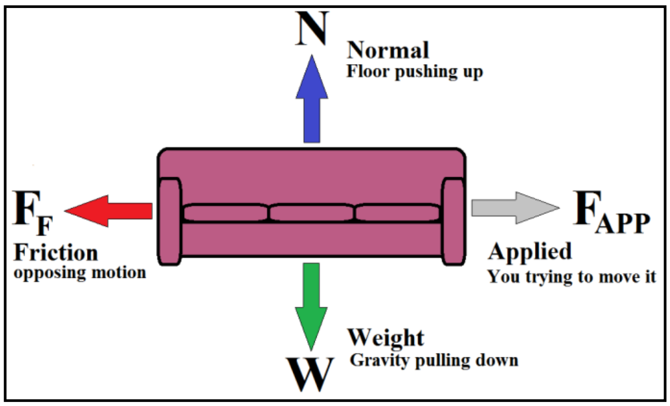
Lecture 7: Friction and Tension

Physics for Engineers & Scientists (Giancoli): Chapters 4 and 5
University Physics V1 (Openstax): Chapters 5 and 6

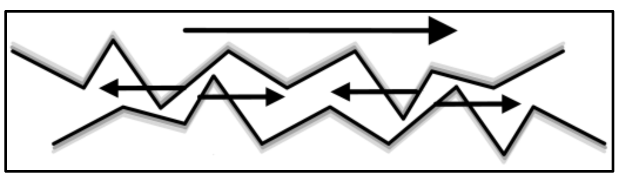
Friction

- When we push on an object (such as a couch) it won't move until a sufficient magnitude of force is applied.
- It doesn't move because the force of friction (static friction) matches the applied force and points in the opposite direction.

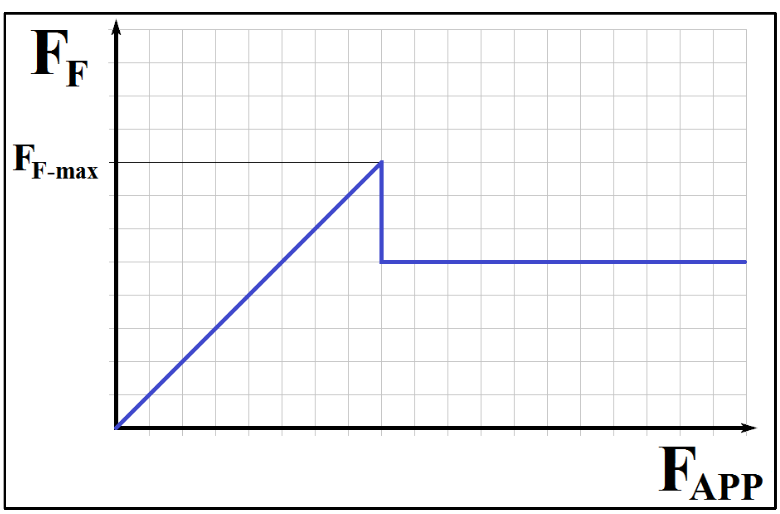
If the acceleration is zero ($a = 0$), then the net force must be zero ($F_{NET} = 0$). For the net force to be equal to zero, the force of friction must be equal in magnitude to the applied force ($F_F = F_{APP}$) and opposite in direction.



- Friction occurs because on the small scale surfaces are rough and numerous tiny normal forces appear when you try to slide the surfaces across each other.

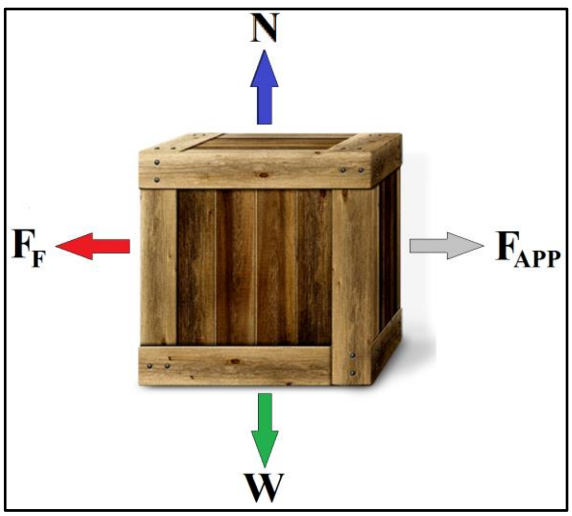


- Once sufficient force is applied to overcome static friction ($F_{F\text{-}Max}$), the force of friction takes on a constant value related to the normal force.



- Friction is parallel to the surface and always opposes motion.
- Solving problems with friction
  - If the object is known to be stationary, use $F_{NET} = 0$ to find F_F.
  - If the object is known to be moving, use $F_F = \mu_k N$
    - N is the normal force related to that point of contact.
    - μ_k is the "coefficient of kinetic friction"
  - If it is not known if the object is stationary or moving, find out.
    - Assume the object is stationary, and use $F_{NET} = 0$ to find F_F
    - Calculate $F_{F\text{-}Max} = \mu_s N$
      - N is the normal force related to that point of contact.
      - μ_s is the "coefficient of static friction"
    - If the calculated value of F_F exceeds $F_{F\text{-}Max}$ ($F_F > F_{F\text{-}Max}$), then the object is moving. The value of F_F previously calculated is not applicable. Set $F_F = \mu_k N$.
    - If the calculated value of F_F does not exceed $F_{F\text{-}Max}$ ($F_F \leq F_{F\text{-}Max}$), then the object is stationary. Keep the value of F_F previously calculated (it is applicable).

Example: A block whose weight is 45.0N rests on a horizontal table. The coefficients of static and kinetic friction are 0.650 and 0.420 respectively. A horizontal force of 36.0N is applied to the block. Will the block move under influence of the force, and if so, what will be the blocks acceleration?



Is the crate moving? We don't know.
Assume it's stationary, find F_F, and compare it to $F_{F\text{-}Max}$.

For F_{NET} to be zero, $F_F = F_{APP} = 36.0$ N
$F_{F\text{-}Max} = \mu_S N = (0.650)(45.0\text{ N}) = 29.25$ N

Is the crate moving? Yes! 36.0 N > 29.25 N
We must use kinetic friction.

$F_F = \mu_k N = (0.420)(45.0\text{ N}) = 18.9$ N
$F_{NET\text{-}x} = F_{APP} - F_F = 36.0\text{ N} - 18.9\text{ N} = 17.1$ N

$m = W/g = (45.0\text{ N})/(9.80\text{ m/s}^2) = 4.5918$ kg
$a = a_x = F_{NET\text{-}x}/m = (17.1\text{ N})/(4.5918\text{ kg}) = 3.724\text{ m/s}^2$
$\Rightarrow 3.72\text{ m/s}^2$

Example: A block of mass M = 10.0 kg rests on an incline sloped at an angle θ = 30.0°. Friction's hold on the box is tentative, and the slightest touch will cause it to slide down the incline. Determine the force of friction, the normal force, and the coefficient of static friction.

First, 'sliding at the slightest touch' means $F_F = F_{F\text{-}max}$.
We will solve this problem three different ways.

Method 1: Brute force

Start by making a force diagram and breaking each vector into components.
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)

The angle of N is defined with respect to the y-axis.
You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$\Sigma F_x = 0 \Rightarrow F_{Fx} = N_x \quad F_F \cdot Cos(\theta) = N \cdot Sin(\theta)$

$\mu_S \cdot N \cdot Cos(\theta) = N \cdot Sin(\theta)$

$\mu_S = N \cdot Sin(\theta)/N \cdot Cos(\theta)$

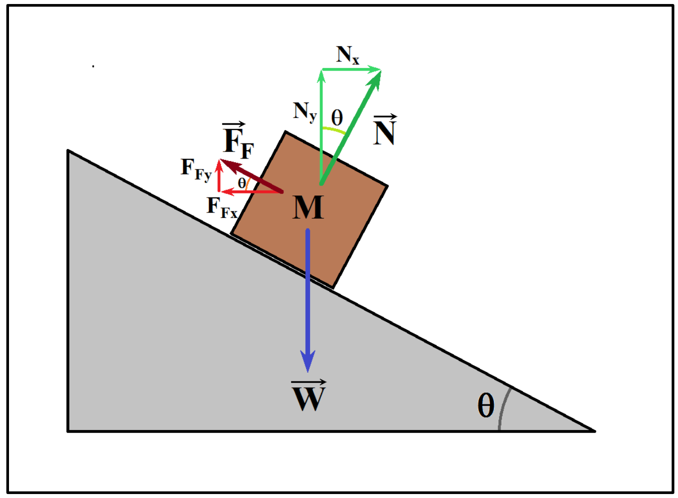
$\mu_S = Tan(\theta) = Tan(30.0°) = 0.57735 \Rightarrow 0.577$

$\Sigma F_y = 0 \Rightarrow F_{Fy} + N_y = W$

$F_F \cdot Sin(\theta) + N \cdot Cos(\theta) = Mg$

$\mu_S \cdot N \cdot Sin(\theta) + N \cdot Cos(\theta) = Mg$

$N \cdot [\mu_S \cdot Sin(\theta) + Cos(\theta)] = Mg$



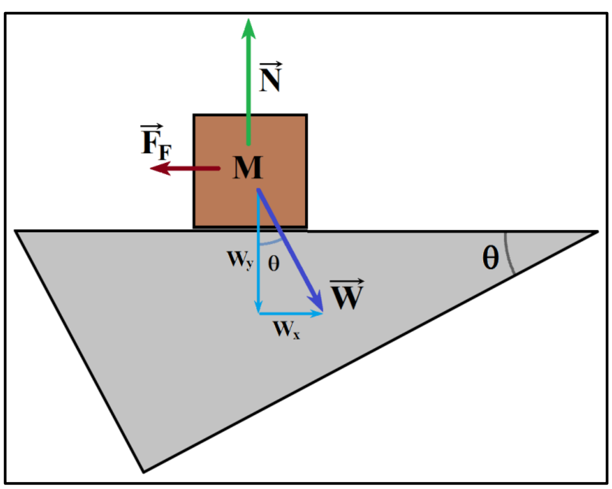
$N = Mg/[\mu_S \cdot Sin(\theta) + Cos(\theta)]$

$N = (10.0\ kg)(9.80\ m/s^2)[(0.57735) \cdot Sin(30.0°) + Cos(30.0°)] = 84.8705\ N \Rightarrow 84.9\ N$

$F_F = \mu_S \cdot N = (0.57735)(84.8705\ N) = 49.0\ N$

<u>Method 2</u>: Use a Different (Rotated) Reference Frame.

Start by making a force diagram and breaking each vector into components.
However, this time let the x-axis run parallel to the slope and the y-axis perpendicular.
This only leaves one vector (W) to break into components instead of two (N and F_F).
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)



$\Sigma F_x = 0 \Rightarrow F_F = W_x = mg \cdot Sin(\theta)$

The angle of W is defined with respect to the y-axis.
You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$F_F = (10.0\ kg)(9.80\ m/s^2) \cdot Sin(30.0°) = 49.0\ N$

$\Sigma F_y = 0 \Rightarrow N = W_y = mg \cdot Cos(\theta)$

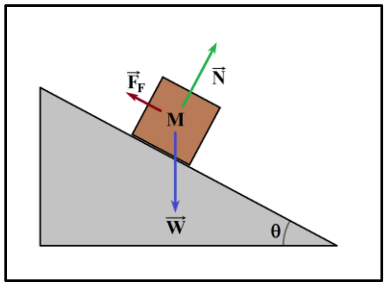
The angle of W is defined with respect to the y-axis.
You must use Cos(θ) or use geometry to find the angle with respect to the positive x-axis.

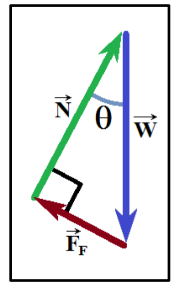
$N = (10.0\ kg)(9.80\ m/s^2) \cdot Cos(30.0°) = 84.8705\ N \Rightarrow 84.9\ N$

$F_F = \mu_S \cdot N \quad \mu_S = F_F/N = [mg \cdot Sin(\theta)] / [mg \cdot Cos(\theta)] = Tan(\theta) = Tan(30.0°) = 0.57735 \Rightarrow 0.577$

Method 3: Trigonometry

Start by making a force diagram. Upon seeing that 3 forces sum to zero, make a triangle out of them. Upon finding N and F_F, use $F_F = \mu_S N$ to get μ_S.





$$N = W\cdot\text{Cos}(\theta) = mg\cdot\text{Cos}(\theta) = (10.0\text{ kg})(9.80\text{ m/s}^2)\cdot\text{Cos}(30.0°) = 84.8705\text{ N} \Rightarrow 84.9\text{ N}$$

$$F_F = W\cdot\text{Sin}(\theta) = mg\cdot\text{Sin}(\theta) = (10.0\text{ kg})(9.80\text{ m/s2})\cdot\text{Sin}(30.0°) = 49.0\text{ N}$$

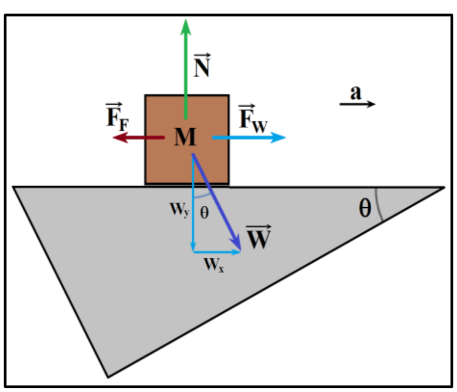
$$\mu_S = F_F/ N = \text{Tan}(\theta) = \text{Tan}(30.0°) = 0.57735 \Rightarrow 0.577$$

On Solving Problems

- Solving the problem by making a triangle was the easiest solution, but this doesn't always work.
  - This can only be done when you can make a triangle.
  - It may not work in non-equilibrium or when a 4^{th} force is present.
- Solving the problem by aligning the coordinate axes with the incline was easier.
  - It's only easier when more vectors align with these coordinate axes.
- Brute force is often the most difficult option, but it always works.
- Find the easiest way to solve each problem!

Example 21: A girl is skiing down a slope that is 30.0° with respect to the horizontal. A moderate wind is aiding the motion by providing a steady force of 105N that is parallel to the motion of the skier. The combined mass of the girl and skis is 65.0kg and the coefficient of kinetic friction between the skis and the snow is 0.150. How much time is required for the skier to travel down a 175m slope, starting from rest?

Start with a force diagram. Laying it flat leaves only one vector (W) to be broken into components. Use the force information to determine the acceleration and then do the kinematics.



This is a non-equilibrium situation $\Rightarrow \Sigma F_x = Ma \quad \Sigma F_y = 0$

$\Sigma F_y = 0 \Rightarrow N = W_y = W\cdot\text{Cos}(\theta) = Mg\cdot\text{Cos}(\theta)$

$F_F = \mu_k N = \mu_k Mg\cdot\text{Cos}(\theta)$

$\Sigma F_x = Ma \quad \Rightarrow \quad F_W + W_x - F_F = Ma$

$F_W + Mg\cdot\text{Sin}(\theta) - \mu_k Mg\cdot\text{Cos}(\theta) = Ma$

$a = F_W/M + g\cdot\text{Sin}(\theta) - \mu_k g\cdot\text{Cos}(\theta)$

$$a = (105\text{ N})/(65.0\text{ kg})+(9.80\text{ m/s}^2)\cdot\text{Sin}(30°)-(0.150)(9.80\text{ m/s}^2)\cdot\text{Cos}(30°) = 5.24233\text{ m/s}^2$$

Extract kinematic data: Let $x_0 = 0$ $x = 175$ m $v_0 = 0$ $v =$ $a = 5.24233$ m/s^2 $t = ???$

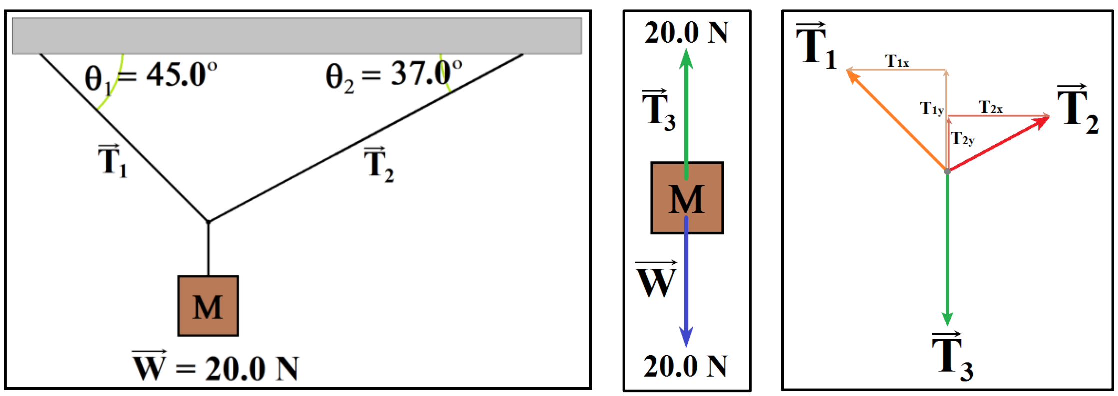
$$x = x_0 + v_0 t + ½·at^2 = ½·at^2 \qquad 2x = at^2 \qquad 2x/a = t^2$$

$$t = (2x/a)^{1/2} = [2(175m)/(5.24233)]^{1/2} = 8.17094 \text{ s} \Rightarrow 8.17 \text{ s}$$

Tension

- Tension is a force that is transmitted along the length of an object, often a flexible object such as a rope, string, or chain.
- Tension always "pulls" and never "pushes".
- The force is typically applied at the connections at each end, pulling along the length of the object.
- The force of tension is felt at each point in the interior of the object (pulled both ways)
- The direction this force is felt can be altered by pulleys. If the pulley has negligible mass and friction the tension is unaltered.

Example: A mass weighing 20.0N hangs from a system of strings as shown. Determine the tensions, T_1 and T_2.



Start by making a force diagram for the hanging mass, labeling the tension in the line above it 'T_3'.
Then make a force diagram at the point of intersection of the three tensions.
Break T_1 and T_2 into components.
This is static equilibrium so the forces must cancel. ***$\Sigma F_x = 0$ $\Sigma F_y = 0$***

Hanging Mass: $\Sigma F_y = 0 \Rightarrow T_3 = W = 20.0$ N

Intersection (x-comp): $\Sigma F_x = 0 \Rightarrow T_{1x} = T_{2x}$ $T_2\text{Cos}(37.0°) = T_1\text{Cos}(45.0°)$ $T_2 = 0.885394·T_1$

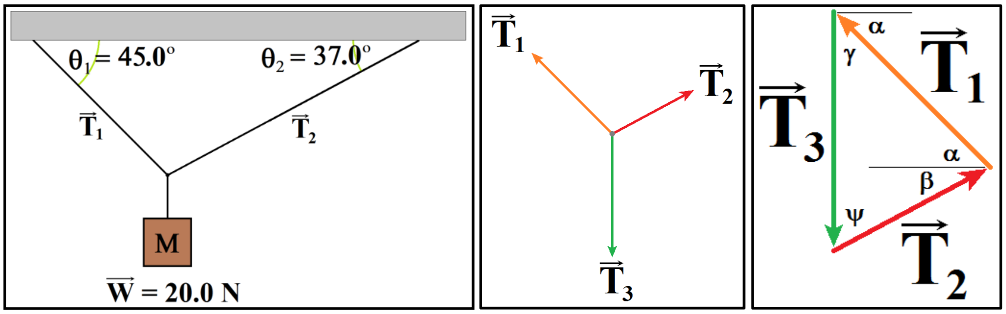
Intersection (y-comp): $\Sigma F_y = 0 \Rightarrow T_3 = T_{1y} + T_{2y}$ $T_3 = T_1\text{Sin}(45.0°) + T_2\text{Sin}(37.0°)$

$$T_3 = T_1\text{Sin}(45.0°) + 0.885394·T_1\text{Sin}(37.0°) = T_1[\text{Sin}(45.0°) + 0.885394·\text{Sin}(37.0°)]$$

$$20.0 \text{ N} = 1.23995·T_1 \qquad T_1 = 16.1297 \text{ N} \Rightarrow 16.1 \text{ N}$$

$$T_2 = 0.885394·T_1 = 0.885394·16.1297 \text{ N} = 14.3 \text{ N}$$

Alternatively, we could have made a triangle and used the law of sines.



$\alpha = 45.0°$ (Given) $\beta = 37.0°$ (Given) $\gamma = 90° - \alpha = 45.0°$ $\psi = 90° - \beta = 53.0°$

$$\frac{\text{Sin}\,\psi}{T_1} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\psi = T_1 \cdot \text{Sin}\,(\alpha+\beta)$$

$$T_1 = \frac{T_3 \cdot \text{Sin}\,\psi}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(82.0°)} = 16.1297\text{ N} \Rightarrow 16.1\text{ N}$$

$$\frac{\text{Sin}\,\gamma}{T_2} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\gamma = T_2 \cdot \text{Sin}\,(\alpha+\beta)$$

$$T_2 = \frac{T_3 \cdot \text{Sin}\,\gamma}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(82.0°)} = 14.2811\text{ N} \Rightarrow 14.3\text{ N}$$