Lecture 32: Ideal Gas Law and the Kinetic Theory of Gas

Physics for Engineers & Scientists (Giancoli): Chapters 17 & 18
University Physics V2 (Openstax): Chapter 2

The Atomic Mass Scale

- The **Atomic Mass Unit** (u) is the mass of a carbon-12 atom (^{12}C) divided by 12. The atomic mass unit is roughly the mass of a proton or neutron.

$$u = 1.66 \times 10^{-27}\ kg$$

- The mass of an atom in atomic mass units is the same as the number of nucleons in the atom.

^{4}He has 4 nucleons (2 protons and 2 neutrons). The mass of a ^{4}He atom is 4u.

- The mass of a molecule is the sum of the masses of its atoms.
- One **Mole** (mol) of a substance contains Avogadro's number (6.022×10^{23}) of particles (could be atoms or molecules), which is the number of atoms in 12.0 g of carbon-12.

$$N_A = 6.022 \times 10^{23}$$

This creates a correspondence between the masses of atoms (atomic mass scale) and masses at the macroscopic scale (the masses of moles in grams).

For example, the mass of 1 mol of a substance with atomic mass 28 u (silicon) is going to be 28 g.

$$N = \#\ of\ particles\ in\ a\ substance \qquad n = \frac{N}{N_A}$$

$$n = \#\ of\ moles\ of\ a\ substance$$

Ideal Gas Law $\quad PV = nRT \quad PV = NkT$

- The ideal gas law can be written in two ways. One uses 'n', the number of moles, and the other uses 'N' the number of particles.
- P is the pressure of the gas.
- V is the container volume.
- T is the gas temperature.
- R is the **Universal Gas Constant**: $R = 8.314\ \frac{J}{mol\cdot K}$
- K is the **Boltzmann Constant**: $k = 1.38 \times 10^{-23}\ \frac{J}{K}$
- For the two equations to be equal, $R = N_A k \quad nR = Nk$

The units of both sides of the ideal gas law are Joules. This is an energy equation!

Example: The Airlander 10 is the largest operational aircraft in the world (as of 2016). It typically contains 38,000 cubic meters of helium at 1.01 atm. If the temperature is 20.0°C, how many moles of helium are needed to fill the craft?

$$P = (1.01\ atm)\left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right) = 102{,}313\ Pa \qquad T(K) = 20.0°\mathrm{C} + 273.15 = 293.15\ K$$

$$PV = nRT \qquad n = \frac{PV}{RT} = \frac{(102{,}313\ Pa)(38000\ m^3)}{\left(R{=}8.314\ \frac{J}{mol\cdot K}\right)(293.15\ K)} = 1.60 \times 10^6\ moles$$

- **Using the Ideal Gas Law**
  - Absolute pressure must be used in the ideal gas law (not gauge pressure). Normally this must also be in Pascal (Pa) except as noted below.
  - Temperature must be in Kelvin.
  - Normally the volume must be in cubic meters (m^3) except as noted below.
  - Often we are comparing two different states of the same gas at two different time periods. If so, we can use that PV/nT (or PV/NT) is a constant.

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \quad \text{...Or if n is constant:} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

    - When comparing two different states of the same gas, multiplicative conversion factors will cancel. This allows most units to be used for volume (liters, cm^3, etc) and pressure (atm).
    - Conversions that involve adding/subtracting cannot be used. Temperature must be in Kelvin, and absolute pressure (not gauge pressure) must be used.
    - If the temperature is held constant, you get **<u>Boyle's</u> <u>Law</u>**: $P_1V_1 = P_2V_2$
    - If the pressure is held constant, you get **<u>Charles'</u> <u>Law</u>**: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
    - If the volume is held constant, you get **<u>Gay-Lussac's</u> <u>Law</u>**: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

<u>Example</u>: A driver in Texas carefully prepares her car for a trip. This includes making sure the tires are inflated to a gauge pressure of 30.0 lbs per square inch (207 kPa). The tires have a volume of 1.64×10^{-2} m^3 and were filled at 95.0°F. Three days later the driver is in Canada at a temperature of 23.0°F. Assuming no air was lost from the tires since they were filled, what is the gauge pressure of the tires in Canada?

$$P_1 = P_{Absolute} = P_{Gauge} + P_{atm} = 207\ kPa + 101.3\ kPa = 308.3\ kPa$$

$$T_1(°C) = \tfrac{5}{9}[T_1(°F) - 32] = \tfrac{5}{9}[95.0°F - 32] = 35.0°C \qquad T_1(K) = 35.0°C + 273.15 = 308.15\ K$$

$$T_2(°C) = \tfrac{5}{9}[T_2(°F) - 32] = \tfrac{5}{9}[23.0°F - 32] = -5.0°C \qquad T_2(K) = -5.0°C + 273.15 = 268.15\ K$$

As no air was lost from the tires, n is constant.
The volume of the tires is given. We can assume that is fixed. So V is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad P_2 = \frac{T_2}{T_1}P_1 = \frac{268.15\ K}{308.15\ k}(308.3\ kPa) = 268.3\ kPa$$

$$P_{Gauge} = P_{Absolute} - P_{atm} = 268.3\ kPa - 101.3\ kPa = 167.0\ kPa \quad \text{(roughly 24.2 lbs/in}^2\text{)}$$

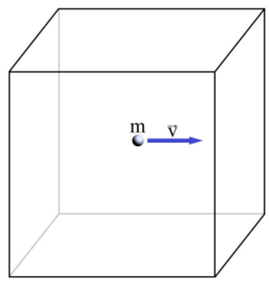
- **<u>Standard</u> <u>Temperature</u> <u>and</u> <u>Pressure</u>** (STP)
  - Standard Temperature is 0°C (or 273.15 K)
  - Standard Pressure is 1 atm (1.013×10^5 Pa)
  - We can determine the volume of 1 mole of an ideal gas at STP.

$$V = \frac{nRT}{P} = \frac{(1\ mol)\left(8.314\ \frac{J}{mol \cdot K}\right)(273.15\ K)}{(1.013\ \times\ 10^5\ Pa)} = 0.02242\ m\ (22.4\ liters)$$

The Kinetic Theory of Gas

Our goal is to find a relationship between the microscopic properties of the particles that make up a gas and the macroscopic properties of the gas.

Let's begin with a cube with sides of length L and the simplest possible gas inside it, a single particle moving with velocity v.



The pressure is the force per area: $P = \frac{F}{A}$

The force is the momentum per time: $F = \frac{\Delta P}{\Delta t}$

The time interval is the time it takes the particle to return:

$v\Delta t = 2L \qquad \Delta t = \frac{2L}{v}$

The change in momentum of the wall is equal and opposite to the change in momentum of the particle.

$$\Delta P_{wall} = -(\Delta P_{particle}) = -(P_{final} - P_{initial}) = -[m(-v) - mv] = 2mv$$

$$F = \frac{\Delta P}{\Delta t} = \frac{2mv}{\frac{2L}{v}} = \frac{mv^2}{L} \qquad P = \frac{\frac{mv^2}{L}}{L^2} = \frac{mv^2}{L^3}$$

This is the pressure created by a single particle. Now let's fill the container with N particles, but we must account for the "equipartition theorem".

Equipartition of Energy: Each degree of freedom (dimension) must have the same amount of energy carried through it.

To account for this, we will have all of our particles moving in one-dimension with a third moving in each direction. Then we will calculate the force and then the pressure on one face.

This may seem rather contrived, but if left for a short time collisions will return this system to a completely random state. Consequently, these states are equivalent (at least for our purposes here).

Instead of v^2, which varies from particles to particle, we must use the average value of v^2. To do this we will use "rms" values (which stands for "root mean square", the square root of the mean of squares).

$$v_{rms} = \sqrt{\overline{v^2}} \qquad \overline{v^2} = v_{rms}^2$$

$$F_{Tot} = (\#\, of\ particles)(F_{avg}) = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L}\right) = \frac{Nmv_{rms}^2}{3L}$$

$$P = \frac{F_{Tot}}{A} = \frac{\frac{Nmv_{rms}^2}{3L}}{L^2} = \frac{Nmv_{rms}^2}{3L^3} = \frac{Nmv_{rms}^2}{3V} \qquad PV = \frac{1}{3}Nmv_{rms}^2 = NkT$$

$$mv_{rms}^2 = 3kT \qquad KE_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

Internal Energy of a Monoatomic Ideal Gas: $U = N\left(\frac{3}{2}kT\right) = \frac{3}{2}NkT = \frac{3}{2}nRT$