Lecture 26: Rotational Dynamics & Static Equilibrium

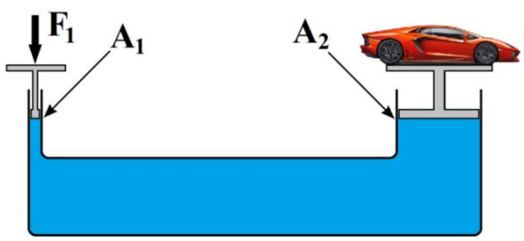
Physics for Engineers & Scientists (Giancoli): Chapter 13
University Physics V1 (Openstax): Chapter 14

Pascal's Principle

"Any change in pressure applied to a completely enclosed fluid is transmitted undiminished to every part of the fluid and to the walls enclosing it."

This is a direct result of $P_2 = P_1 + \rho g h$.

Any change to P_1 changes P_2 as well, and P_2 could be anywhere in the fluid.



The two pistons are at the same height. They must have the same pressure.

$$P_1 = P_{atm} + \frac{F_1}{A_1} \qquad P_2 = P_{atm} + \frac{F_2}{A_2}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_1 = \frac{A_1}{A_2} F_2$$

The mechanical advantage created in this way is the basis of hydraulics.

Example: The input piston of a hydraulic lift in an automotive repair shop has a radius of 1.00 cm. The output piston has a radius of 32.0 cm. What force is needed at the input piston to lift the 1570 kg car on the output piston?

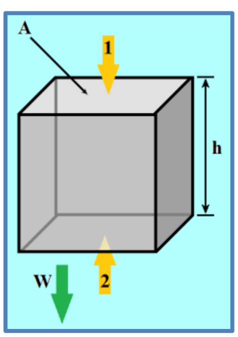
$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi r_1^2}{\pi r_2^2} F_2 = \left(\frac{r_1}{r_2}\right)^2 F_2 = \left(\frac{r_1}{r_2}\right)^2 mg = \left(\frac{1.00\ cm}{32.0\ cm}\right)^2 (1570\ kg)\left(9.80 \frac{m}{s^2}\right) = 15.0\ N$$

Energy is still conserved. For every millimeter the car moves, the input piston must move a meter.

Archimedes' Principle

"The buoyant force is equal to the weight of the fluid displaced."

- The net sum of the force of pressure on all sides of an object is called the Buoyant Force, and it points upward.



Let's take a large mass of a static fluid and place and object inside. We can calculate the buoyant force on this object.

$$F_B = F_2 - F_1 = P_2 A - P_1 A = [P_2 - P_1]A =$$

$$F_B = [(P_1 + \rho_F g h) - P_1]A = \rho_F g h A = \rho_F g V =$$

$$F_B = m_{DF} g = W_{DF}$$

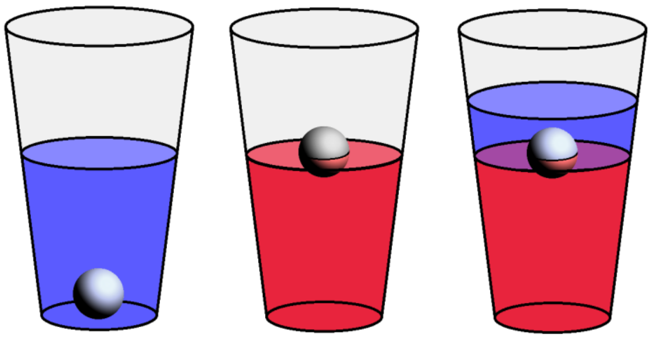
Archimedes' Principle: The buoyant force on an object is equal to the weight of the fluid displaced.

If the object is replaced with fluid (the same fluid), the fluid filling the boundaries is static. If the fluid is static, then the net force must be zero. If the net force is zero, then the buoyant force must equal to the weight. The buoyant force doesn't change when we return the original object.

- To get the Apparent Weight (W_{App}) of an object subtract the buoyant force from the weight.

Objects in water are easier to lift as the buoyant force helps us.

Conceptual Example: If blue fluid is added to the glass with red fluid, will the ball rise or fall?



The grey ball sinks to the bottom when it is placed in the glass with the blue fluid.

The grey ball floats with half of its volume submerged when placed in the glass with the red fluid.

If the blue fluid is slowly added to the top of the red fluid (with the ball floating on top) will the ball rise or fall?

Before the blue fluid is added, the ball has half of its volume submerged.

$$F_B = W_{DF} = \rho_{RF}\left(\tfrac{1}{2}V_{Ball}\right)g + \rho_{Air}\left(\tfrac{1}{2}V_{Ball}\right)g = \left(\tfrac{1}{2}\rho_{RF} + \tfrac{1}{2}\rho_{Air}\right)V_{Ball}\cdot g = W_{Ball}$$

If the ball remains in the same place (when the blue fluid is added), here is the buoyant force:

$$F_B = W_{DF} = \rho_{RF}\left(\tfrac{1}{2}V_{Ball}\right)g + \rho_{BF}\left(\tfrac{1}{2}V_{Ball}\right)g = \left(\tfrac{1}{2}\rho_{RF} + \tfrac{1}{2}\rho_{BF}\right)V_{Ball}\cdot g$$

As $\rho_{BF} > \rho_{Air}$, then: $\left(\tfrac{1}{2}\rho_{RF} + \tfrac{1}{2}\rho_{BF}\right)V_{Ball}\cdot g > \left(\tfrac{1}{2}\rho_{RF} + \tfrac{1}{2}\rho_{Air}\right)V_{Ball}\cdot g = W_{Ball}$

As the buoyant force is greater than the mass of the ball, the ball will rise until the buoyant force is equal to the weight.

Example: A duck is floating on the water with half of its volume underwater. Determine the density of the duck.

$$F_B = W_{Duck} = m_{Duck}\cdot g = \rho_{Duck}\cdot V_{Duck}\cdot g$$

$$F_B = W_{DF} = m_{DF}\cdot g = \rho_{DF}\cdot V_{DF}\cdot g = \rho_{DF}\cdot\left(\frac{1}{2}V_{Duck}\right)\cdot g = \frac{1}{2}\rho_{DF}\cdot V_{Duck}\cdot g$$

$$\rho_{Duck}\cdot V_{Duck}\cdot g = \frac{1}{2}\rho_{DF}\cdot V_{Duck}\cdot g$$

$$\rho_{Duck} = \frac{1}{2}\rho_{DF} = \frac{1}{2}\left(1000\frac{kg}{m^3}\right) = 500\frac{kg}{m^3}$$

Fluids in Motion

- In **Steady Flow**, all the particles are moving at the same speed as they pass a given point.
- In **Unsteady Flow**, particles are moving at different speeds as they pass a given point.
- In **Turbulent Flow**, velocities can change radically at a given point.
- A fluid is **Compressible** if the density of the fluid changes with pressure.
- A fluid is **Incompressible** if the density of the fluid is constant with changes with pressure.

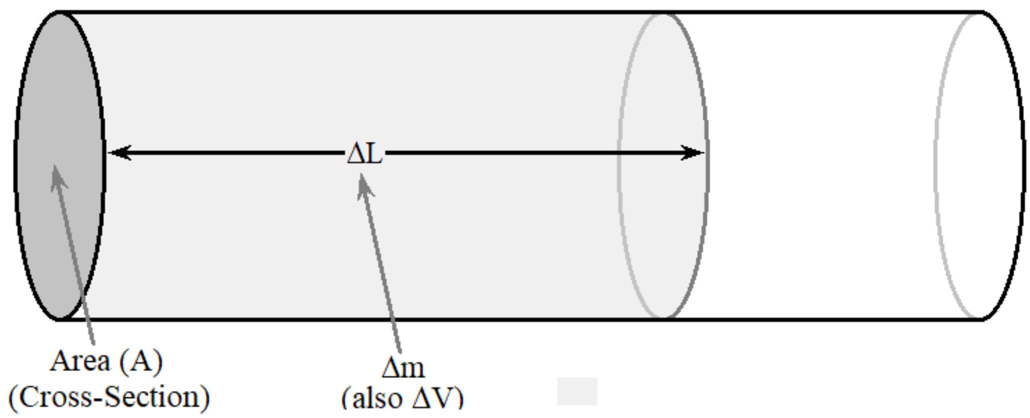
Fluids are nearly incompressible/ Gases are compressible.

- A **Viscous** fluid doesn't flow readily.
- A **Non-viscous** fluid flows readily.

We will assume that all fluids are Ideal Fluids, meaning they are incompressible and non-viscous. We will also assume that the flow is steady.

Mass Flow Rate

- **Mass Flow Rate** is the amount of mass that passes through a cross-section of the pipe in a given time interval.



$$MFR = \frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta L}{\Delta t} = \rho A v$$

ΔV is the volume occupied by the mass that will move through the cross section. ΔL is the length of ΔV, and if this masses through the cross-section in Δt, then the velocity of the fluid must be $\Delta L / \Delta t$.

- Contained fluid (such as in a pipe) has a fixed volume. If we assume it is an ideal fluid with a constant density, then it also has a fixed mass. Any mass that enters one end implies that an equal mass leave the other end. In other words, the mass flow rate is constant.

$$\rho A_1 v_1 = \rho A_2 v_2$$

- Dividing out the density gives the **Volume Flow Rate**, which is also constant.

$$A_1 v_1 = A_2 v_2$$

Example: Water flows through a pipe of diameter 0.500 m at a velocity 0.250 m/s. The water flow is then constricted to a pipe of diameter 0.250 m. Determine A) the mass flow rate, and B) the velocity in the narrow pipe.

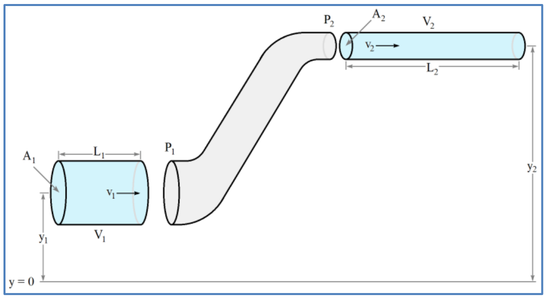
$$MFR = \rho A v = \left(1000 \frac{kg}{m^3}\right) \pi \left(\frac{0.500\, m}{2}\right)^2 \left(0.250 \frac{m}{s}\right) = 49.1 \frac{kg}{s}$$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{0.500 \text{ m}}{0.250 \text{ m}}\right)^2 \left(0.250 \frac{\text{m}}{\text{s}}\right) = 1.00 \frac{m}{s}$$

The Bernoulli Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$



If we push water into the pipe, an equal amount of water must come out the other side. ($V_1=V_2$)

If there is steady flow through the pipe, then energy must be conserved in this process.

The initial and final states have kinetic and gravitational potential energy, but energy is also added by work done by pressure.

$$E_{Init} = \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}\rho V_1 v_1^2 + \rho V_1 g y_1$$

$$E_{Final} = \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}\rho V_2 v_2^2 + \rho V_2 g y_2$$

$$E_{added} = W_1 + W_2 = F_1 L_1 - F_2 L_2 = P_1 A_1 L_1 - P_2 A_2 L_2 = P_1 V_1 - P_2 V_2$$

$$E_{Init} + E_{added} = E_{Final}$$

$$\frac{1}{2}\rho V_1 v_1^2 + \rho V_1 g y_1 + P_1 V_1 - P_2 V_2 = \frac{1}{2}\rho V_2 v_2^2 + \rho V_2 g y_2$$

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 - P_2 = \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\boldsymbol{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$