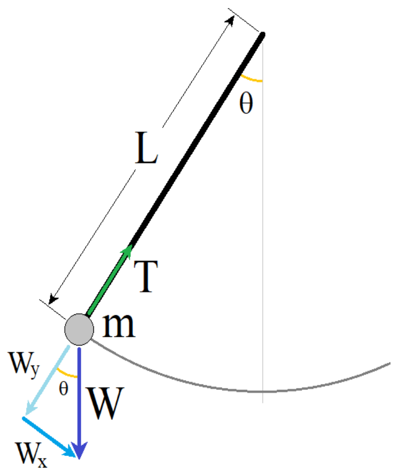
Lecture 28: Pendulums and Wave Motion

Physics for Engineers & Scientists (Giancoli): Chapters 14 & 15
University Physics V1 (Openstax): Chapters 15 & 16

Pendulum

- We will treat this as 1-dimensional simple harmonic motion along the arc made by the hanging mass.



Weight (W) and Tension (T) act on the hanging mass.

One component of the weight (W_y) cancels out the tension.

The other component of the weight (W_x) acts as the restoring force.

If we can find k, then we can use $\omega = \sqrt{\frac{k}{m}}$

$$k = -\frac{F}{x} = -\frac{W_x}{-L\theta} = \frac{mg\sin\theta}{L\theta} \approx \frac{mg\theta}{L\theta} = \frac{mg}{L}$$

In the small angle approximation, sin θ ≈ θ

$$\omega = \sqrt{\frac{mg}{Lm}} = \sqrt{\frac{g}{L}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

- The period/frequency of a pendulum is independent of mass.

Example: The pendulum in Big Ben has a 299 kg bob and a period of 2 seconds. What is the length of the arm of this pendulum?

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \frac{T}{2\pi} = \sqrt{\frac{L}{g}} \qquad \frac{T^2}{4\pi^2} = \frac{L}{g} \qquad L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\frac{m}{s^2}\right)(2.00\,s)^2}{4\pi^2} = 99.3\,cm$$

Example: Wilson Hall, the picturesque administrative building at Fermilab, used to have a rather slow-moving pendulum hanging from the very top of the building, 16 floors high (roughly 160 ft.). How long does it take this pendulum to make one complete cycle?

$$L = (160\,ft.)\frac{(0.3048\,m)}{(1\,ft.)} = 48.768\,m$$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{48.768\,m}{9.80\frac{m}{s^2}}} = 14.0\,s$$

Damping, Driving, and Resonance

- Springs are useful for absorbing impacts.
- Choosing the spring constant allows smaller deceleration to occur over a larger distance, reducing the force of impact.
- Springs absorb and release a portion of the impact energy, which can lead to unwanted harmonic motion.
- In **Damped Harmonic Motion**, energy is steadily removed from the system resulting in decreasing amplitude.

Cars are suspended on springs so that when you drive over a bump the deceleration is gentler. If the absorbed collision energy is not dissipated by damping (shock absorbers), then your vehicle would continue to bounce.

- In **Driven Harmonic Motion**, energy is added from an outside source.
  - How a driven harmonic oscillator behaves is dependent upon both the frequency of the driving force and the natural frequency of the oscillator.
  - **Resonance** occurs when the driving frequency and the oscillator frequency match. When this happen, energy is continually added to the system.

When pushing a child in a swing, timing your pushes to the timing of the swing results in the child swinging higher and higher (ever increasing amplitude) even as the friction in the system (damping) causes their swinging to slow down and lose height.

The Wave Equation and its Solutions

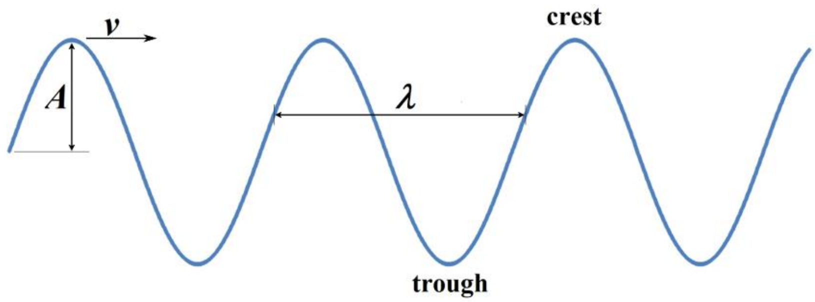
- In some instances, Newton's laws lead to a partial differential equation known as the Wave Equation.

$$\frac{\partial^2 D}{\partial t^2} - v^2 \frac{\partial^2 D}{\partial x^2} = 0$$

D = Displacement as measured from equilibrium.
The constant (v^2) is the square of the velocity of the wave.

- The solutions to this equation are travelling waves, either a sine function, a cosine function, or a combination of the two depending upon the initial phase.

$$D = A\sin(kx - \omega t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$



  - The Wave Number (k) is the spatial frequency of the wave (cycles per unit distance).

$$k = \frac{2\pi}{\lambda} \qquad v = \frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda = \frac{\lambda}{T}$$

  - The velocity (v) is determined by the properties of the medium through which the wave moves.
  - The frequency (f), angular frequency (ω), period (T), wavelength (λ), and wave number (k) are all inter-related and determined by whatever excitation created the wave.