# **Lecture 29**: Wave Motion and Sound

***Physics for Engineers & Scientists* (Giancoli): Chapters 15 & 16**
***University Physics V1* (Openstax): Chapters 16 & 17**

- Travelling waves obey **Superposition**, meaning that the displacements of two different waves simply add together, superimposing one wave on top of the other.
- When waves of the same frequency combine (superimpose), it is called **Interference** and can create a resultant wave of greater (**Constructive Interference**) or lower (**Destructive Interference**) amplitude.
- Driven travelling waves may also experience **Resonance**.
- Travelling waves are **Longitudinal** if the movement of particles making the wave is parallel/anti-parallel with the direction of the wave's motion.
- Travelling waves are **Transverse** if the movement of particles making the wave is perpendicular to the direction of the wave's motion.
- Travelling waves often reflect back when they encounter boundaries and may invert (180° phase shift) upon reflection.

**Example**: The amplitude of an ocean swell is 1.50 m with crests separated 33.8 m. A wave crest strikes the beach once every 5.70 s. Determine (A) the frequency of the waves, (B) the speed of the waves, and (C) the wave number.

A = 1.50 m    λ = 33.8 m    T = 5.70 s.

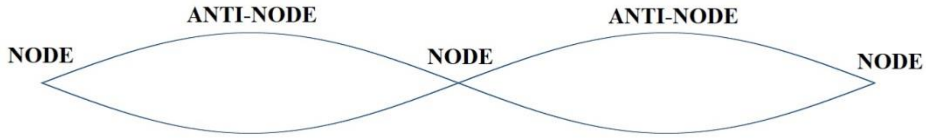
$$f = \frac{1}{T} = \frac{1}{5.70s} = 0.175\,Hz \qquad v = \frac{\lambda}{T} = \frac{33.8\,m}{5.70\,s} = 5.93\,\frac{m}{s} \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{33.8\,m} = 0.186\,m^{-1}$$

## Transverse Waves on Strings

- The velocity of waves on a string are given by: $v = \sqrt{\frac{F_T}{\mu}}$
  - $F_T$ is the tension in the string (so as not to be confused with the period T).
  - $\mu$ is the mass per unit length of the string.

  *The length of a string determines the wavelengths allowed. If we assume λ is fixed, then the frequency of the wave increases with the velocity.* $f = v/\lambda$.

  - Increasing the tension (typically by turning a tuning nut) increases the velocity of the waves, creating higher pitched notes.
  - Thicker strings have higher values of μ, and higher values of μ lead to lower velocities and lower pitched notes.
- Waves travelling down a string will reflect back from the ends of the string interfering with the original waves.
  - The majority of frequencies experience destructive interference.
  - Only a few specific modes of vibration (also known as **Harmonics**) experience constructive interference. In these cases, the superposition of the original wave and its reflections results in a **Standing Wave**, a wave that oscillates in time at amplitudes that are fixed in space.




- In each mode, certain positions called on the string **<u>Nodes</u>** have zero amplitude (no vibration). As the ends of strings are typically held in place (as the string is under tension), these must become nodes.
- Certain other positions called **<u>Anti-Nodes</u>** are vibrating with the maximum amplitude.

*All modes begin vibrating when a string is plucked. Normally the amplitudes of these various modes fall as you move to higher harmonics (making the higher modes significantly quieter). The various amplitudes are also affected by how and where a string is struck or plucked. The sum total of all of these modes produces the sound that you hear.*

- Each mode of vibration (n) occurs at a specific wavelength (and frequency) related to the string's length (L).

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{v_0}{\lambda_n} = \frac{nv_0}{2L} \qquad \#Nodes = n + 1 \qquad \#Anti\text{-}Nodes = n$$

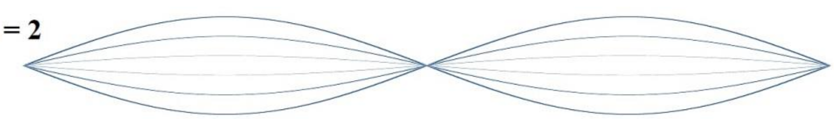
  - The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has a node at each end, and one anti-node in between.

**n = 1**

$$L = \frac{1}{2}\lambda_1 \qquad \lambda_1 = 2L \qquad f_1 = \frac{v_0}{\lambda_1} = \frac{v_0}{2L}$$

  - The second harmonic (n = 2) has a node at each end, another node in the middle, and two anti-nodes (halfway between each adjacent pair of nodes).

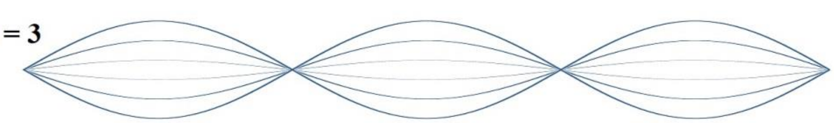
**n = 2**



$$\lambda_2 = L \qquad f_2 = \frac{v_0}{\lambda_2} = \frac{v_0}{L}$$

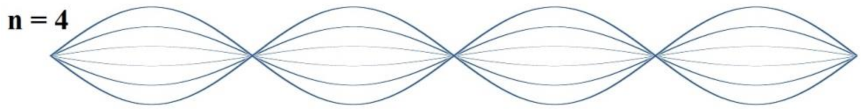
  - The third harmonic (n = 3) has a node at each end, two more nodes evenly spaced in between, and three anti-nodes (halfway between each adjacent pair of nodes).

**n = 3**



$$L = \frac{3}{2}\lambda_3 \qquad \lambda_3 = \frac{2L}{3} \qquad f_3 = \frac{v_0}{\lambda_3} = \frac{3v_0}{2L}$$

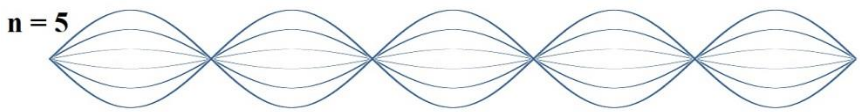
- The fourth harmonic (n = 4) has 5 nodes and 4 anti-nodes.




$$L = 2\lambda_4 \qquad \lambda_4 = \frac{L}{2} \qquad f_4 = \frac{v_0}{\lambda_4} = \frac{2v_0}{L}$$

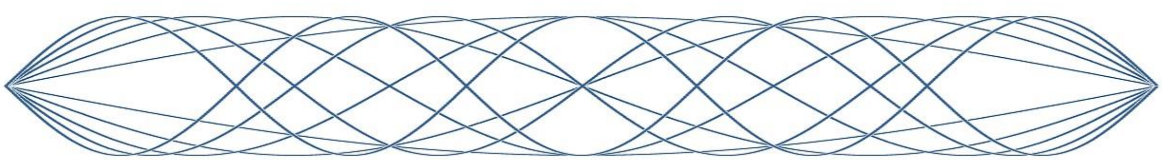
*Sometimes a guitarist will play a "harmonic" by holding their finger gently on the string without pressing it against the fret board (often to tune). This forces a node at that position, suppressing all modes of vibration that don't have this node (including the fundamental). If done at the 5th fret sounds are dominated by the 4th harmonic mode. The 7th fret gives you the 3rd harmonic mode, and the 12th fret gives you the second.*

- The fifth harmonic (n = 5) has 6 nodes and 5 anti-nodes.




$$L = \frac{5}{2}\lambda_5 \qquad \lambda_5 = \frac{2L}{5} \qquad f_5 = \frac{v_0}{\lambda_5} = \frac{5v_0}{2L}$$

*When the envelopes of the first five harmonics of a string are superimposed, it makes a rather striking image.*



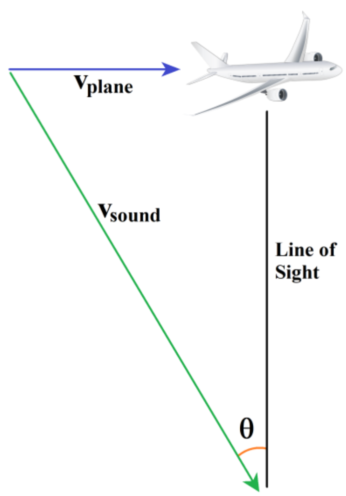
## Sound Waves

- Sound is a pressure wave (also called a compression wave).
- The velocity of sound waves in liquid or gas: $v_{sound} = \sqrt{\frac{\beta}{\rho}}$
  - $\beta$ is the bulk modulus and $\rho$ is the mass density of the medium.
  - The speed of sound in air varies with temperature. At 20°C: $v_{sound} = 343\,\frac{m}{s}$
  - The speed of sound in water depends on pressure (depth), temperature, and salinity. On average the speed of sound is 1560 m/s in saltwater and 1435 m/s in freshwater.
  - In liquids and gases, sound is strictly a longitudinal wave with alternating bands of high pressure (crests) and low pressure (troughs).
- The velocity of sound waves in a solid: $v_{sound} = \sqrt{\frac{Y}{\rho}}$
  - $Y$ is the Young's modulus and $\rho$ is the mass density of the medium.

- The speed of sound in iron is roughly 5130 m/s.
- In solids sound can propagate as either a longitudinal or a transverse wave. The transverse waves are alternating shear stress at a right angle to the propagation, and the longitudinal waves are alternating bands of high pressure (crests) and low pressure (troughs).

*Seismic activity (earthquakes) generates both (primary) longitudinal waves (P-waves) and (secondary) transverse waves (S-waves). The longitudinal waves travel at roughly 5000 m/s (in granite) while the slower transverse waves travel at only 3000 m/s. Typically the transverse waves do greater damage as they are typically created with larger amplitudes.*

**Example:** As a plane flies overhead you notice that the sound of the engines appears to be coming from a spot 20.0° behind the aircraft. How fast is the airplane moving?




$$v_{airplane} = v_{sound} \sin\theta = \left(343 \frac{m}{s}\right) \sin 20.0° = 117 \frac{m}{s}$$

*117 m/s is about 262 mph.*
*At altitudes below 10,000 ft., aircraft are limited to a maximum speed of 250 knots (288 mph).*

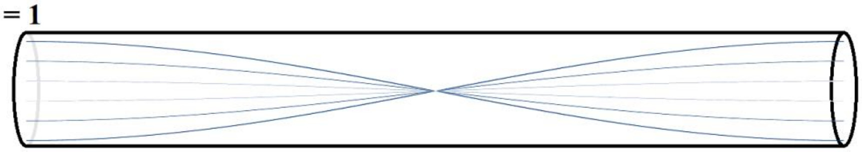
## Longitudinal Waves in a Pipe

- A closed end of a pipe creates a node while an open end creates an anti-node.
- Each mode of vibration (n) occurs at a specific wavelength (and frequency) related to the pipe's length (L), depending upon whether one end or both ends are open.
- Harmonics in pipe with two open ends:

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{v_{sound}}{\lambda_n} = \frac{n v_{sound}}{2L} \qquad \#Nodes = n \qquad \#Anti\text{-}Nodes = n + 1$$

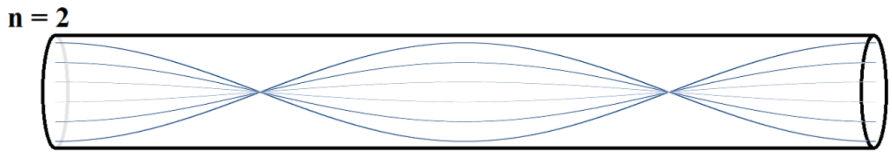
  - The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has an anti-node at each end, and one node in between.

**n = 1**



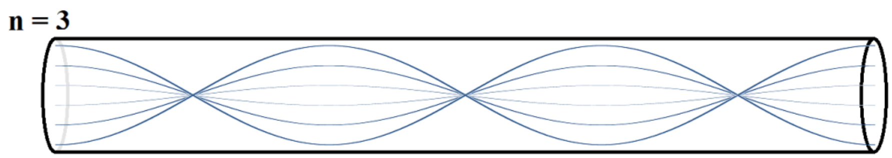
$$L = \frac{1}{2}\lambda_1 \qquad \lambda_1 = 2L \qquad f_1 = \frac{v_{sound}}{\lambda_1} = \frac{v_{sound}}{2L}$$

- The second harmonic (n = 2) has an anti-node at each end, another anti-node in the middle, and two nodes (halfway between each adjacent pair of anti-nodes).



$$\lambda_2 = L \qquad f_2 = \frac{v_{sound}}{\lambda_2} = \frac{v_{sound}}{L}$$

- The third harmonic (n = 3) has an anti-node at each end, two more anti-nodes evenly spaced in between, and three nodes (halfway between each adjacent pair of anti-nodes).

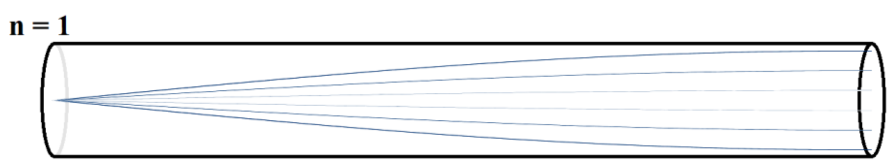


$$L = \frac{3}{2}\lambda_3 \qquad \lambda_3 = \frac{2L}{3} \qquad f_3 = \frac{v_{sound}}{\lambda_3} = \frac{3v_{sound}}{2L}$$

- Harmonics in pipe with one end open and one end closed:

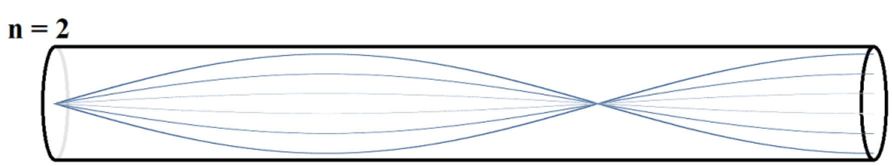
$$\lambda_n = \frac{4L}{2n-1} \qquad f_n = \frac{v_{sound}}{\lambda_n} = \frac{(2n-1)v_{sound}}{4L} \qquad \#Nodes = n \qquad \#Anti\text{-}Nodes = n$$

- The first harmonic or fundamental mode (n = 1) is the simplest and usually the loudest tone heard (largest amplitude). It has a node at one end and an anti-node at the other.



$$L = \frac{1}{4}\lambda_1 \qquad \lambda_1 = 4L \qquad f_1 = \frac{v_{sound}}{\lambda_1} = \frac{v_{sound}}{4L}$$

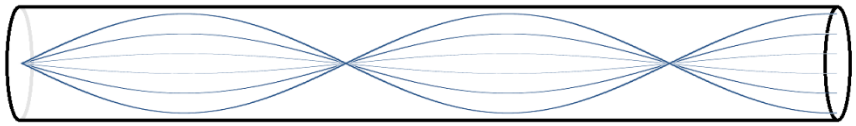
- The second harmonic (n = 2) has a node and an anti-nodes at either end, with another node and anti-node in the middle.



$$L = \frac{3}{4}\lambda_2 \qquad \lambda_2 = \frac{4}{3}L \qquad f_2 = \frac{v_{sound}}{\lambda_2} = \frac{3v_{sound}}{4L}$$

- The third harmonic (n = 3) has three nodes and three anti-nodes.

n = 3



$$L = \frac{5}{4}\lambda_3 \qquad \lambda_3 = \frac{4L}{5} \qquad f_3 = \frac{v_{sound}}{\lambda_3} = \frac{5v_{sound}}{4L}$$

*Sometimes musicians playing horns will cover the opening, providing a drop in frequency (one octave) and a change in the harmonic frequencies.*

**Example**: Two pipes are the same length. The first pipe has both ends open, and the second has one closed end. If the both pipes have their 4th harmonic excited, which pipe produces the higher pitch?

1st Pipe: $f_n = \frac{nv_{sound}}{2L} = \frac{4v_{sound}}{2L} = 2\frac{v_{sound}}{L}$

2nd Pipe: $f_n = \frac{(2n-1)v_{sound}}{4L} = \frac{[2(4)-1]v_{sound}}{4L} = \frac{7}{4}\frac{v_{sound}}{L}$

As $2 > \frac{7}{4}$, the 1st pipe produces the higher pitch.

## Intensity (I): $I = \frac{P}{A}$

- The **Intensity (I)** of a wave is the power per unit area carried by the wave. Units: $\frac{W}{m^2}$
- Typically a time averaged value is used for power.
- As sound tends to radiate spherically, the intensity will drop with the square of the radius.

## Sound Levels ($\beta$) $\beta(dB) = 10log\left(\frac{I}{I_0}\right)$

- Normal human hearing is sensitive to frequencies from 20 Hz to 20 kHz, but is most sensitive to sounds between 1 kHz and 4 kHz.
- The minimum intensity that we can hear is $I_0 = 10^{-12}\frac{W}{m^2}$
- As human hearing is sensitive to a wide range of intensity, a log scale is used for sound levels.

$$\beta(dB) = 10log\left(\frac{I}{I_0}\right)$$

- Sound below 75 dB typically does no damage to hearing.
    - Breathing (10 dB) is barely audible.
    - Whispering or a Quiet Rural Area (30 dB) is very quiet.
    - Conversation in restaurant, office background music, or an air conditioner at 100 ft. (60 dB) is fairly quiet.
    - Vacuum Cleaner (70 dB)

- Intense sounds (85 dB and above) can damage a person's ability to hear depending upon the duration of exposure.
    - 8 hours of exposure to sounds over 90 dB can possibly cause damage. This includes the sound of a power mower (96 dB), being 25 ft. from a motorcycle (90 dB), or being 1 nautical mile (6080 ft.) from a landing commercial aircraft (97 dB)
    - Sounds at the average human pain threshold of 110 dB such as a car horn at 1m (110 dB) or live music at a rock concert (108 to 114 dB) can cause damage in minutes.
    - Sounds at 120dB (such as a chainsaw) are painful and can damage an ear after only seconds.
    - Sounds at 150 dB (such a jet taking off at 25 meters) can rupture eardrums.