Lecture 2: One-Dimensional Kinematics

Physics for Engineers & Scientists (Giancoli): Chapter 2
University Physics V1 (Openstax): Chapter 3

Example: To make it to an important interview on time, a driver needs to average 65.0 mph over the 240 miles trip from Dallas to Houston. At the halfway point the driver has only averaged 55.0 mph. How fast does he need to go the rest of the way to reach his appointment on time?

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

Distance Travelled = (240 miles)/2 = 120 miles

Time (2nd half) = Time (total) – Time (1st half)

$$\text{Average Speed} \times \text{Elapsed Time} = \text{Total Distance Travelled}$$

$$\text{Elapsed Time (first half)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{120\text{ miles}}{55.0\frac{\text{miles}}{\text{hr}}} = 2.182\text{ hrs} \Rightarrow 2.2\text{ hrs}$$

$$\text{Elapsed Time (whole trip)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{240\text{ miles}}{65.0\frac{\text{miles}}{\text{hr}}} = 3.692\text{ hrs} \Rightarrow 3.7\text{ hrs}$$

Time (2nd half) = Time (total) – Time (1st half) = 3.692 hrs – 2.182 hrs = 1.510 hrs ⇒ 1.5 hrs

$$\text{Average Speed(2nd half)} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} = \frac{120\text{ miles}}{1.510\text{ hrs}} = 79.47\text{ mph} \Rightarrow 79\text{ mph}$$

You might think that the answer is 75 mph, but this is wrong!

At 75 mph it takes 1.6 hours to cover 120 miles. This would make the total trip time 3.782 hours instead of the desired 3.692 hours. You would be 5 minutes late!

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

This is a ratio, and ratios don't work like that!

Suppose during target practice you hit the target on 2 of your first 8 shots, and after a break, you hit it once more on your last 2 shots. Then you hit the target on 3 out of 10 shots. You do NOT get to say that you hit 25% of your shots in the first batch and 50% in the second batch and average these together to say you hit 37.5%

$$\frac{2+1}{8+2} = \frac{3}{10} \neq \left(\frac{2}{8} + \frac{1}{2}\right)\frac{1}{2} = \frac{3}{8}$$

You can only take the average of velocities when they all have the same time interval.

Acceleration **Average acceleration** $= a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$$\text{(Instantaneous)}\mathbf{Acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- The units of acceleration are m/s^2 (length per time squared).

Knowing the units can help identify the quantities in problems.

- Acceleration can be positive or negative.
- Usually the word "instantaneous" is dropped. It is the default.
- In one-dimensional motion, when the sign on the acceleration and velocity match, the object is said to be "accelerating" (i.e. its speed is increasing). When the signs on acceleration and velocity differ, the object is said to be "decelerating" (i.e. its speed is decreasing).

Example: A top fuel dragster is capable of accelerating from rest to 160 km/hr (~100 mph) in 0.80 seconds. What is the average acceleration of a dragster that does this?

$$160\,\frac{km}{hr}\left\{\frac{1000\,m}{1\,km}\right\}\left\{\frac{1\,hr}{60\,min}\right\}\left\{\frac{1\,min}{60\,s}\right\} = 44.444\,\frac{m}{s} \quad\Rightarrow\quad 44\,\frac{m}{s}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} = \frac{44.444\frac{m}{s} - 0\frac{m}{s}}{0.80\ s} = 55.555\,\frac{m}{s^2} \quad\Rightarrow\quad 56\,\frac{m}{s^2}$$

Example: An object's position is given by $x(t) = (5.0\ m) + (3.0\ m/s)t + (2.5\ m/s^2)t^2$. Determine the velocity and acceleration as a function of t.

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{(5.0\ m) + \left(3.0\frac{m}{s}\right)t + \left(2.5\frac{m}{s^2}\right)t^2\right\} = \left(3.0\frac{m}{s}\right) + \left(5.0\frac{m}{s^2}\right)t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{\left(3.0\frac{m}{s}\right) + \left(5.0\frac{m}{s^2}\right)t\right\} = 5.0\frac{m}{s^2}$$

Example: An object accelerates uniformly at $5.0\ m/s^2$ with an initial velocity of 3.0 m/s and an initial position of 5.0 m. Determine the objects position as a function of time.

$$a = \frac{dv}{dt} \qquad a \cdot dt = dv \qquad \int a \cdot dt = \int dv = v$$

$$v = \int a \cdot dt = at + C = at + v_0 = \left(5.0\frac{m}{s^2}\right)t + 3.0\frac{m}{s}$$

$$v = \frac{dx}{dt} \qquad v \cdot dt = dx \qquad \int v \cdot dt = \int dx = x$$

$$x = \int v \cdot dt = \int (at + v_0) \cdot dt = \frac{1}{2}at^2 + v_0 t + C = \frac{1}{2}at^2 + v_0 t + x_0$$

$$x = \left(2.5\frac{m}{s^2}\right)t^2 + \left(3.0\frac{m}{s}\right)t + (5.0\ m)$$

Constant Acceleration (A special case)

- When acceleration is constant, the average acceleration and the instantaneous acceleration are the same ($a = a_{avg}$).
  - $a = a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$
- When acceleration is constant, $v_{avg} = \frac{(v + v_0)}{2}$

This is true because the velocity is increasing linearly.
The average of 3, 4, 5, 6, and 7 is just (3+7)/2 = 5.

- We can derive a set of **four equations**.
  - Solve the acceleration equation for v to get the first equation.

$$a = \frac{v - v_0}{t} \qquad at = v - v_0 \qquad at + v_0 = v$$

$$\boxed{\mathbf{v = v_0 + at}}$$

v, a, and t are variables. v_0 is a constant.

  - To get our second equation, solve the equation for average velocity for x and then plug in the previous equation for v_{avg}.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} \qquad (v_{avg})t = x - x_0 \qquad (v_{avg})t + x_0 = x \qquad \frac{1}{2}(v + v_0)t + x_0 = x$$

$$\boxed{\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}}$$

x, v and t are variables. x_0 and v_0 are constants.

  - To get our third equation, plug the value of v from the first equation into the second.

$$x = x_0 + \frac{1}{2}\{v + v_0\}t \qquad x = x_0 + \frac{1}{2}\{(v_0 + at) + v_0\}t \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\boxed{\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}}$$

x, a and t are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

  - To get our fourth (and final) equation, solve the first equation for t, plug that into the 2nd equation, and solve for v^2.

$$v = v_0 + at \qquad v - v_0 = at \qquad t = \frac{v - v_0}{a}$$

$$x = x_0 + \frac{1}{2}(v + v_0)t \qquad x = x_0 + \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$x - x_0 = \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right) \qquad 2a(x - x_0) = (v + v_0)(v - v_0)$$

$$2a(x - x_0) = v^2 - v_0^2 \qquad v_0^2 + 2a(x - x_0) = v^2$$

$$\boxed{\mathbf{v^2 = v_0^2 + 2a(x - x_0)}}$$

x, v and a are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

- There are 4 variables (x, v, a, and t). Each equation has only 3 of them.

$$\mathbf{v = v_0 + at}$$ v, a, and t $\Rightarrow$ **No x**

$$\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}$$ x, v, and t $\Rightarrow$ **No a**

$$\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}$$ x, a, and t $\Rightarrow$ **No v**

$$\mathbf{v^2 = v_0^2 + 2a(x - x_0)}$$ x, v, and a $\Rightarrow$ **No t**

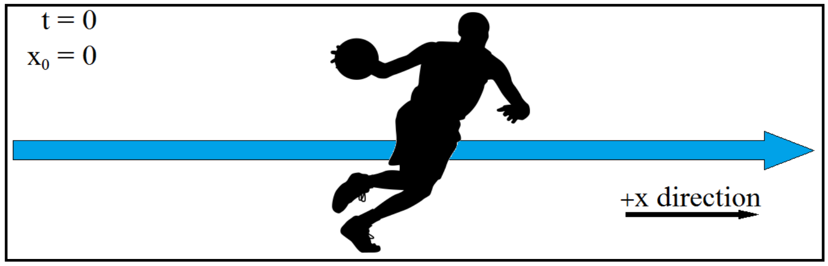
When given the values of two variables and looking for a value of a third, find the equation with all three variables. This is done easily after you determine the 4th variable (missing).

- This set of 4 equations is valid for one object moving with constant acceleration.
  - If acceleration is not constant, you must use the definitions rather than these equations.
  - If there is more than one object moving, you may need two sets of these equations (one for each object).
  - If an object moves with constant acceleration but then suddenly shifts to a new constant acceleration, then you must use a separate set of these four equations for before and after. Usually it's best to treat the before and after as two separate problems.

- Problem solving
  - First, draw a diagram if needed. It can help.
  - Second, set your reference frame (when is t = 0/where is x = 0).
  - Third, extract the data (values of variables and constants) from the problem. Units will help you determine what variables to use for each number.
  - Fourth, determine the formula (or formulas) that you need (i.e. find the path that leads to your answer).
  - Fifth, do the math.

Example: In getting ready to slam-dunk a ball, a player starts from rest and sprints to a speed of 6.0 m/s in 1.5s. Assuming he accelerates uniformly, determine the distance he runs.

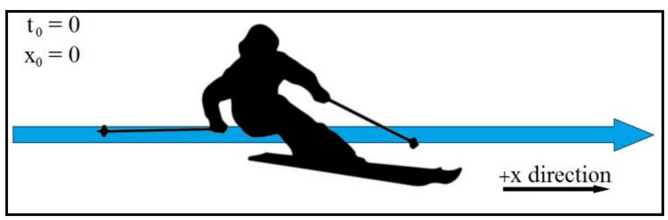
- Draw a diagram.
- Set reference frame.



- Extract data : $x_0 = 0$ $v_0 = 0$ $v = 6.0$ m/s $t = 1.5$s $x=?$
- Determine formula: "a" is missing $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
- Do the math: $x = \cancel{x_0}^{0} + \frac{1}{2}(v + \cancel{v_0}^{0})t = \frac{1}{2}vt = \frac{1}{2}\left(6.0\,\frac{m}{s}\right)(1.5\ s) = 4.5\ m$

Example: (a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0m/s when going down a slope for 5.0s (b) How far does the skier travel in this time?

- Part A
    - Draw a diagram.
    - Set reference frame.



- Extract data : $x_0 = 0$ $a=?$ $v_0 = 0$ $v = 8.0$ m/s $t = 5.0$s
- Determine formula: "x" is missing $\Rightarrow$ $v = v_0 + at$
- Do the math: $v = \cancel{v_0}^{0} + at = at$ $\quad a = \frac{v}{t} = \frac{8.0\frac{m}{s}}{5.0\ s} = 1.6\,\frac{m}{s^2}$

- Part B
    - Determine formula: Any formula with "x" will do $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
    - Do the math: $x = \cancel{x_0}^{0} + \frac{1}{2}(v + \cancel{v_0}^{0})t = \frac{1}{2}vt = \frac{1}{2}\left(8.0\,\frac{m}{s}\right)(5.0\ s) = 20.\ m$