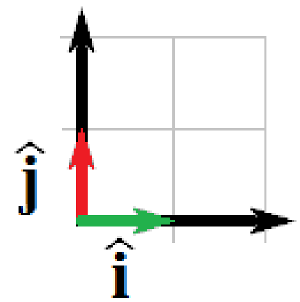
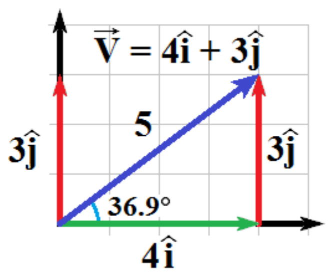
Lecture 7: Vectors

Physics for Engineers & Scientists (Giancoli): Chapter 3
University Physics V1 (Openstax): Chapter 2

Unit Vectors

- Unit vectors are vectors of magnitude 1. Usually these vectors align with coordinate axes.
- Multiplying a unit vector by a scalar creates a vector with length equal to the scalar pointing along a coordinate axis.
- Any vector may be represented as the sum of it's horizontal and vertical components multiplied by their respective unit vectors. $\vec{V} = V\angle\theta_V = V_x\hat{\imath} + V_y\hat{\jmath}$





For example,

$$\vec{V} = V\angle\theta_V = (5.00\text{ m})\angle 36.9° = (4.00\text{ m})\hat{\imath} + (3.00\text{m})\hat{\jmath}$$

- This creates a convenient notation for vector addition.

$$\vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) = (A_x\hat{\imath} + B_x\hat{\imath}) + (A_y\hat{\jmath} + B_y\hat{\jmath})$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} = C_x\hat{\imath} + C_y\hat{\jmath} = \vec{C}$$

Example: Find the resultant of 3 displacement vectors by means of the component method. The three vectors are $\vec{A} = (5.00\text{ m})\angle 160°$, $\vec{B} = (5.00\text{ m})\angle 60°$, and $\vec{C} = (4.00\text{ m})\angle 270°$.

$$\vec{A} = (5.00\text{ m})\angle 160° = (5.00\text{ m})\text{Cos}(160°)\hat{\imath} + (5.00\text{ m})\text{Sin}(160°)\hat{\jmath} = (-4.70\text{ m})\hat{\imath} + (1.71\text{ m})\hat{\jmath}$$

$$\vec{B} = (5.00\text{ m})\angle 60° = (5.00\text{ m})\text{Cos}(60°)\hat{\imath} + (5.00\text{ m})\text{Sin}(60°)\hat{\jmath} = (2.50\text{ m})\hat{\imath} + (4.33\text{ m})\hat{\jmath}$$

$$\vec{C} = (4.00\text{ m})\angle 270° = (4.00\text{ m})\text{Cos}(270°)\hat{\imath} + (4.00\text{ m})\text{Sin}(270°)\hat{\jmath} = (-4.00\text{m})\hat{\jmath}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = \underbrace{\{(-4.70\text{ m})\hat{\imath} + (1.71\text{ m})\hat{\jmath}\}}_{\vec{A}} + \underbrace{\{(2.50\text{ m})\hat{\imath} + (4.33\text{ m})\hat{\jmath}\}}_{\vec{B}} + \underbrace{\{(-4.00\text{m})\hat{\jmath}\}}_{\vec{C}}$$

$$\vec{R} = \{(-4.70\text{ m}) + (2.50\text{ m})\}\hat{\imath} + \{(1.71\text{ m}) + (4.33\text{ m}) + (-4.00\text{m})\}\hat{\jmath}$$

$$\vec{R} = (-2.20\text{ m})\hat{\imath} + (2.04\text{ m})\hat{\jmath}$$

$$R = |\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.20\text{ m})^2 + (2.04\text{ m})^2} = 3.00\text{ m}$$

$$\theta_R = \text{Tan}^{-1}\left(\frac{R_y}{R_x}\right) + 180° = \text{Tan}^{-1}\left(\frac{2.04\text{ m}}{-2.20\text{ m}}\right) + 180° = -42.8° + 180° = 137.2°$$

$$\vec{R} = (-2.20\text{ m})\hat{\imath} + (2.04\text{ m})\hat{\jmath} = (3.00\text{ m})\angle 137.2°$$

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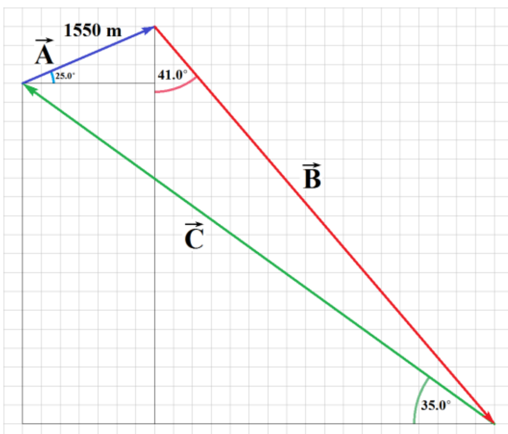
Example: The route followed by a hiker consists of 3 displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. Vector $\vec{A}$ is along a measured trail and is 1550m in a direction 25.0° north of east. Vector $\vec{B}$ is not along a measured trail but the hiker uses a compass and knows that the direction is 41.0° east of south. Similarly the direction of vector $\vec{C}$ is 35° north of west. The hiker ends up back where she started, so the resulting displacement is zero (or $\vec{A} + \vec{B} + \vec{C} = 0$). Find the magnitude of vector $\vec{B}$ and vector $\vec{C}$.

We can solve this is two different ways. The first option, by using components, requires one to solve a system of two equations. The second option, using the law of sines, requires a good bit of geometry. We will solve this one both ways.

First make a diagram (not easy when most lengths aren't given).

Solution 1 (by components)

Note: The angles given for $\vec{B}$ and $\vec{C}$ are not defined with respect to the positive x-axis. You must make triangles and use trigonometry to find components.



A_x = (1550 m)Cos(25.0°) = 1404.8 m

A_y = (1550 m)Sin(25.0°) = 655.1 m

B_x = B·Sin(41.0°) *{B_x is the opposite side}*

B_y = -B·Cos(41.0°) *{B_y is pointed down}*

C_x = -C·Cos(35.0°) *{C_x is pointed left}*

C_y = C·Sin(35.0°)

$$A_x + B_x + C_x = 0$$

$$1404.8\text{ m} + B\cdot\sin(41.0°) - C\cdot\cos(35.0°) = 0$$

$$A_y + B_y + C_y = 0$$

$$655.1\text{ m} - B\cdot\cos(41.0°) + C\cdot\sin(35.0°) = 0$$

2 equations, 2 unknowns (B and C).

Multiply the first equation by Sin (35.0°), multiply the second equation by Cos (35.0°), and then add he two equations. The C terms will disappear.

$$(1404.8\text{ m})\cdot\sin(35.0°) + B\cdot\sin(41.0°)\cdot\sin(35.0°) - C\cdot\cos(35.0°)\cdot\sin(35.0°) = 0$$
$$\underline{(655.1\text{ m})\cdot\cos(35.0°) - B\cdot\cos(41.0°)\cdot\cos(35.0°) + C\cdot\cos(35.0°)\cdot\sin(35.0°) = 0}$$
$$(1404.8\text{ m})\cdot\sin(35.0°) + (655.1\text{ m})\cdot\cos(35.0°) + B\cdot\sin(41.0°)\cdot\sin(35.0°) - B\cdot\cos(41.0°)\cdot\cos(35.0°) = 0$$

$$(1404.8\text{ m})\cdot\sin(35.0°) + (655.1\text{ m})\cdot\cos(35.0°) = B\cdot\cos(41.0°)\cdot\cos(35.0°) - B\cdot\sin(41.0°)\cdot\sin(35.0°)$$

$$(1404.8\text{ m})\cdot\sin(35.0°) + (655.1\text{ m})\cdot\cos(35.0°) = B\cdot\{\cos(41.0°)\cdot\cos(35.0°) - \sin(41.0°)\cdot\sin(35.0°)\}$$

$$B = \{(1404.8\text{ m})\cdot\sin(35.0°) + (655.1\text{ m})\cdot\cos(35.0°)\}/\{\cos(41.0°)\cdot\cos(35.0°) - \sin(41.0°)\cdot\sin(35.0°)\}$$

$$B = \{805.76\text{ m} + 536.63\text{ m}\}/\cos(76.0°) = 1342.39\text{ m}/\cos(76.0°) = 5548.86\text{ m} \Rightarrow 5550\text{ m}$$

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Solve first equation for C.

$$1404.8 \text{ m} + B \cdot \text{Sin}(41.0°) - C \cdot \text{Cos}(35.0°) = 0$$

$$1404.8 \text{ m} + (5548.86 \text{ m}) \cdot \text{Sin}(41.0°) - C \cdot \text{Cos}(35.0°) = 0$$

$$1404.8 \text{ m} + 3640.4 = C \cdot \text{Cos}(35.0°)$$

$$5045.2 \text{ m} = C \cdot \text{Cos}(35.0°)$$

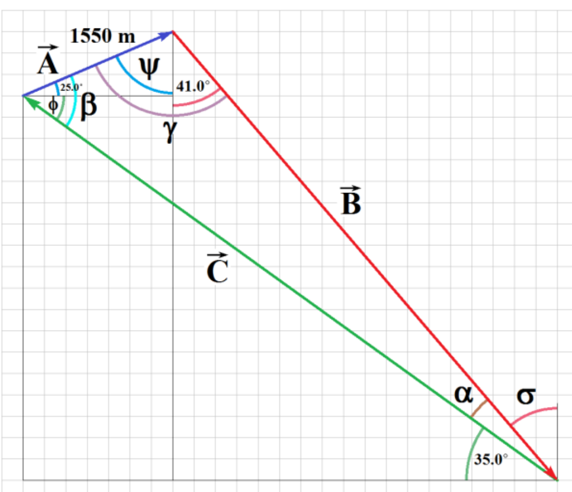
$$C = 5045.2 \text{ m}/\text{Cos}(35.0°) = 6159.1 \text{ m} \Rightarrow 6160 \text{ m}$$

$$B = 5550 \text{ m} \quad \text{and} \quad C = 6160 \text{ m}$$

Solution 2 (law of sines)

To use the law of sines we need one length (which we have from side A), and all the interior angles. So, first we need to find α, β, and γ.

Start by adding some extra angles to our diagram.



$\beta = 25.0° + \phi = 25.0° + 35.0° = 60.0°$

$\psi = 90° - 25.0° = 65.0°$ *{Interior angles}*
$\gamma = \psi + 41.0° = 65.0° + 41.0° = 106.0°$

$\sigma = 41.0°$ *{Vertical lines are parallel}*
$\alpha = 90° - 35.0° - 41.0° = 14.0°$

Check: $60.0° + 106.0° + 14.0° = 180.0°$

$$\frac{\text{Sin}\,\alpha}{A} = \frac{\text{Sin}\,\beta}{B} = \frac{\text{Sin}\,\gamma}{C}$$

$B \cdot \text{Sin}(\alpha) = A \cdot \text{Sin}(\beta)$
$B = A \cdot \text{Sin}(\beta)/\text{Sin}(\alpha) = (1550 \text{ m}) \cdot \text{Sin}(60.0°)/\text{Sin}(14.0°)$
$B = 5548.65 \text{ m} \Rightarrow 5550 \text{ m}$

$C \cdot \text{Sin}(\alpha) = A \cdot \text{Sin}(\gamma)$
$C = A \cdot \text{Sin}(\gamma)/\text{Sin}(\alpha) = (1550 \text{ m}) \cdot \text{Sin}(106.0°)/\text{Sin}(14.0°)$
$C = 6158.83 \text{ m} \Rightarrow 6160 \text{ m}$

$\phi = 35.0°$ *{Horizontal lines are parallel}*

It is usually a good idea to determine what method of solving a problem is the most simple and expedient.

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