Lecture 3: One-Dimensional Kinematics

Physics for Engineers & Scientists (Giancoli): Chapter 2
University Physics V1 (Openstax): Chapter 3

Example: An object's position is given by $x(t) = (5.0\text{ m}) + (3.0\text{ m/s})t + (2.5\text{ m/s}^2)t^2$. Determine the velocity and acceleration as a function of t.

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{(5.0\text{ m}) + \left(3.0\frac{m}{s}\right)t + \left(2.5\frac{m}{s^2}\right)t^2\right\} = \left(3.0\frac{m}{s}\right) + \left(5.0\frac{m}{s^2}\right)t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{\left(3.0\frac{m}{s}\right) + \left(5.0\frac{m}{s^2}\right)t\right\} = 5.0\frac{m}{s^2}$$

Example: An object accelerates uniformly at 5.0 m/s^2 with an initial velocity of 3.0 m/s and an initial position of 5.0 m. Determine the objects position as a function of time.

$$a = \frac{dv}{dt} \qquad a\cdot dt = dv \qquad \int a\cdot dt = \int dv = v$$

$$v = \int a\cdot dt = at + C = at + v_0 = \left(5.0\frac{m}{s^2}\right)t + 3.0\frac{m}{s}$$

$$v = \frac{dx}{dt} \qquad v\cdot dt = dx \qquad \int v\cdot dt = \int dx = x$$

$$x = \int v\cdot dt = \int (at + v_0)\cdot dt = \frac{1}{2}at^2 + v_0t + C = \frac{1}{2}at^2 + v_0t + x_0$$

$$x = \left(2.5\frac{m}{s^2}\right)t^2 + \left(3.0\frac{m}{s}\right)t + (5.0\text{ m})$$

Constant Acceleration (A special case)

- When acceleration is constant, the average acceleration and the instantaneous acceleration are the same ($a = a_{avg}$).
  - $a = a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$
- When acceleration is constant, $v_{avg} = \frac{(v + v_0)}{2}$

This is true because the velocity is increasing linearly.
The average of 3, 4, 5, 6, and 7 is just (3+7)/2 = 5.

- We can derive a set of **four equations**.
  - Solve the acceleration equation for v to get the first equation.

$$a = \frac{v - v_0}{t} \qquad at = v - v_0 \qquad at + v_0 = v$$

$$\boxed{\mathbf{v = v_0 + at}}$$

v, a, and t are variables. v_0 is a constant.

  - To get our second equation, solve the equation for average velocity for x and then plug in the previous equation for v_{avg}.

.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} \qquad (v_{avg})t = x - x_0 \qquad (v_{avg})t + x_0 = x \qquad \frac{1}{2}(v + v_0)t + x_0 = x$$

$$\boxed{\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}}$$

x, v and t are variables. x_0 and v_0 are constants.

- To get our third equation, plug the value of v from the first equation into the second.

$$x = x_0 + \frac{1}{2}\{v + v_0\}t \qquad x = x_0 + \frac{1}{2}\{(v_0 + at) + v_0\}t \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\boxed{\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}}$$

x, a and t are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

- To get our fourth (and final) equation, solve the first equation for t, plug that into the 2nd equation, and solve for v^2.

$$v = v_0 + at \qquad v - v_0 = at \qquad t = \frac{v - v_0}{a}$$

$$x = x_0 + \frac{1}{2}(v + v_0)t \qquad x = x_0 + \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$x - x_0 = \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right) \qquad 2a(x - x_0) = (v + v_0)(v - v_0)$$

$$2a(x - x_0) = v^2 - v_0^2 \qquad v_0^2 + 2a(x - x_0) = v^2$$

$$\boxed{\mathbf{v^2 = v_0^2 + 2a(x - x_0)}}$$

x, v and a are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

- There are 4 variables (x, v, a, and t). Each equation has only 3 of them.

$\boxed{\mathbf{v = v_0 + at}}$	v, a, and t $\Rightarrow$ **No x**
$\boxed{\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}}$	x, v, and t $\Rightarrow$ **No a**
$\boxed{\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}}$	x, a, and t $\Rightarrow$ **No v**
$\boxed{\mathbf{v^2 = v_0^2 + 2a(x - x_0)}}$	x, v, and a $\Rightarrow$ **No t**

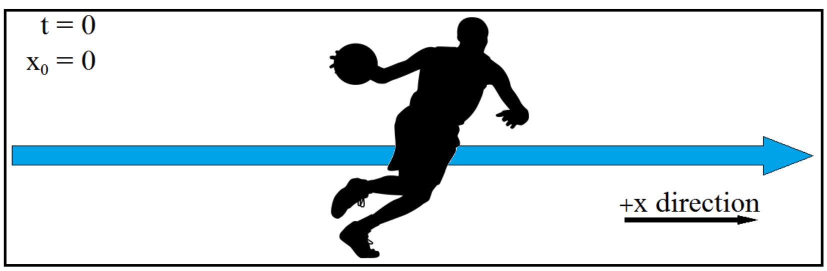
When given the values of two variables and looking for a value of a third, find the equation with all three variables. This is done easily after you determine the 4th variable (missing).

- This set of 4 equations is valid for one object moving with constant acceleration.
  - If acceleration is not constant, you must use the definitions rather than these equations.
  - If there is more than one object moving, you may need two sets of these equations (one for each object).
  - If an object moves with constant acceleration but then suddenly shifts to a new constant acceleration, then you must use a separate set of these four equations for before and after. Usually it's best to treat the before and after as two separate problems.

- Problem solving
  - First, draw a diagram if needed. It can help.
  - Second, set your reference frame (when is t = 0/where is x = 0).
  - Third, extract the data (values of variables and constants) from the problem. Units will help you determine what variables to use for each number.
  - Fourth, determine the formula (or formulas) that you need (i.e. find the path that leads to your answer).
  - Fifth, do the math.

Example: In getting ready to slam-dunk a ball, a player starts from rest and sprints to a speed of 6.0 m/s in 1.5s. Assuming he accelerates uniformly, determine the distance he runs.

- Draw a diagram.
- Set reference frame.



- Extract data : $x_0 = 0$ $v_0 = 0$ $v = 6.0$ m/s $t = 1.5$s $x=?$
- Determine formula: "a" is missing $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
- Do the math: $x = \cancel{x_0}^{0} + \frac{1}{2}(v + \cancel{v_0}^{0})t = \frac{1}{2}vt = \frac{1}{2}\left(6.0\,\frac{m}{s}\right)(1.5\ s) = 4.5\ m$