neither is the sum of the forces)

  Newton's 2nd law acts as a link between force problems and kinematic problems.

- **<u>Newton's 3rd Law of Motion</u>**: Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction.

  "For every action, there is an equal and opposite reaction."

  The force that a nail experiences when hit with a hammer (driving it into the wood) is equal in magnitude and opposite in direction to the force that the hammer experiences from being hit by the nail, slowing the hammer down.

<u>Example</u>: An F-14 has a mass of $3.1\text{x}10^4$ kg and takes off under the influence of a constant net force of $3.7\text{ x}10^4$ N. What is the net force that acts upon the 78 kg pilot?

The forces and accelerations in this problem are collinear. We may treat it 1-dimensionally.

$$\sum \vec{F} = m\vec{a} \qquad F_{F-14} = m_{F-14} \cdot a \qquad a = \frac{F_{F-14}}{m_{F-14}} = \frac{3.7\times10^4\text{ N}}{3.1\times10^4\text{ kg}} = 1.19355\text{ m/s}^2$$

$$F_{Pilot} = m_{Pilot} \cdot a = (78\text{ kg})\left(1.19355\,\frac{\text{m}}{\text{s}^2}\right) = 93.0969\text{ N} \Rightarrow 93\text{ N}$$