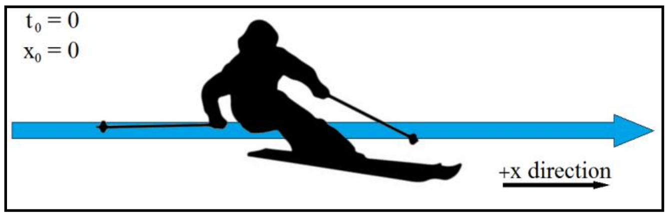
<u>Lecture 4</u>: One-Dimensional Kinematics

***Physics for Engineers & Scientists* (Giancoli): Chapter 2**
***University Physics V1* (Openstax): Chapter 3**

<u>Example</u>: (a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0m/s when going down a slope for 5.0s (b) How far does the skier travel in this time?

- Part A
    - Draw a diagram.
    - Set reference frame.



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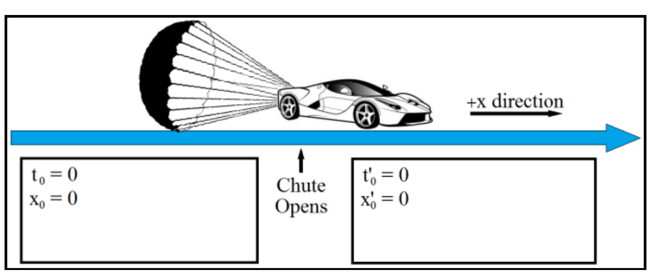
    - Extract data : $x_0 = 0$ a=? $v_0 = 0$ v = 8.0 m/s t = 5.0s
    - Determine formula: "x" is missing $\Rightarrow$ $v = v_0 + at$
    - Do the math: $v = \cancel{v_0}^{\,0} + at = at$ $a = \frac{v}{t} = \frac{8.0\frac{m}{s}}{5.0\ s} = 1.6\frac{m}{s^2}$
- Part B
    - Determine formula: Any formula with "x" will do $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
    - Do the math: $x = \cancel{x_0}^{\,0} + \frac{1}{2}(v + \cancel{v_0}^{\,0})t = \frac{1}{2}vt = \frac{1}{2}\left(8.0\frac{m}{s}\right)(5.0\ s) = 20.\ m$

<u>Example</u>: A drag racer, starting from rest, speeds up for 402 m with acceleration of +17.0 m/s². A parachute then opens, slowing the car down with an acceleration of -6.10 m/s². How fast is the racer moving 350 m after the chute opens?

- Draw a diagram.
- Set reference frame.
    - We have an object where the acceleration changes from once constant value to another.
    - This requires two sets of equations, one before the chute opens and one after.
    - Essentially this is like working two separate problems.
    - We can use two separate reference frames, S = (x,t) for before the chute opens and S' = (x', t') for after.



- Extract data : $x_0 = 0$ $v_0 = 0$ $x = 402$ m $a = 17.0$ m/s^2
  $x'_0 = 0$ $a' = -6.10$ m/s^2 $x' = 350$ m $v' = ?$
- Determine formulas
  - 2nd half with open chute: t' is missing $\Rightarrow$ $v'^2 = v_0'^2 + 2a'(x' - x'_0)$
    - We have a', x' and x'_0, but we don't have v'_0.
    - v'_0 (the initial velocity after the chute opens) is the same as v, the final velocity before the chute opens. So we need to find v (or actually v^2).
  - 2nd half with no chute: t is missing $\Rightarrow$ $v^2 = v_0^2 + 2a(x - x_0)$
- Do the math:

$$v^2 = \cancelto{0}{v_0^2} + 2a(x - \cancelto{0}{x_0}) = 2ax = 2\left(17.0\frac{m}{s^2}\right)(402\ m) = 13{,}668\frac{m^2}{s^2}$$

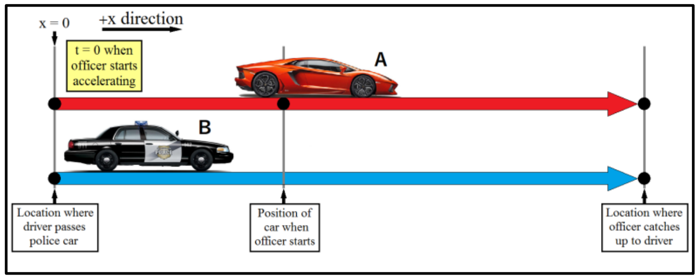
$$v = \sqrt{13{,}668\frac{m^2}{s^2}} = 116.92\frac{m}{s}$$

$$v'^2 = v_0'^2 + 2a'(x' - \cancelto{0}{x'_0}) = v_0'^2 + 2a'x' = 13{,}668\frac{m^2}{s^2} + 2\left(-6.10\frac{m}{s^2}\right)(350\ m) = 9{,}398\frac{m^2}{s^2}$$

$$v' = \sqrt{9{,}398\frac{m^2}{s^2}} = 97\frac{m}{s}$$

Example: A driver of a speeding car travelling down a deserted road passes a hidden police car. The driver is moving at a constant speed of 89.5 mph (40.0 m/s) when he passes the police officer. It takes the officer 5.00 seconds to begin pursuit. Once he gives chase he accelerates at a uniform 7.50 m/s^2. What distance does the officer go before he catches the car?

- Draw a diagram. There are three important reference points.
  - 1) The location where the speeding car passes the police car.
  - 2) The location of the speeding car when the police car begins to give chase.
  - 3) The location when the police car catches up to the speeding car.
- Set reference frame.
  - x = 0 is best set at the location where the speeding car passes the police car with the positive x-axis pointing in the direction of motion.
  - There are two reasonable options for t = 0.
    - 1) When the speeding car passes the police car.
      - With this option you must account for the delay of the police car by calculating an artificial x_0 and v_0 that will place him at rest at the origin at t = 5.00s.
      - To get v_0: Set v = 0, a = 7.50 m/s^2, and t = 5.00 s in $v = v_0 + at$
      - To get x_0: Set x = 0, a = 7.50 m/s^2, t = 5.00, and v_0 in $x = x_0 + v_0 t + \frac{1}{2}at^2$
    - 2) When the police car begins to accelerate.
      - With this option you must calculate the position of the car at t=0, which is rather straight forward.



- Extract data
  - Police car: $x_{B0} = 0$ $v_{B0} = 0$ $a_B = 7.50\ \text{m/s}^2$
  - Speeding car: $x_{A0} = ?$ $v_{A0} = 40.0\ \text{m/s}$ $a_A = 0$
- Determine formulas
  - To catch up means to be at the same place (x) at the same time (t). Need an equation with both x and t in them. As both accelerations are known, use the equation with x, a, and t (having no v) $\Rightarrow$ $x = x_0 + v_0 t + \frac{1}{2}at^2$
    - Need two versions. One for each vehicle.
    - $x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2 = x_{A0} + v_{A0}t$
    - $x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 = \frac{1}{2}a_B t^2$
  - We also need to determine x_{A0}, which is the distance a car travelling at a constant 40.0 m/s covers in 5.00 s. ($x_{A0} = v_{A0}{\cdot}\Delta t$)
  - When the police car catches up: $x_A = x_B$
- Do the math:
  - Determine x_{A0}: $x_{A0} = v_{A0}{\cdot}\Delta t = \left(40.0\frac{\text{m}}{\text{s}}\right)(5.00\ \text{s}) = 200.\ \text{m}$
  - Set the distances equal to each other: $x_A = x_B$
  - Plug in formulas for each vehicle and solve for t: $x_{A0} + v_{A0}t = \frac{1}{2}a_B t^2$

$$\frac{1}{2}a_B t^2 - v_{A0}t - x_{A0} = 0$$

$$\frac{1}{2}\left(7.50\frac{\text{m}}{\text{s}^2}\right)t^2 - \left(40.0\frac{\text{m}}{\text{s}}\right)t - (200.\ \text{m}) = 0$$

$$\left(3.75\frac{\text{m}}{\text{s}^2}\right)t^2 - \left(40.0\frac{\text{m}}{\text{s}}\right)t - (200.\ \text{m}) = 0$$

  - Quadratic equation is needed:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{\left(40.0\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(-40.0\frac{\text{m}}{\text{s}}\right)^2 - 4\left(3.75\frac{\text{m}}{\text{s}^2}\right)(-200.\ \text{m})}}{2\left(3.75\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$t = \frac{40.0\,\frac{m}{s} \pm \sqrt{1600\,\frac{m^2}{s^2} + 3000\,\frac{m^2}{s^2}}}{7.50\,\frac{m}{s^2}} = \frac{40.0\,\frac{m}{s} \pm \sqrt{4600\,\frac{m^2}{s^2}}}{7.50\,\frac{m}{s^2}} = \frac{40.0\,\frac{m}{s} \pm 67.823\,\frac{m}{s}}{7.50\,\frac{m}{s^2}}$$

- Two roots, one from positive sign (t_1) and one from negative sign (t_2)

$$t_1 = \frac{40.0\,\frac{m}{s} + 67.823\,\frac{m}{s}}{7.50\,\frac{m}{s^2}} = \frac{107.823\,\frac{m}{s}}{7.50\,\frac{m}{s^2}} = 14.376\ s$$

$$t_2 = \frac{40.0\,\frac{m}{s} - 67.823\,\frac{m}{s}}{7.50\,\frac{m}{s^2}} = \frac{-27.823\,\frac{m}{s}}{7.50\,\frac{m}{s^2}} = -3.710\ s$$

The negative value of time is an extraneous root and can be ignored.

- Plug t_1 back into distance formulas to determine the answer.

Plugging into both provides an additional check.

$$x_A = x_{A0} + v_{A0}t = (200.\,m) + \left(40.0\,\frac{m}{s}\right)(14.376\ s) = 775\ m$$

$$x_B = \frac{1}{2}a_B t^2 = \frac{1}{2}\left(7.50\,\frac{m}{s^2}\right)(14.376\ s)^2 = 775\ m$$

Freely Falling Bodies

- In the absence of air resistance, all objects at (or near) the surface of the Earth accelerate downward at the same rate when released. $|a| = g = 9.80\ m/s^2$
  - g is positive and is sometimes used as a measure of acceleration.

Air force pilots must be able of withstanding a "g-force" of 9g = (9)(9.80 m/s²) = 88.2 m/s²

Example: How long does it take a bowling ball to fall from rest at a height of 10.0 m?

- We'll skip the diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data: $v_0 = 0$ $y_0 = 10.0$ m $y = 0$ $a = -9.80\ m/s^2$ $t = ?$
- Determine formula: v is missing ⇒ $x = x_0 + v_0 t + \frac{1}{2}at^2$
- Do the math:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad 0 = x_0 + \tfrac{1}{2}at^2 \qquad 0 = x_0 - \tfrac{1}{2}gt^2 \qquad \tfrac{1}{2}gt^2 = x_0$$

$$gt^2 = 2x_0 \qquad t^2 = \frac{2x_0}{g} \qquad t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2(10.0\ m)}{9.80\frac{m}{s^2}}} = 1.43\ s$$

Would a feather fall at the same rate? What if it was in a vacuum with no air resistance?
In a vacuum with no air resistance, everything falls at the same rate!