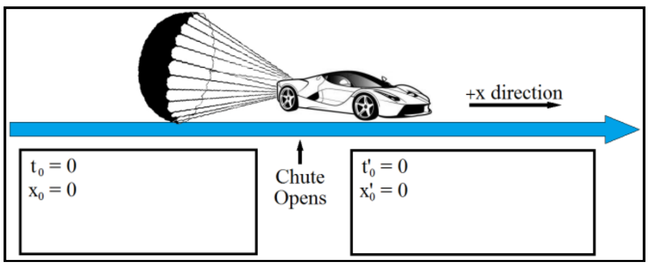
Lecture 3: One-Dimensional Kinematics, Free Fall, & Graphical Analysis

Physics for Engineers & Scientists (Giancoli): Chapter 2
University Physics V1 (Openstax): Chapter 3

Example: A drag racer, starting from rest, speeds up for 402 m with acceleration of +17.0 m/s². A parachute then opens, slowing the car down with an acceleration of -6.10 m/s². How fast is the racer moving 350 m after the chute opens?

- Draw a diagram.
- Set reference frame.
    - We have an object where the acceleration changes from once constant value to another.
    - This requires two sets of equations, one before the chute opens and one after.
    - Essentially this is like working two separate problems.
    - We can use two separate reference frames, S = (x,t) for before the chute opens and S' = (x', t') for after.



- Extract data : $x_0 = 0$ $v_0 = 0$ $x = 402$ m $a = 17.0$ m/s^2
  $x'_0 = 0$ $a' = -6.10$ m/s^2 $x' = 350$ m $v' = ?$

- Determine formulas
    - 2nd half with open chute: t' is missing $\Rightarrow$ $v'^2 = v'^2_0 + 2a'(x' - x'_0)$
        - We have a', x' and x'_0, but we don't have v'_0.
        - v'_0 (the initial velocity after the chute opens) is the same as v, the final velocity before the chute opens. So we need to find v (or actually v^2).
    - 2nd half with no chute: t is missing $\Rightarrow$ $v^2 = v_0^2 + 2a(x - x_0)$

- Do the math:

$$v^2 = \cancelto{0}{v_0^2} + 2a(x - \cancelto{0}{x_0}) = 2ax = 2\left(17.0\frac{m}{s^2}\right)(402\ m) = 13{,}668\frac{m^2}{s^2}$$

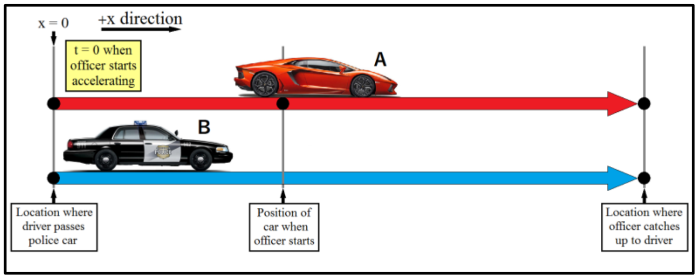
$$v = \sqrt{13{,}668\frac{m^2}{s^2}} = 116.92\frac{m}{s}$$

$$v'^2 = v'^2_0 + 2a'(x' - \cancelto{0}{x'_0}) = v'^2_0 + 2a'x' = 13{,}668\ \frac{m^2}{s^2} + 2\left(-6.10\ \frac{m}{s^2}\right)(350\ m) = 9{,}398\ \frac{m^2}{s^2}$$

$$v' = \sqrt{9{,}398\ \frac{m^2}{s^2}} = 97\ \frac{m}{s}$$

Example: A driver of a speeding car travelling down a deserted road passes a hidden police car. The driver is moving at a constant speed of 89.5 mph (40.0 m/s) when he passes the police officer. It takes the officer 5.00 seconds to begin pursuit. Once he gives chase he accelerates at a uniform 7.50 m/s^2. What distance does the officer go before he catches the car?

- Draw a diagram. There are three important reference points.
  - 1) The location where the speeding car passes the police car.
  - 2) The location of the speeding car when the police car begins to give chase.
  - 3) The location when the police car catches up to the speeding car.
- Set reference frame.
  - x = 0 is best set at the location where the speeding car passes the police car with the positive x-axis pointing in the direction of motion.
  - There are two reasonable options for t = 0.
    - 1) When the speeding car passes the police car.
      - With this option you must account for the delay of the police car by calculating an artificial x_0 and v_0 that will place him at rest at the origin at t = 5.00s.
      - To get v_0: Set v = 0, a = 7.50 m/s^2, and t = 5.00 s in $v = v_0 + at$
      - To get x_0: Set x = 0, a = 7.50 m/s^2, t = 5.00, and v_0 in $x = x_0 + v_0 t + \frac{1}{2}at^2$
    - 2) When the police car begins to accelerate.
      - With this option you must calculate the position of the car at t=0, which is rather straight forward.



- Extract data
  - Police car: $x_{B0} = 0$ $v_{B0} = 0$ $a_B = 7.50\ m/s^2$
  - Speeding car: $x_{A0} = ?$ $v_{A0} = 40.0\ m/s$ $a_A = 0$
- Determine formulas
  - To catch up means to be at the same place (x) at the same time (t). Need an equation with both x and t in them. As both accelerations are known, use the equation with x, a, and t (having no v) $\Rightarrow$ $x = x_0 + v_0 t + \frac{1}{2}at^2$
    - Need two versions. One for each vehicle.
    - $x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2 = x_{A0} + v_{A0}t$
    - $x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 = \frac{1}{2}a_B t^2$
  - We also need to determine x_{A0}, which is the distance a car travelling at a constant 40.0 m/s covers in 5.00 s. ($x_{A0} = v_{A0} \cdot \Delta t$)

- When the police car catches up: $x_A = x_B$
- Do the math:
  - Determine x_{A0}: $x_{A0} = v_{A0} \cdot \Delta t = \left(40.0\frac{m}{s}\right)(5.00\ s) = 200.\ m$
  - Set the distances equal to each other: $x_A = x_B$
  - Plug in formulas for each vehicle and solve for t: $x_{A0} + v_{A0}t = \frac{1}{2}a_B t^2$

$$\frac{1}{2}a_B t^2 - v_{A0}t - x_{A0} = 0$$

$$\frac{1}{2}\left(7.50\frac{m}{s^2}\right)t^2 - \left(40.0\frac{m}{s}\right)t - (200.\ m) = 0$$

$$\left(3.75\frac{m}{s^2}\right)t^2 - \left(40.0\frac{m}{s}\right)t - (200.\ m) = 0$$

  - Quadratic equation is needed:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{\left(40.0\frac{m}{s}\right) \pm \sqrt{\left(-40.0\frac{m}{s}\right)^2 - 4\left(3.75\frac{m}{s^2}\right)(-200.\ m)}}{2\left(3.75\ \frac{m}{s^2}\right)}$$

$$t = \frac{40.0\frac{m}{s} \pm \sqrt{1600\frac{m^2}{s^2} + 3000\frac{m^2}{s^2}}}{7.50\frac{m}{s^2}} = \frac{40.0\frac{m}{s} \pm \sqrt{4600\frac{m^2}{s^2}}}{7.50\frac{m}{s^2}} = \frac{40.0\frac{m}{s} \pm 67.823\frac{m}{s}}{7.50\frac{m}{s^2}}$$

  - Two roots, one from positive sign (t_1) and one from negative sign (t_2)

$$t_1 = \frac{40.0\frac{m}{s} + 67.823\frac{m}{s}}{7.50\frac{m}{s^2}} = \frac{107.823\frac{m}{s}}{7.50\frac{m}{s^2}} = 14.376\ s$$

$$t_2 = \frac{40.0\frac{m}{s} - 67.823\frac{m}{s}}{7.50\frac{m}{s^2}} = \frac{-27.823\frac{m}{s}}{7.50\frac{m}{s^2}} = -3.710\ s$$

The negative value of time is an extraneous root and can be ignored.

  - Plug t_1 back into distance formulas to determine the answer.

Plugging into both provides an additional check.

$$x_A = x_{A0} + v_{A0}t = (200.\ m) + \left(40.0\frac{m}{s}\right)(14.376\ s) = 775\ m$$

$$x_B = \frac{1}{2}a_B t^2 = \frac{1}{2}\left(7.50\frac{m}{s^2}\right)(14.376\ s)^2 = 775\ m$$

Freely Falling Bodies

- In the absence of air resistance, all objects at (or near) the surface of the Earth accelerate downward at the same rate when released. $|a| = g = 9.80\ m/s^2$
  - g is positive and is sometimes used as a measure of acceleration.

Air force pilots must be able of withstanding a "g-force" of 9g = (9)(9.80 m/s²) = 88.2 m/s²

Example: How long does it take a bowling ball to fall from rest at a height of 10.0 m?

- We'll skip the diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data: $v_0 = 0$ $y_0 = 10.0$ m $y = 0$ $a = -9.80\ m/s^2$ $t = ?$
- Determine formula: v is missing $\Rightarrow$ $x = x_0 + v_0 t + \frac{1}{2}at^2$
- Do the math:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad 0 = x_0 + \frac{1}{2}at^2 \qquad 0 = x_0 - \frac{1}{2}gt^2 \qquad \frac{1}{2}gt^2 = x_0$$

$$gt^2 = 2x_0 \qquad t^2 = \frac{2x_0}{g} \qquad t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2(10.0\ m)}{9.80\frac{m}{s^2}}} = 1.43\ s$$

Would a feather fall at the same rate? What if it was in a vacuum with no air resistance?
In a vacuum with no air resistance, everything falls at the same rate!

Example: A wrecking ball is hanging at rest from a crane when suddenly the cable breaks. The times it takes the ball to fall halfway to the ground is 1.2s. Find the time it takes for the ball to fall from rest all the way to the ground.

- Diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data:
  - At every location: $a = -9.80\ m/s^2$
  - At the top: $t = 0$ $v_0 = 0$ $y_0 =$
  - In the middle: $t_1 = 1.2$ s $v_1 =$ $y_1 = \frac{1}{2} y_0$
  - At the bottom: $t_2 = ???$ $v_2 =$ $y_2 = 0$
- Determine formulas:
  - We know a & y_2 and want to find t_2. v is missing $\Rightarrow$ $y_2 = y_0 + v_{0y}t_2 + \frac{1}{2}at_2^2$
  - $v_0 = 0$ and $y_2 = 0$: $0 = y_0 + \frac{1}{2}at_2^2$

I don't have y_0. How do I get it? Let's look at t_1.

  - We know a & t_2 and want to find y_0. v is missing $\Rightarrow$ $y_1 = y_0 + v_{0y}t_1 + \frac{1}{2}at_1^2$
  - $v_0 = 0$ and $y_1 = \frac{1}{2} y_0$: $\frac{1}{2}y_0 = y_0 + \frac{1}{2}at_1^2$ (with a and t_1 known, this will get us y_0)
- Do the math:
  - Find y_0: $\frac{1}{2}y_0 = y_0 - \frac{1}{2}gt_1^2$ $\qquad \frac{1}{2}y_0 - y_0 = -\frac{1}{2}gt_1^2$ $\qquad -\frac{1}{2}y_0 = -\frac{1}{2}gt_1^2$ $\qquad y_0 = gt_1^2$
  - Use y_0 to get t_2: $0 = y_0 - \frac{1}{2}gt_2^2$ $\qquad \frac{1}{2}gt_2^2 = y_0$ $\qquad gt_2^2 = 2y_0$ $\qquad gt_2^2 = 2gt_1^2$

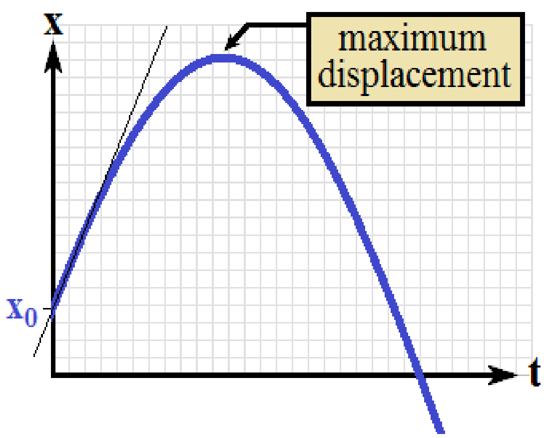
$$t_2^2 = 2t_1^2 \qquad t_2 = \sqrt{2t_1^2} = (\sqrt{2})t_1 = (\sqrt{2})(1.2\ s) = 1.7\ s$$

Graphical Analysis

- Drawing graphs of a variable can offer insights into some problems. It is especially useful on problems where little numerical information is given.

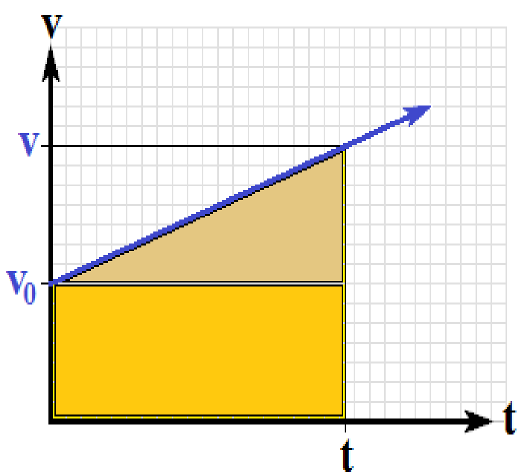
Position graph with no acceleration (a=0)

- $x = mt + b$
- $m = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} = v$
- Velocity is the slope ($v = v_{avg} = v_0 = const.$)
- x_0 is the intercept
- $x = vt + x_0$
- When a = 0, position graph is straight lines.



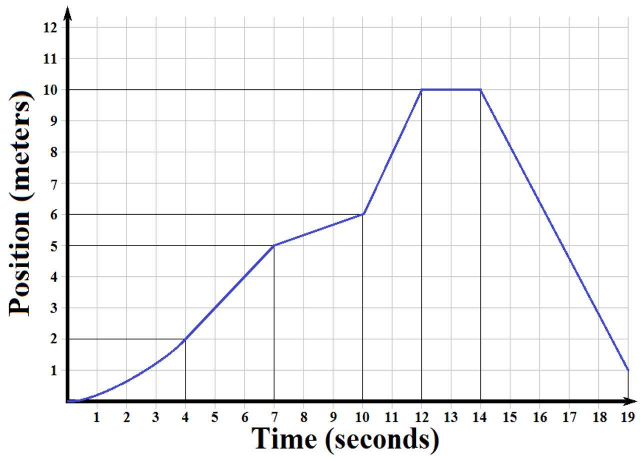
Position graph with const. acceleration (a < 0)

- $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Parabola)
- x_0 is the intercept
- v_0 is the slope at t=0.
- The slope (v) decreases with time (a < 0)
- Maximum displacement occurs when the slope is zero (v = 0)



Velocity graph with constant acceleration (a > 0)

- $m = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a$
- Acceleration is the slope (a = const.)
- Constant acceleration creates straight lines.
- v_0 is the intercept
- $v = at + v_0$
- Rectangle: $LW = v_0 t$
- Triangle: $½bh = ½t(v-v_0) = ½t(at) = ½at^2$
- Area under the curve = $\Delta x = v_0 t + ½at^2$
- The area under the curve is displacement.
- When v is negative (v < 0) so is area ($\Delta x < 0$)



Example:

1) What is x at t = 5? x = 3 m

2) What is v_{avg} between 4 s and 7 s?

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(7s) - x(4s)}{7s - 4s} = \frac{5m - 2m}{3s} = 1\,m/s$$

3) What is v at t = 11 s? $v = \bar{v}$ $(10s \le t \le 12s)$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(12) - x(10)}{12\,s - 10\,s} = \frac{10\,m - 6\,m}{2\,s} = 2.0\,m/s$$

4) What is v at t = 13s? $v = 0$

5) What is a at t = 11s? a=0

6) What is a between t = 0s and t = 4s?

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(4s) - v(0s)}{4s - 0s} = \frac{1m/s - 0}{4s} = 0.25\,m/s^2$$