Lecture 5: Graphical Analysis

Physics for Engineers & Scientists (Giancoli): Chapter 2
University Physics V1 (Openstax): Chapter 3

Example: A wrecking ball is hanging at rest from a crane when suddenly the cable breaks. The times it takes the ball to fall halfway to the ground is 1.2s. Find the time it takes for the ball to fall from rest all the way to the ground.

- Diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data:
  - At every location: $a = -9.80\ m/s^2$
  - At the top: $t = 0$ $v_0 = 0$ $y_0 =$
  - In the middle: $t_1 = 1.2\ s$ $v_1 =$ $y_1 = ½\ y_0$
  - At the bottom: $t_2 = ???$ $v_2 =$ $y_2 = 0$
- Determine formulas:
  - We know a & y_2 and want to find t_2. v is missing $\Rightarrow$ $y_2 = y_0 + v_{0y}t_2 + \frac{1}{2}at_2^2$
  - $v_0 = 0$ and $y_2 = 0$: $0 = y_0 + \frac{1}{2}at_2^2$

I don't have y_0. How do I get it? Let's look at t_1.

  - We know a & t_2 and want to find y_0. v is missing $\Rightarrow$ $y_1 = y_0 + v_{0y}t_1 + \frac{1}{2}at_1^2$
  - $v_0 = 0$ and $y_1 = ½\ y_0$: $\frac{1}{2}y_0 = y_0 + \frac{1}{2}at_1^2$ (with a and t_1 known, this will get us y_0)
- Do the math:
  - Find y_0: $\frac{1}{2}y_0 = y_0 - \frac{1}{2}gt_1^2$ $\frac{1}{2}y_0 - y_0 = -\frac{1}{2}gt_1^2$ $-\frac{1}{2}y_0 = -\frac{1}{2}gt_1^2$ $y_0 = gt_1^2$
  - Use y_0 to get t_2: $0 = y_0 - \frac{1}{2}gt_2^2$ $\frac{1}{2}gt_2^2 = y_0$ $gt_2^2 = 2y_0$ $gt_2^2 = 2gt_1^2$

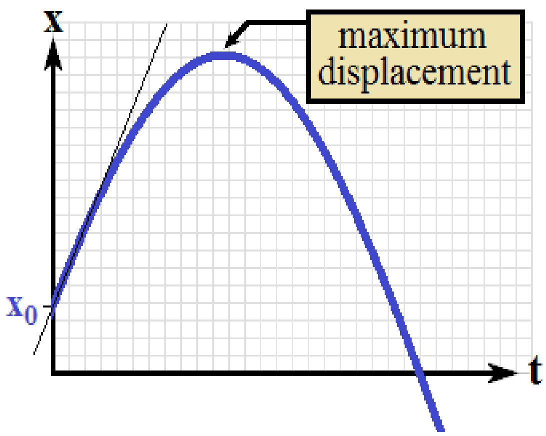
$$t_2^2 = 2t_1^2 \qquad t_2 = \sqrt{2t_1^2} = (\sqrt{2})t_1 = (\sqrt{2})(1.2\ s) = 1.7\ s$$

Graphical Analysis

- Drawing graphs of a variable can offer insights into some problems. It is especially useful on problems where little numerical information is given.

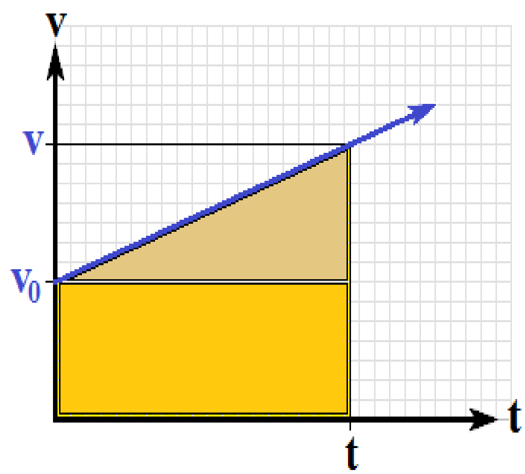
Position graph with no acceleration (a=0)

- $x = mt + b$
- $m = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} = v$
- Velocity is the slope ($v = v_{avg} = v_0 =$ const.)
- x_0 is the intercept
- $x = vt + x_0$
- When a = 0, position graph is straight lines.



Position graph with const. acceleration ($a < 0$)

- $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Parabola)
- x_0 is the intercept
- v_0 is the slope at t=0.
- The slope (v) decreases with time ($a < 0$)
- Maximum displacement occurs when the slope is zero ($v = 0$)



Velocity graph with constant acceleration ($a > 0$)

- $m = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a$
- Acceleration is the slope (a = const.)
- Constant acceleration creates straight lines.
- v_0 is the intercept
- $v = at + v_0$
- Rectangle: $LW = v_0 t$
- Triangle: $\frac{1}{2}bh = \frac{1}{2}t(v-v_0) = \frac{1}{2}t(at) = \frac{1}{2}at^2$
- Area under the curve = $\Delta x = v_0 t + \frac{1}{2}at^2$
- The area under the curve is displacement.
- When v is negative ($v < 0$) so is area ($\Delta x < 0$)

Example:

1) What is x at t = 5? x = 3 m
2) What is v_{avg} between 4 s and 7 s?

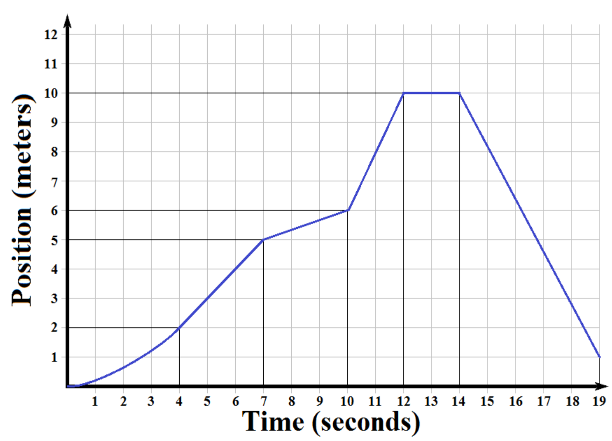
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(7s) - x(4s)}{7s - 4s} = \frac{5m - 2m}{3s} = 1\,m/s$$

3) What is v at t = 11 s? $v = \bar{v}$ $(10s \le t \le 12s)$

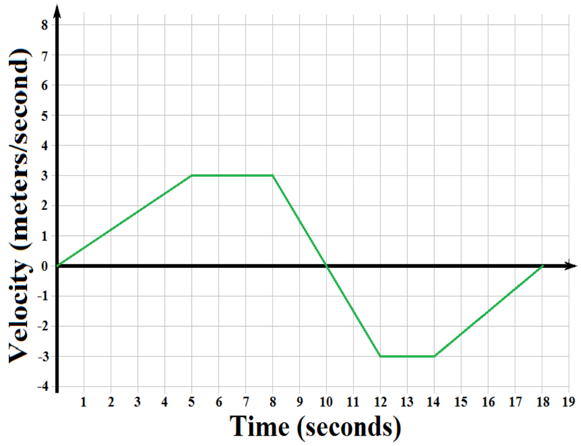
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(12) - x(10)}{12\,s - 10\,s} = \frac{10\,m - 6\,m}{2\,s} = 2.0\,m/s$$

4) What is v at t = 13s? $v = 0$
5) What is a at t = 11s? a=0
6) What is a between t = 0s and t = 4s?

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(4s) - v(0s)}{4s - 0s} = \frac{1m/s - 0}{4s} = 0.25\,m/s^2$$



Example: From the velocity graph shown determine the following if **x_0=10 m**:

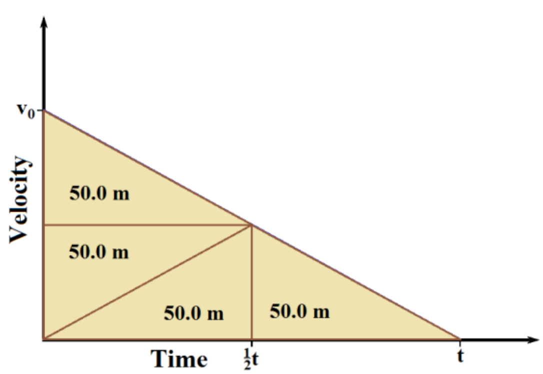


1) What is '**v**' at t = 5?
2) What is '**a**' between 0 s and 5 s?
3) What is '**a**' at t = 6 s?
4) What is '**Δx**' at t = 5s?
5) What is '**x**' at t = 8s?
6) What is '**x**' at t = 14s?

1) $v = 3\ m/s$
2) $a = \frac{\Delta v}{\Delta t} = \frac{v(5s) - v(0s)}{5s - 0s} = \frac{3m/s - 0m/s}{5s} = 0.6\,m/s^2$
3) $a = 0$
4) $\Delta x = ½bh = ½(5s)(3\ m/s) = 7.5\ m$
5) $x = x_0 + \frac{1}{2}bh + LW$ $\quad x = 10\ m + 7.5\ m + (3s)\left(3\frac{m}{s}\right) = 26.5\ m$
6) $x = x(t = 8) + \Delta x$ $\quad x = 26.5\ m + \frac{1}{2}bh - \frac{1}{2}bh - LW$

$x = 26.5\ m + 3m - 3m - 6m = 20.5\ m$

Example: A hockey player passes the puck. After uniform deceleration, the puck comes to rest 200.0 m away. How far does it travel in half this time?



- Draw a graph of velocity
- It starts at v_0 and falls to zero in time t.
- The area under the curve = Δx = 200.0 m
- The distance it covers in half the time is area left of the ½t line.
- We break the area into 4 equal triangles
- The sum of the 4 equal areas must be 200.0 m
- Each triangle is then 50.00 m (a quarter of 200.0 m)
- As 3 triangles make up the area left of the ½t line, the answer is 150.0 m (3 × 50.00 m)