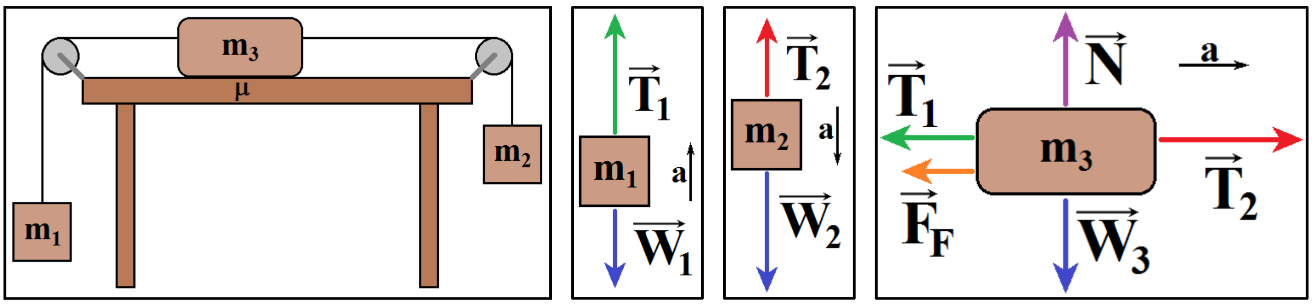
Lecture 8: Apparent Gravity and Uniform Circular Motion

***Physics for Engineers & Scientists* (Giancoli): Chapter 5**
***University Physics V1* (Openstax): Chapter 6**

Example: A box of mass m_3 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_2 = 25.0 kg. On the other side a cord attaches it to a hanging weight of mass m_1 = 5.00 kg. If the coefficient of kinetic friction between the box and table is 0.300, determine the acceleration of the system. Assume the mass and friction of the pulleys is negligible.



Can we go straight to m_3, treating the two tensions as m_1g and m_2g?
No! $a \neq 0 \Rightarrow T \neq W$.

What direction is m_3 accelerating? To the right ($m_2 > m_1$). So, friction points to the left.

Start by making force diagrams for all 3 masses.
There are 4 unknowns (a, F_F, T_1, and T_2).
We need 4 equations (ΣF_y for m_1, ΣF_y for m_2, ΣF_y for m_3, ΣF_x for m_3).

$$\Sigma F_y = m_1a: \quad T_1 - m_1g = m_1a \qquad T_1 = m_1a + m_1g$$

$$\Sigma F_y = m_2a: \quad m_2g - T_2 = m_2a \qquad T_2 = m_2g - m_2a$$

m_2 accelerates downward (typically negative), but a is being treated as positive.
You either need to make down positive or use '-a' as the acceleration to correct for this.

$$\Sigma F_y = 0 \text{ for } m_3: \quad N = W_3 = m_3g \qquad F_F = \mu N = \mu m_3 g$$

$$\Sigma F_x = m_3a: \quad T_2 - T_1 - F_F = m_3a \qquad m_2g - m_2a - m_1a - m_1g - \mu m_3g = m_3a$$

$$m_2g - m_1g - \mu m_3g = m_1a + m_2a + m_3a$$

$$(m_2 - m_1 - \mu m_3)g = (m_1 + m_2 + m_3)a$$

$$a = (m_2 - m_1 - \mu m_3)g / (m_1 + m_2 + m_3)$$

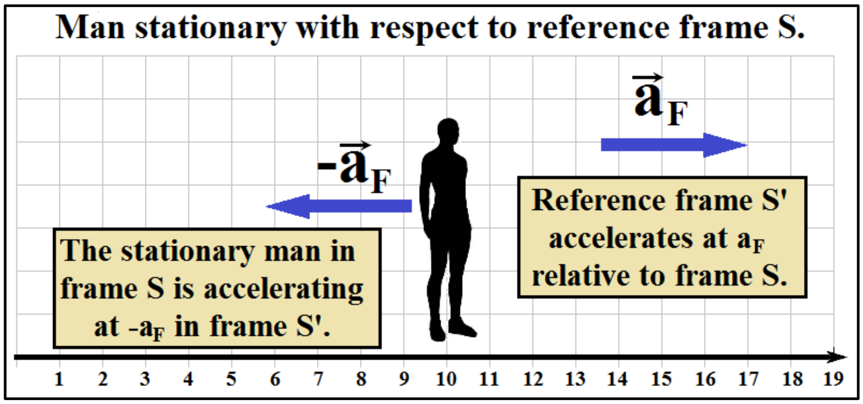
$$a = (25.0 \text{ kg} - 5.00 \text{ kg} - 0.300 \cdot 10.0 \text{ kg}) \cdot (9.80 \text{ m/s}^2) / (5.00 \text{ kg} + 25.0 \text{ kg} + 10.0 \text{ kg}) = 4.17 \text{ m/s}^2$$

Apparent Gravity

- One method of dealing with an accelerating reference frame (a non-inertial reference frame) is to 'package' the effects created by acceleration in combination with gravity (g) into apparent gravity (g_{app}).

- The acceleration (a_F) of a non-inertial reference frame relative to an inertial reference frame, causes every object in that inertial reference frame to have an additional acceleration ($-a_F$) when viewed in the non inertial frame.

For example, the man below is stationary with respect to his original reference frame. If our coordinate system accelerates to the right at a_F, then he must accelerate in the opposite direction at $-a_F$.



- This additional acceleration ($-a_F$) can be combined (vector addition) to gravity to get apparent gravity.

$$\vec{g}_{APP} = \vec{g} - \vec{a}_F$$

- Problems in the non-inertial frame can then be treated as if they were an inertial frame with a new value (and possibly direction) of gravity.

<u>Example</u>: A person is holding a 5.0 kg package in an elevator that begins accelerating upwards as 1.2 m/s^2. What force must the person exert to hold the package in the same relative position?

You could treat this as a person, elevator and package accelerating in an inertial frame.

$$F - mg = ma \Rightarrow F = ma + mg = (5.0\text{ kg})(1.2\text{ m/s}^2) + (5.0\text{ kg})(9.8\text{ m/s}^2) = 6\text{N} + 49\text{N} = 55\text{ N}$$

Or you could treat this as a person and package stationary in an accelerating reference frame.

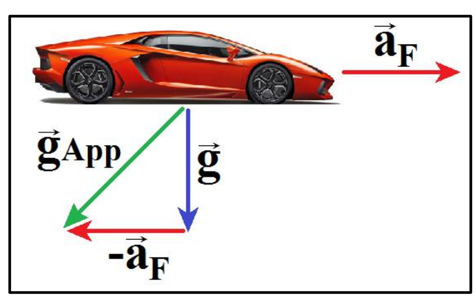
Note that as a_F points upwards, - a_F points downward (in the same direction as gravity). This indicates that you should add the magnitudes.

$$g_{app} = g - a_F = 9.8\text{ m/s}^2 - (-1.2\text{ m/s}^2) = 11\text{ m/s}^2$$

$$W_{app} = mg_{app} = (5.0\text{ kg})(11\text{ m/s}^2) = 55\text{N}$$

<u>Conceptual Example</u>: When you are in an accelerating car, you feel like you are being pushed back into your seat.

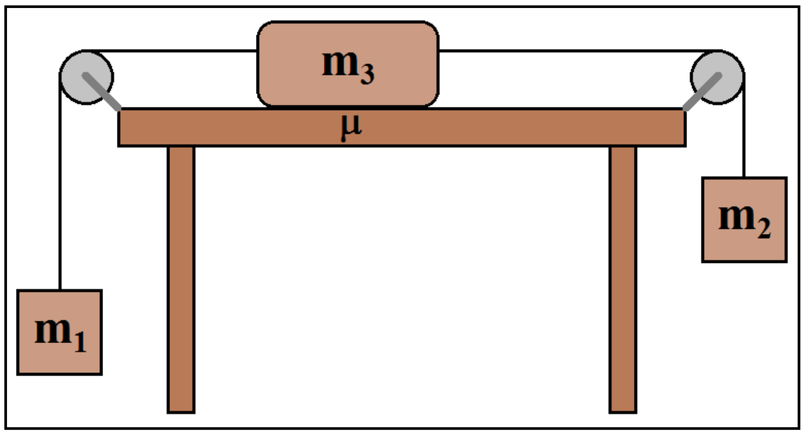
$$|g_{APP}| = \sqrt{g^2 + a_F^2}$$

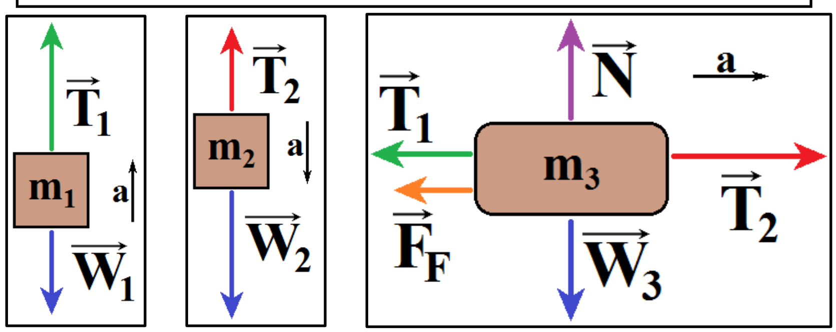


Please don't measure the acceleration of your car by noting the angle of something hanging from the mirror while driving!

<u>Example</u>: A box of mass m_3 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_2 = 25.0 kg. On the other side a cord attaches it to a hanging weight of mass m_1 = 5.00 kg. The coefficient of kinetic friction between the box and table is 0.300. *If the table is placed in an elevator that is accelerating downward at 1.20 m/s²*, determine the horizontal acceleration of the box on the table. Assume the mass and friction of the pulleys is negligible.

This is identical to the problem we solved earlier with one exception. Now it's in an elevator accelerating downward.





We could solve this without apparent gravity, making modifications in each force diagram.

Decrease the acceleration of m_1 (in $\Sigma F_y = m_1 a$): $T_1 - m_1 g = m_1(a - 1.20\ \text{m/s}^2)$

Increase the acceleration of m_2 (in $\Sigma F_y = m_2 a$): $m_2 g - T_2 = m_2(a + 1.20\ \text{m/s}^2)$

Account for the acceleration of m_3 (ΣF_y is not zero): $W_3 - N = m_3(1.20\ \text{m/s}^2)$

Which changes friction to: $F_F = \mu N = \mu m_3 g - \mu m_3(1.20\ \text{m/s}^2)$

Then solve for a after plugging T_1, T_2, and F_F into: $T_2 - T_1 - F_F = m_3 a$

Or we can solve it as we did before and replace g with $g_{app} = g - a_F$

$$g_{app} = g - a_F = 9.80\ \text{m/s}^2 - 1.20\ \text{m/s}^2 = 8.6\ \text{m/s}^2$$

$$a = (m_2 - m_1 - \mu m_3) g_{app} / (m_1 + m_2 + m_3)$$

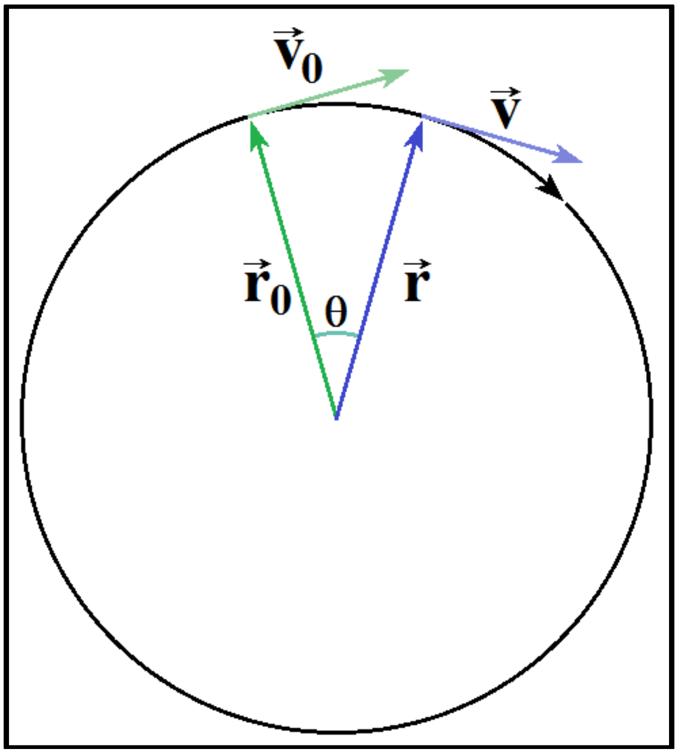
$$a = (25.0\ \text{kg} - 5.00\ \text{kg} - 0.300 \cdot 10.0\ \text{kg}) \cdot (8.60\ \text{m/s}^2) / (5.00\ \text{kg} + 25.0\ \text{kg} + 10.0\ \text{kg}) = 3.655\ \text{m/s}^2$$

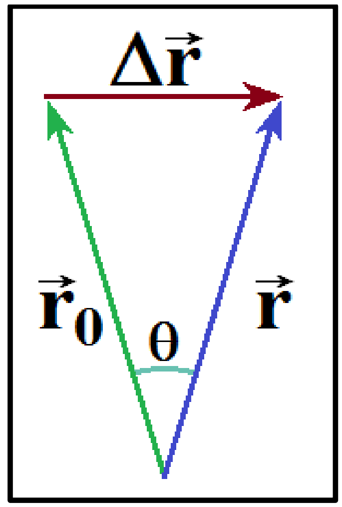
FYI, one of the major breakthroughs of Albert Einstein was the 'equivalence principle', essentially stating that gravity was no different than any other acceleration. This lead to his prediction that light would bend due to gravity and eventually to general relativity.

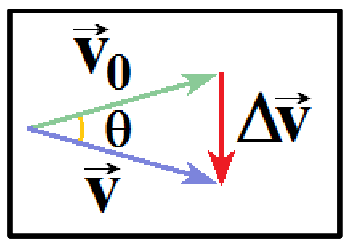
Uniform Circular Motion

- **Uniform Circular Motion** is moving in a circular path at a constant speed.
- **The Period (T)** is the time needed to complete one full cycle (once around the circle).
- **The Frequency (f)** is the number of cycles complete divided by the time interval.

$$\mathbf{T = \frac{1}{f} \qquad v = \frac{2\pi r}{T} = 2\pi r f}$$







The distance triangle (upper right) and the velocity triangle are similar triangles (identical interior angles). The ratios of corresponding sides of similar triangles are equal.

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

For small time intervals ($\Delta t \to 0$), the angle theta is very small. This allows us to approximate Δr.

$$\Delta r \approx v \cdot \Delta t$$

Plugging into the previous equation leads to our result.

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v} \qquad \frac{v^2 \Delta t}{r} = \Delta v \qquad \frac{v^2}{r} = \frac{\Delta v}{\Delta t}$$

For small time intervals ($\Delta t \to 0$), $a = \Delta v/\Delta t$. In this case, the acceleration is referred to as "centripetal acceleration (a_c) or radial acceleration (a_r). Centripetal acceleration always points in the direction of Δv, radially inward towards the center of the circle.

$$a_c = \frac{v^2}{r}$$

As the centripetal acceleration a_c is perpendicular to velocity with no component in the direction of the velocity, it only changes the direction and not the magnitude of the velocity.

Example: Computer-controlled display screens provide drivers in the Indianapolis 500 with a variety of information about how their cars are performing. For instance, as a car is going through a turn, a speed of 221 mi/h (98.8m/s) and a centripetal acceleration of 3.00g are displayed. Determine the radius of the turn.

$$3.00g = (3.00)\left(9.80\frac{m}{s^2}\right) = 29.4\ \frac{m}{s^2}$$

$$a_c = \frac{v^2}{r} \qquad a_c \cdot r = v^2 \qquad r = \frac{v^2}{a_c} = \frac{\left(98.8\frac{m}{s}\right)^2}{29.4\frac{m}{s^2}} = 332\ m$$

Centripetal Forces

- The **Centripetal Force** is the force that gives rise to the centripetal acceleration that causes an object to move in a curved path.

$$F_C = ma_C = \frac{mv^2}{r}$$

- The centripetal force is not a new force. Any of the other forces we've studied (or a combination) may act as the centripetal force.
- The centripetal force is always directed at the center of the circle, perpendicular to the velocity.

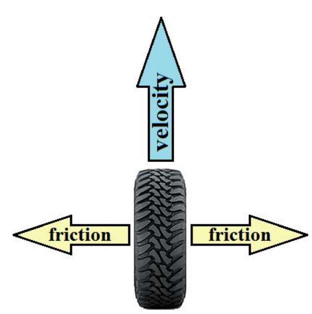
Example: In a skating stunt known as "crack-the-whip", a number of skaters hold hands and form a straight line. They skate so that the line rotates around the skater at one end who acts as a pivot. The skater farthest out has a mass of 80.0kg and is 6.10m from the pivot. He is skating at a speed of 6.80m/s. Determine the magnitude of the centripetal force that acts on him.

$$F_c = \frac{mv^2}{r} = \frac{(80.0\ kg)\left(6.80\frac{m}{s}\right)^2}{6.10\ m} = 606\ \text{N}$$

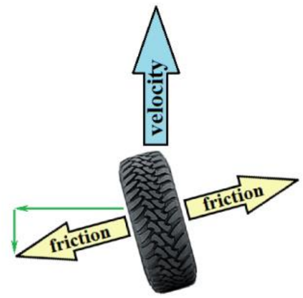
Example: Compare the maximum speeds at which a car can safely negotiate an unbanked turn (radius = 50.0m) in dry weather (μ_s = 0.900) and icy weather (μ_s = 0.100)

Note: Rolling (not sliding) wheels are dependent upon μ_s not μ_k

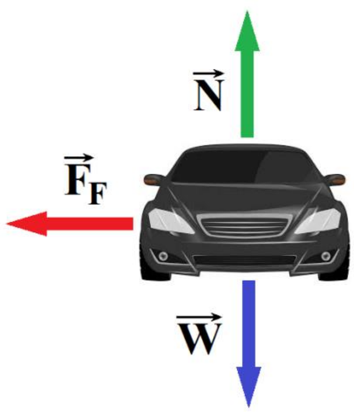
The bottom of a rolling tire is stationary on the road's surface. This is evident when it rolls through water as the tread of the tire is visible in the water trail it leaves behind. This rolling motion prevents any friction forces from being felt in the direction the tire rolls (unless brakes are applied disrupting the rolling motion). Consequently, a rolling tire can only feel friction forces parallel to the tires axle.



When are car turns, the tires can only exert a friction force on one side or the other. One side would have a friction component in the direction of motion (which is not allowed). The friction force is thus felt on the other side with two components. One component is perpendicular to the motion, causing the car to turn. The other is opposite the velocity, causing the car to decelerate.



Note: Anti-lock brakes keep a car from skidding. Skidding cars are affected by the coefficient of kinetic friction, which is typically smaller than the coefficient of static friction. A non-skidding car affected by the larger static coefficient will stop faster.



There are three forces acting on the car: the weight, the normal force, and the friction force.

Step 1: Determine which force is acting as F_C. Remember this force points to the center of the circle the object moves in. In this case, the friction force is acting as the centripetal force.

Step 2: Solve for F_C: $F_C = F_F = \mu_s N = \mu_s mg$

Step 3: Set F_C equal to $\frac{mv^2}{r}$:

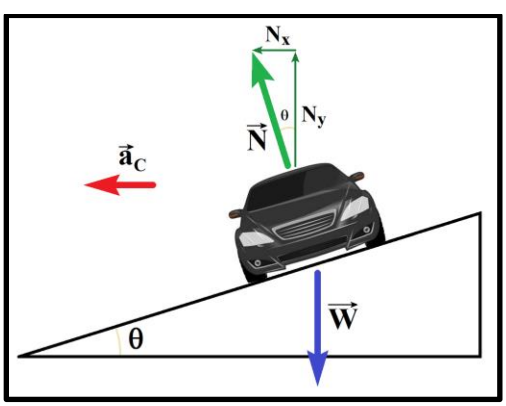
$$\frac{mv^2}{r} = \mu_s mg \qquad \frac{v^2}{r} = \mu_s g \qquad v^2 = \mu_s gr \qquad v = \sqrt{\mu_s gr}$$

For dry weather ($\mu_S = 0.900$): $v = \sqrt{\mu_s gr} = \sqrt{(0.900)\left(9.80\frac{m}{s^2}\right)(50.0\ m)} = 21.0\frac{m}{s}$ (47 mph)

For icy weather ($\mu_S = 0.100$): $v = \sqrt{\mu_s gr} = \sqrt{(0.100)\left(9.80\frac{m}{s^2}\right)(50.0\ m)} = 7.00\frac{m}{s}$ (15.7 mph)

Frictionless Banked Curve

There are only two forces on a frictionless banked curve (weight and the normal force). When combined these forces result in an acceleration that is inward.



Step 1: Determine which force is acting as F_C.

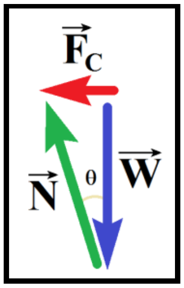
What force points to the center? $N_x = F_C$.

Step 2: Solve for F_C: $\text{Tan}\,\theta = \frac{N_x}{N_y} = \frac{F_C}{W} = \frac{F_C}{mg}$

$$F_C = mg\text{Tan}\,\theta$$

Step 3: Set $F_C = mv^2/r$: $F_C = mg\text{Tan}\,\theta = \frac{mv^2}{r}$

$$g\text{Tan}\,\theta = \frac{v^2}{r} \qquad v^2 = rg\text{Tan}\,\theta \qquad v = \sqrt{rg\text{Tan}\,\theta}$$



Alternatively, we could have made a triangle from a simple vector equation:

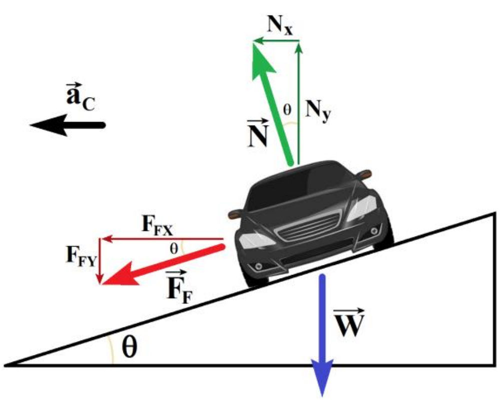
$$\vec{W} + \vec{N} = \vec{F}_C$$

$$\text{Tan}\,\theta = \frac{F_C}{W} = \frac{F_C}{mg}$$

Banked Curve with Friction

There are three forces on a frictionless banked curve (weight, the normal force, and friction). When combined these forces result in an acceleration that is inward.

The direction of the friction force is dependent upon circumstances. A slow moving car would tend to slide down the incline (opposing this, the friction would point up the incline). Alternatively, a fast moving car would tend to skid outward and up the incline. In this latter case the friction would point down the incline. As we will be considering maximum velocities, we will have the friction pointing down the slope.



Step 1: Determine which force(s) is acting as F_C.

What force(s) points to the center? $F_C = N_x + F_{Fx}$.

Step 2: Solve for F_C: start with x-components

$$F_c = N_x + F_{Fx} = N\text{Sin}\,\theta + F_F\text{Cos}\,\theta$$

$$F_c = N\text{Sin}\,\theta + \mu_S N\text{Cos}\,\theta = (\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta)N$$

Need N from y-components: $N_y = W + F_{Fy}$

$$N\text{Cos}\,\theta = mg + F_F\text{Sin}\,\theta = mg + \mu_S N\text{Sin}\,\theta$$

$$N\text{Cos}\,\theta - \mu_S N\text{Sin}\,\theta = mg \qquad (\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta)N = mg \qquad N = \frac{mg}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}$$

Plug N (from y-components) back in to the equation for F_C (from the x-components):

$$F_c = (\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta)N = (\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta)\left(\frac{mg}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right)$$

$$F_c = mg\left(\frac{\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right)$$

Step 3: Set $F_C = mv^2/r$: $\quad F_c = mg\left(\frac{\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right) = \frac{mv^2}{r} \qquad g\left(\frac{\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right) = \frac{v^2}{r}$

$$v^2 = rg\left(\frac{\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right) \qquad v = \sqrt{rg\left(\frac{\text{Sin}\,\theta + \mu_S\text{Cos}\,\theta}{\text{Cos}\,\theta - \mu_S\text{Sin}\,\theta}\right)}$$

Earlier we solved an unbanked turn with friction and got the maximum velocity $v = \sqrt{\mu_s g r}$.

This should be equivalent to the banked turn with friction if we let the banking angle be zero.

$$v = \sqrt{rg\left(\frac{\text{Sin } 0 + \mu_S \text{Cos } 0}{\text{Cos } 0 - \mu_S \text{Sin } 0}\right)} = \sqrt{rg\left(\frac{0 + \mu_S \cdot 1}{1 - \mu_S \cdot 0}\right)} = \sqrt{\mu_s g r}$$

Earlier we solved a banked turn without friction and got the velocity $v = \sqrt{rgTan\,\theta}$

This should be equivalent to the banked turn with friction if we let μ_S *be zero.*

$$v = \sqrt{rg\left(\frac{\text{Sin } \theta + 0 \cdot \text{Cos } \theta}{\text{Cos } \theta - 0 \cdot \text{Sin } \theta}\right)} = \sqrt{rg\left(\frac{\text{Sin } \theta}{\text{Cos } \theta}\right)} = \sqrt{rg\text{Tan } \theta}$$