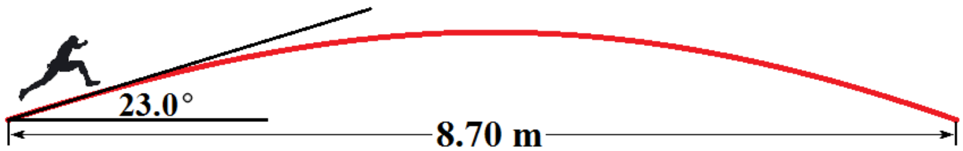
Lecture 6: Dynamics, Weight, and Normal Forces

Physics for Engineers & Scientists (Giancoli): Chapters 3 and 4
University Physics V1 (Openstax): Chapters 4 and 5

Example: An Olympic jumper leaves the ground at an angle of 23.0° and travels through the air for a horizontal distance of 8.70 m before landing. What is the takeoff speed of the jumper?



$x_o = 0$ $x = 8.70m$ $a_x = 0$ $v_{ox} = v_x = v_o Cos(23.0°)$

$y_o = 0$ $y = 0$ $v_{oy} = v_o Sin(23.0°)$ $v_y = -v_{oy} = -v_o Sin(23.0°)$ $a_y = -g$

The x and y equations are linked via t (so we need equations with t), but we have 2 unknowns (v_0 and t). Need 2 equations (x-comp and y-comp) that have both v_0 and t.

x-components: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t = v_0 Cos(23.0°)t$ $t = \frac{x}{v_0 Cos(23.0°)}$

y-components: $v_y = v_{0y} + a_y t$ $-v_0 Sin(23.0°) = v_0 Sin(23.0°) - gt$

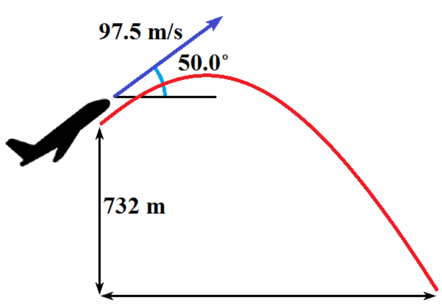
$-2v_0 Sin(23.0°) = -gt$ $2v_0 Sin(23.0°) = gt$

Plug in t (from x-comp): $2v_0 Sin(23.0°) = g\left(\frac{x}{v_0 Cos(23.0°)}\right)$ $2v_0 Sin(23.0°) = \frac{gx}{v_0 Cos(23.0°)}$

$v_0 = \frac{gx}{v_0 2Sin(23.0°)Cos(23.0°)}$ $v_0^2 = \frac{gx}{2Sin(23.0°)Cos(23.0°)}$

$$v_0 = \sqrt{\frac{gx}{2Sin(23.0°)Cos(23.0°)}} = \sqrt{\frac{\left(9.80\frac{m}{s^2}\right)(8.70\ m)}{2Sin(23.0°)Cos(23.0°)}} = 10.887\frac{m}{s} \Rightarrow 10.9\frac{m}{s}$$

Example: An airplane with a speed of 97.5m/s is climbing upwards at an angle of 50.0° with respect to the horizontal. When the planes altitude is 732m the pilot releases a package. (a) Calculate the distance along the ground measured from a point directly beneath the point of release, to where the package hits the earth. (b) Relative to the ground, determine the angle of the velocity vector just before impact.



$x_0 = 0$ $x = ???$ $a_x = 0$

$v_{0x} = (97.5\ m/s)Cos(50.0°) = 62.6718\ m/s$

$v_x = v_{0x} = 62.6718\ m/s$

$y_0 = 732\ m$ $y = 0$

$v_{0y} = (97.5\ m/s)Sin(50.0°) = 74.6893\ m/s$

$v_y =$ $a_y = -g$ $t =$

x-components (find x): $x = x_0 + v_x t = v_x t$ *{Need t.}*

y-components (find t, no v): $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ *{Solve with quadratic equation}*

$0 = 732\ m + \left(74.6893\frac{m}{s}\right)t - \frac{1}{2}\left(9.80\frac{m}{s^2}\right)t^2$ $0 = 732\ m + \left(74.6893\frac{m}{s}\right)t - \left(4.90\frac{m}{s^2}\right)t^2$

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$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-74.6893\frac{m}{s} \pm \sqrt{\left(74.6893\frac{m}{s}\right)^2 - 4\left(-4.90\frac{m}{s^2}\right)(732\ m)}}{2\left(-4.90\frac{m}{s^2}\right)}$$

$$t = \frac{-74.6893\frac{m}{s} \pm \sqrt{5578.4915\frac{m^2}{s^2} + 14347.2\frac{m^2}{s^2}}}{-9.80\frac{m}{s^2}} \qquad t = \frac{-74.6893\frac{m}{s} \pm \sqrt{19925.6915\frac{m^2}{s^2}}}{-9.80\frac{m}{s^2}}$$

$$t = \frac{-74.6893\frac{m}{s} \pm 141.1584\frac{m}{s}}{-9.80\frac{m}{s^2}} \qquad t_1 = \frac{-74.6893\frac{m}{s} + 141.1584\frac{m}{s}}{-9.80\frac{m}{s^2}} = \frac{66.4691\frac{m}{s}}{-9.80\frac{m}{s^2}} = -6.78256\ s$$

$$t_2 = \frac{-74.6893\frac{m}{s} - 141.1584\frac{m}{s}}{-9.80\frac{m}{s^2}} = \frac{-215.8477\frac{m}{s}}{-9.80\frac{m}{s^2}} = 22.0253\ s$$

Now plug the value of t into the original equation to get x:

$$x = v_x t = \left(62.6718\frac{m}{s}\right)(22.0253\ s) = 1380.36 \Rightarrow 1380\ m$$

The value of t can also be used to get v_y, which is needed to produce the velocities angle (part b).

$$v_y = v_{0y} + a_y t = \left(74.6893\frac{m}{s}\right) + \left(-9.80\frac{m}{s^2}\right)(22.0253\ s) = -141.159\frac{m}{s}$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{v_y}{v_x}\right) = \mathrm{Tan}^{-1}\left(\frac{-141.159\frac{m}{s}}{62.6718\frac{m}{s}}\right) = -66.0597° \Rightarrow -66.1°$$

Alternatively (instead of using quadratic equation) find v first, then t.

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = \left(74.6893\frac{m}{s}\right)^2 + 2\left(-9.80\frac{m}{s^2}\right)(0 - 732\ m) = 19925.69\frac{m^2}{s^2}$$

$$v_y = -\sqrt{19925.69\frac{m^2}{s^2}} = -141.158\frac{m}{s} \qquad v_y = v_{0y} + a_y t \qquad v_y - v_{0y} = a_y t = -gt$$

$$t = \frac{v_y - v_{0y}}{-g} = \frac{-141.158\frac{m}{s} - 74.6893\frac{m}{s}}{-9.80\frac{m}{s^2}} = 22.0252\ s$$

$$x = v_x t = \left(62.6718\frac{m}{s}\right)(22.0252\ s) = 1380.36 \Rightarrow 1380\ m$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{v_y}{v_x}\right) = \mathrm{Tan}^{-1}\left(\frac{-141.158\frac{m}{s}}{62.6718\frac{m}{s}}\right) = -66.0596° \Rightarrow -66.1°$$

If the quadratic equation gives you trouble, you can find your answer in another way!

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Forces

- Intuitively, a force is a push or a pull.
- Forces are vectors with both magnitude and direction.
- Contact forces are created when two objects are in physical contact.
- Non-contact forces are felt between objects that are not in contact.

Mass is a measure of the amount of matter in an object.

Newton's Laws of Motion

- **Newton's 1st Law of Motion**: **"The Law of Inertia"**

  Every object continues in its state of rest, or of uniform speed in a straight line, as long as no net force acts on it.

  - The tendency for an object to move at a constant speed in a straight line is called inertia.
  - Mass is a qualitative measure of the inertia of an object.
  - Velocity is a vector. A change in the speed or direction of motion is a change in inertia.
  - If something is speeding up, slowing down or changing direction Newton's First Law says there is an external FORCE acting on it.
  - An inertial reference frame is a coordinate system that obeys Newton's First Law. Any reference frame moving with constant velocity (or stationary) with respect to an inertia reference frame is also an inertia reference frame

- **Newton's 2nd Law of Motion**: "**The Law of Force and Acceleration.**"

  The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportionally to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

  - The standard SI unit of mass is the kilogram (kg)
  - The standard SI unit of force is the Newton (N): $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$.
  - This is a vector equation.
    1) The acceleration points in the same direction as the sum of the forces.
    2) The magnitude of the net force equals the mass multiplied by the magnitude of the acceleration.
  - There are two cases
    1) In static equilibrium the acceleration (and the sum of the forces) is zero.
    2) In non-equilibrium the acceleration is not zero (and neither is the sum of the forces)

  Newton's 2nd law acts as a link between force problems and kinematic problems.

- **Newton's 3rd Law of Motion**: Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction.

  "For every action, there is an equal and opposite reaction."

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The force that a nail experiences when hit with a hammer (driving it into the wood) is equal in magnitude and opposite in direction to the force that the hammer experiences from being hit by the nail, slowing the hammer down.

Example: An F-14 has a mass of $3.1\text{x}10^4$ kg and takes off under the influence of a constant net force of $3.7\text{ x}10^4$ N. What is the net force that acts upon the 78 kg pilot?

The forces and accelerations in this problem are collinear. We may treat it 1-dimensionally.

$$\sum \vec{F} = m\vec{a} \qquad F_{F-14} = m_{F-14} \cdot a \qquad a = \frac{F_{F-14}}{m_{F-14}} = \frac{3.7\times10^4\text{ N}}{3.1\times10^4\text{ kg}} = 1.19355\text{ m/s}^2$$

$$F_{Pilot} = m_{Pilot} \cdot a = (78\text{ kg})\left(1.19355\frac{\text{m}}{\text{s}^2}\right) = 93.0969\text{ N} \Rightarrow 93\text{ N}$$

Example: When a 58g tennis ball is served, it accelerates from rest to a speed of 45m/s. The impact of the racket gives the ball a constant acceleration over a distance of 44cm. What is the magnitude of the net force acting on the ball?

The forces and accelerations in this problem are collinear. We may treat it 1-dimensionally.

The acceleration is needed to solve this problem (F=ma). We can find the acceleration from kinematics.

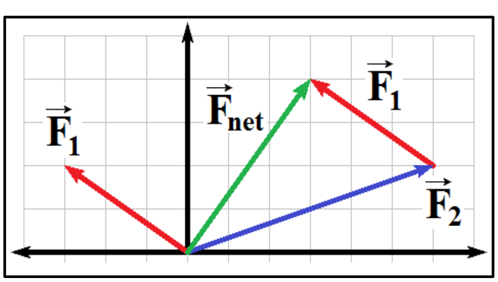
$$x_0 = 0 \qquad x = 44\text{ cm} = 0.44\text{ m} \qquad v_0 = 0 \qquad v = 45\text{ m/s} \qquad a = ???$$

$$v^2 = v_0^2 + 2a(x - x_0) = 2ax \qquad a = \frac{v^2}{2x} = \frac{\left(45\frac{\text{m}}{\text{s}}\right)^2}{2(0.44\text{ m})} = 2301.14\frac{\text{m}}{\text{s}^2}$$

$$F = ma = (0.058\text{ kg})\left(2301.14\frac{\text{m}}{\text{s}^2}\right) = 133.466\text{ N} \Rightarrow 130\text{ N}$$

Example: Two forces, $\vec{F}_1 = (-3\text{ N})\hat{\imath} + (2\text{ N})\hat{\jmath}$ and $\vec{F}_2 = (6\text{ N})\hat{\imath} + (2\text{ N})\hat{\jmath}$ are acting on an object with a mass of 1 kg. What is the magnitude of that object's acceleration?

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \underbrace{(-3\text{ N})\hat{\imath} + (2\text{ N})\hat{\jmath}}_{\vec{F}_1} + \underbrace{(6\text{ N})\hat{\imath} + (2\text{ N})\hat{\jmath}}_{\vec{F}_2} = (3\text{ N})\hat{\imath} + (4\text{ N})\hat{\jmath} = (5\text{ N})\angle 53.13°$$



$$|\vec{F}_{net}| = \sqrt{F_{net-x}^2 + F_{net-y}^2} = \sqrt{(3\text{ N})^2 + (4\text{ N})^2} = 5\text{ N}$$

$$\theta = \text{Tan}^{-1}\left(\frac{F_{net-y}}{F_{net-x}}\right) = \text{Tan}^{-1}\left(\frac{4\text{ N}}{3\text{ N}}\right) = 53.13°$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{(5\text{ N})\angle 53.13°}{1\text{ kg}} = \left(5\frac{\text{m}}{\text{s}^2}\right)\angle 53.13°$$

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Example: A marksman fires a rifle. The 9.50 gram bullet accelerates from rest to 851 m/s in 1.31 ms at which point it leaves the barrel of the rifle. What is the average recoil force on the gun from the bullet?

Determine the force on the bullet from its acceleration.
The magnitude of the force on the rifle is the same.

$x_0 = 0$ $x =$ $v_0 = 0$ $v = 851$ m/s $a = ???$ $t = 0.00131$ s

$v = v_0 + at = at$ $a = v/t = (851 \text{ m/s})/(0.00131 \text{ s}) = 649{,}618 \text{ m/s}^2$

$F = ma = (0.00950 \text{ kg})(649{,}618 \text{ m/s}^2) = 6171.37 \text{ N} \Rightarrow 6.17 \text{ kN}$

Weight

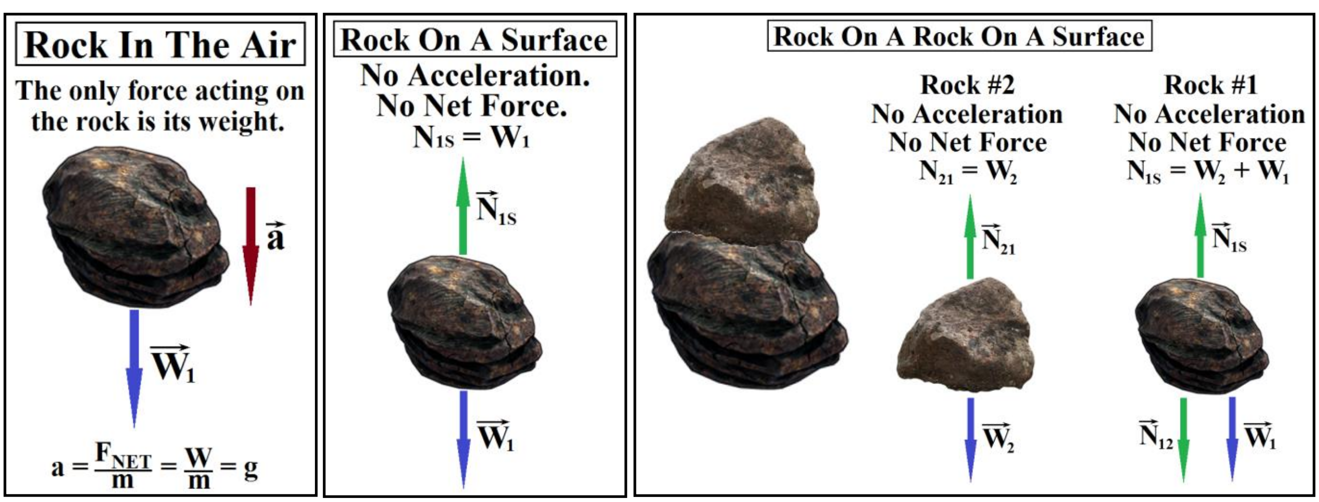
- Objects accelerate downward due to the gravitational force from the Earth.
- This force from gravity is called weight (W). $\vec{W} = m\vec{g}$
- Be careful not to confuse weight (a force) with mass (not a force).
  - The mass of objects in space doesn't change, but they have no weight.
  - On the moon, the gravitational acceleration is $g = 1.622 \text{ m/s}^2$. Masses are the same as they are on Earth, but weights are only a sixth of that on Earth.

Normal Forces

- When two objects come in contact with each other, each surface repels the other.
- As these forces are directed perpendicular to the surface, they are called "Normal Forces."
- The strength of a given normal force is dependent upon the circumstances. Its value can change as the circumstances change.

Force Diagrams

- To account for the various forces acting objects, we typically make a force diagram showing the forces acting on each object individually.



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Example: A large ceramic planter when filled with dirt has a mass of 86.0 kg. A second identical dirt-filled planter is placed on top of it. A third dirt-filled planter, which has a mass of 16.5 kg, is placed on top. Determine A) the normal force of the floor on the bottom planter, and B) the normal force the second planter exerts on the dirt at the top of the bottom planter.

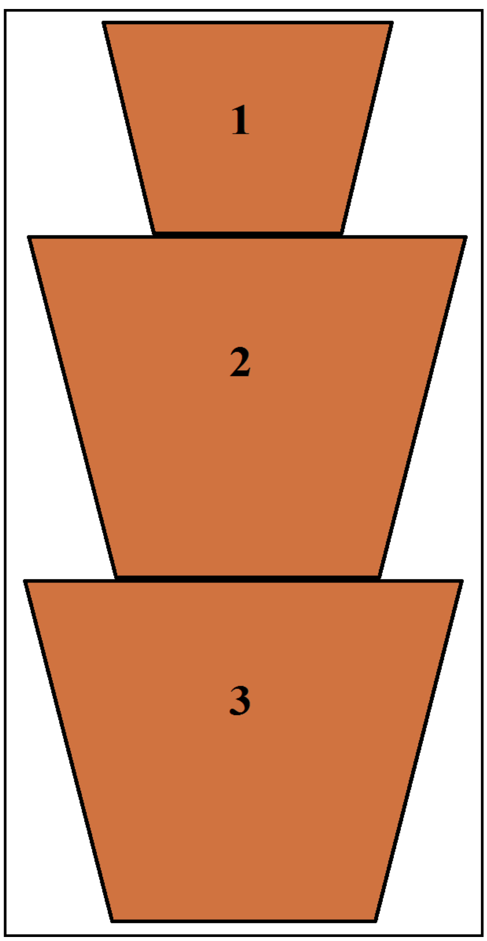
- Start by making a diagram of the problem.
- Next make a force diagram for each object in the problem.
  - Force diagrams must include every force that acts upon that object.
  - All objects will have weight in a gravitational field
  - At every point of contact between objects, each object experiences a normal force repelling it from the other. These are equal and opposite.

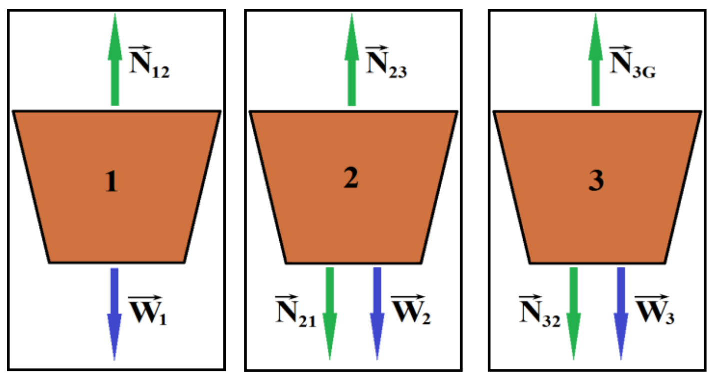
An object in contact with more than one object will experience more than one normal force.

  - At every point of contact between objects, each object experiences a friction force opposing its motion. These are also equal and opposite.

An object in contact with more than one object may experience more than one friction force.

  - Any ropes, cords, chains, etc. will produce a force called tension (discussed later)
  - Any other applied forces must also be included.
- In this problem, only vertical forces are relevant. The horizontal friction forces are zero as there are no other horizontal forces.





Starting with the top planter: Vertical forces must sum to zero.

$$N_{12} = W_1 = m_1g = (16.5\text{ kg})(9.80\text{ m/s}^2) = 161.7\text{ N}$$

Middle planter: Vertical forces must sum to zero.

The top planter creates a normal force pushing down.

$$N_{23} = N_{21} + W_2 = N_{12} + m_2g = 161.7\text{ N} + (86.0\text{ kg})(9.80\text{ m/s}^2) = 161.7\text{ N} + 842.8\text{ N} = 1004.5\text{ N}$$

Bottom Planter: Vertical forces must sum to zero.

The middle planter creates a normal force pushing down.

$$N_{3G} = N_{32} + W_3 = N_{23} + m_3g = 1004.5\text{ N} + (86.0\text{ kg})(9.80\text{ m/s}^2) = 1004.5\text{ N} + 842.8\text{ N} = 1847.3\text{ N}$$

A) $N_{3G} = 1847.3$ N B) $N_{32} = N_{23} = 1004.5$ N

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