

Lecture 2: One Dimensional Kinematics.

Physics for Engineers & Scientists (Giancoli): Chapter 2

University Physics V1 (Openstax): Chapter 3

Mechanics, the study of motion, has two parts

- **Kinematics** is the mathematical description of motion. (Our starting point)
- **Dynamics** is the study of the causes of motion (including forces, energy, etc.)

There are 3 types of motion

- **Translational motion** is moving from one location in space to another.
- **Rotational Motion** is changing orientation (direction) without necessarily changing location.
- **Vibrational Motion** is oscillatory motion about an average position.

Translational Kinematics is the Mathematical description of how position changes with time.

- **Position** is given by three variables, the point (x, y, z).
- We use a specific point in each object (or group of objects) called “**the center of mass**”.
 - All motion can be described as a combination of translation of the center of mass, rotation about the center of mass, and vibration about the center of mass.
 - This allows us to treat each and **every object as a point**. It doesn't matter what the object is.

How to find the center of mass will be discussed later in the semester.

- To determine position we must also have a **Reference Frame**, a coordinate axis (mathematics) that is overlaid on top of our reality.
 - There is no fixed way to place a reference frame. You may choose the location of the origin and the directions of the axes. In some cases the origin may even move.
 - While all choices of reference frames are equally valid, some choices can greatly simplify the math needed to solve a problem.
- Each of the position variables must be a function of time.

$$x = x(t) \quad y = y(t) \quad z = z(t)$$

One-Dimensional (1D) Translational Kinematics (simplest case with only x, no y or z)

- x = position at time t , the “current position” for whatever time you prefer (a variable).
- x_0 = position at time $t=0$, the initial position for the reference frame you've chosen (a constant).
 - When looking at the position of the same object at different times (apart from $t=0$) we will use x_1, x_2, x_3 , etc. to denote the position at times t_1, t_2, t_3 , etc, respectively.
- $\Delta x = x - x_0 =$ **Displacement**.
 - Displacement is a vector. In 1D motion this just means it has a sign attached (+ or –)
 - If x_0 is greater than x , Δx will be negative. (*Displacement can be negative*)

Position (x) and initial position (x_0) are dependent upon your choice of reference frame. Displacement (Δx) is not (at least not until we hit special relativity)

Speed and Velocity (similar but different things)

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} \quad \text{Average Velocity} = \frac{\text{Displacement}}{\text{Elapsed Time}} = v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- The units of speed and velocity are **m/s** (length per time).

Knowing the units can help identify the quantities in problems.

- Speed (distance) is always positive while velocity (displacement) can be negative.

In NASCAR races the average speed sometimes exceeds 200 mph, but as they return to their starting position (zero displacement) their average velocity is zero.

$$(\text{Instantaneous}) \text{Velocity} = v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (\text{Instantaneous}) \text{Speed} = |v|$$

- Usually the word “instantaneous” is dropped. It is the default.
- Instantaneous Speed is also always positive while instantaneous velocity can be negative.

Example: Pat is in a race. Assume the track runs along the x-axis with the finish line marked as $x = 0$ (exact). During a 3.00 s time interval, Pat’s position changes from $x_1 = 60.0$ m to $x_2 = 40.5$ m. A) What is Pat’s average velocity during this time? B) If he continues to run with the same average velocity, how long does it take him to finish the race?

$$\text{A) } \Delta x = x_2 - x_1 = 40.5 \text{ m} - 60.0 \text{ m} = -19.5 \text{ m} \quad v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-19.5 \text{ m}}{3.00 \text{ s}} = -6.50 \frac{\text{m}}{\text{s}}$$

$$\text{B) } v_{\text{avg}} = \frac{\Delta x}{\Delta t} \quad v_{\text{avg}} \cdot \Delta t = \Delta x \quad \Delta t = \frac{\Delta x}{v_{\text{avg}}} = \frac{x_f - x_2}{v_{\text{avg}}} = \frac{0 \text{ m} - 40.5 \text{ m}}{-6.50 \frac{\text{m}}{\text{s}}} = 6.23 \text{ s}$$

Example: A car travels 274 miles from Dallas to San Antonio in 4.50 hours and then turns around and drives 81 miles back from San Antonio to Austin in 1.40 hours. Over this full trip from Dallas to Austin, determine A) the car’s average speed, and B) the magnitude of the car’s average velocity.

$$\text{A) Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} = \frac{274 \text{ miles} + 81 \text{ miles}}{4.50 \text{ hours} + 1.40 \text{ hours}} = \frac{355 \text{ miles}}{5.90 \text{ hours}} = 60.2 \text{ mph}$$

$$\text{B) Average Velocity} = \frac{\text{Displacement}}{\text{Elapsed Time}} = \frac{274 \text{ miles} - 81 \text{ miles}}{4.50 \text{ hours} + 1.40 \text{ hours}} = \frac{193 \text{ miles}}{5.90 \text{ hours}} = 32.7 \text{ mph}$$

Example: To make it to an important interview on time, a driver needs to average 65.0 mph over the 240 miles trip from Dallas to Houston. At the halfway point the driver has only averaged 55.0 mph. How fast does he need to go the rest of the way to reach his appointment on time?

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

$$\text{Distance Travelled} = (240 \text{ miles})/2 = 120 \text{ miles}$$

$$\text{Time (2}^{\text{nd}} \text{ half)} = \text{Time (total)} - \text{Time (1}^{\text{st}} \text{ half)}$$

$$\text{Average Speed} \times \text{Elapsed Time} = \text{Total Distance Travelled}$$

$$\text{Elapsed Time (first half)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{120 \text{ miles}}{55.0 \frac{\text{miles}}{\text{hr}}} = 2.182 \text{ hrs} \Rightarrow 2.2 \text{ hrs}$$

$$\text{Elapsed Time (whole trip)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{240 \text{ miles}}{65.0 \frac{\text{miles}}{\text{hr}}} = 3.692 \text{ hrs} \Rightarrow 3.7 \text{ hrs}$$

Time (2nd half) = Time (total) – Time (1st half) = 3.692 hrs – 2.182 hrs = 1.510 hrs $\Rightarrow$ 1.5 hrs

$$\text{Average Speed (2nd half)} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} = \frac{120 \text{ miles}}{1.510 \text{ hrs}} = 79.47 \text{ mph} \Rightarrow 79 \text{ mph}$$

You might think that the answer is 75 mph, but this is wrong!

At 75 mph it takes 1.6 hours to cover 120 miles. This would make the total trip time 3.782 hours instead of the desired 3.692 hours. You would be 5 minutes late!

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

This is a ratio, and ratios don't work like that!

Suppose during target practice you hit the target on 2 of your first 8 shots, and after a break, you hit it once more on your last 2 shots. Then you hit the target on 3 out of 10 shots. You do NOT get to say that you hit 25% of your shots in the first batch and 50% in the second batch and average these together to say you hit 37.5%

$$\frac{2+1}{8+2} = \frac{3}{10} \neq \left(\frac{2}{8} + \frac{1}{2}\right) \frac{1}{2} = \frac{3}{8}$$

You can only take the average of velocities when they all have the same time interval.

Acceleration

$$\text{Average acceleration} = a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$(\text{Instantaneous}) \text{Acceleration} = a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2 x}{dt^2}$$

- The units of acceleration are m/s² (length per time squared).

Knowing the units can help identify the quantities in problems.

- Acceleration can be positive or negative.
- Usually the word “instantaneous” is dropped. It is the default.
- In one-dimensional motion, when the sign on the acceleration and velocity match, the object is said to be “accelerating” (i.e. its speed is increasing). When the signs on acceleration and velocity differ, the object is said to be “decelerating” (i.e. its speed is decreasing).

Example: A top fuel dragster is capable of accelerating from rest to 160 km/hr (~100 mph) in 0.80 seconds. What is the average acceleration of a dragster that does this?

$$160 \frac{\text{km}}{\text{hr}} \left\{ \frac{1000 \text{ m}}{1 \text{ km}} \right\} \left\{ \frac{1 \text{ hr}}{60 \text{ min}} \right\} \left\{ \frac{1 \text{ min}}{60 \text{ s}} \right\} = 44.444 \frac{\text{m}}{\text{s}} \Rightarrow 44 \frac{\text{m}}{\text{s}}$$

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} = \frac{44.444 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{0.80 \text{ s}} = 55.555 \frac{\text{m}}{\text{s}^2} \Rightarrow 56 \frac{\text{m}}{\text{s}^2}$$