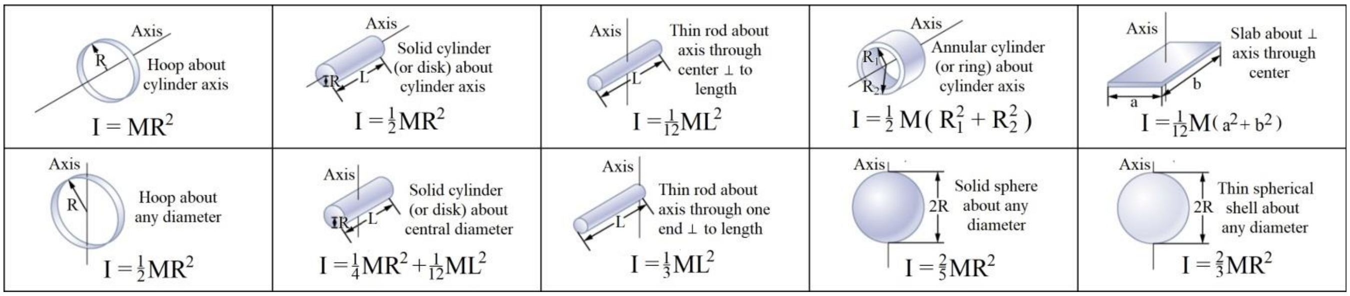
<u>Lecture 22</u>: Rotational Dynamics

Physics for Engineers & Scientists (Giancoli): Chapter 10
University Physics V1 (Openstax): Chapter 10

- Most of the time you will simply pull a formula from a table of standard shapes.



- Moment of inertia is dependent upon the choice of axis.
- If you have multiple objects with the same axis of rotation you simply add their moments of inertia. (Moments of inertia sum)
- The <u>Parallel Axis Theorem</u> allows you to calculate the moment of inertia of an object rotating around an axis that doesn't pass through its center of mass. To do this you need the moment of inertia for an axis parallel to the axis of rotation and passing through the center of mass (I_{CM}) and the separation of the two axes (h).

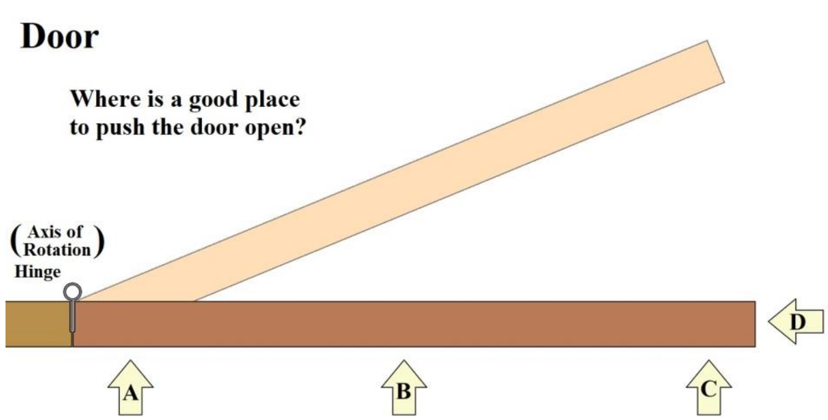
$$I = I_{cm} + Mh^2$$

<u>Example</u>: Find the moment of inertia of a thin rod about axis through one end ⊥ to length via the parallel axis theorem.

$$I = I_{cm} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

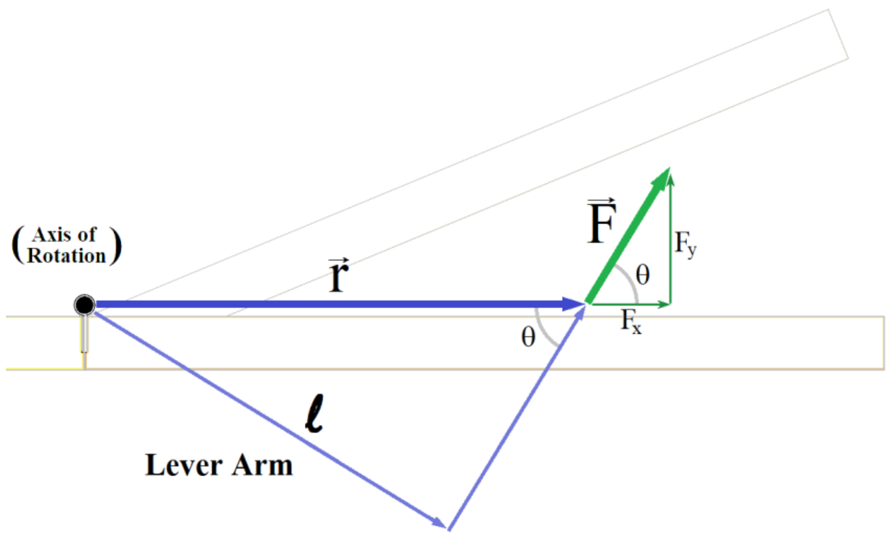
<u>Torque</u> (τ): $\tau = rF_y = rF\sin\theta \qquad \tau = Fl = Fr\sin\theta \qquad |\vec{\tau}| = |\vec{r}\times\vec{F}| = Fr\sin\theta$

Moment of inertia of most objects is fixed (constant). In those cases, Newton's 2nd law ($\vec{\tau} = I\vec{\alpha}$) indicates torque (τ) and angular acceleration (α) are proportional. This allows us to use the behavior of an object (its angular acceleration) to indicate how much torque was delivered when various forces are applied to it. For our example we will use a door.



Force A doesn't work very well. A lot of force leads to little movement of the door.
Force C is the best option. A little force here is usually enough to open the door.
Force B requires more force than C, but not as much as A.
Force D doesn't open the door at all.

- Any force that acts through the axis of rotation generates no torque.
    - To generate torque a force must have a a component $\perp$ to the line connecting the axis of rotation to the point where the force acts.
- The further from the axis of rotation the force is applied the greater the torque.



If we apply a force $\vec{F}$ to an object, we define the vector $\vec{r}$ to start at the axis of rotation and at the position where the force is applied. θ is defined to be the angle between $\vec{F}$ and $\vec{r}$.

- Breaking $\vec{F}$ into component (parallel and perpendicular to $\vec{r}$) we find:
    - F_x (the component parallel to $\vec{r}$) generates NO torque.
    - F_y (the component perpendicular to $\vec{r}$) generates positive torque as it rotates the door counter clockwise (CCW).
        - The magnitude of the torque ($\tau = |\vec{\tau}|$) is given by: $\tau = rF_y = rF\sin\theta$
    - Forces that create clockwise (CW) rotations are generating negative torque.
- Breaking $\vec{r}$ into component (parallel and perpendicular to $\vec{F}$) we find:
    - The component parallel to $\vec{F}$ has no bearing on the torque at all.
    - The component perpendicular to $\vec{F}$ is called the <u>Lever Arm</u> (l), and it is directly related to the torque. Any increase in the lever arm gives a proportional increase in torque ($\vec{\tau}$).
        - The magnitude of the torque ($\tau = |\vec{\tau}|$) is given by: $\tau = Fl = Fr\sin\theta$

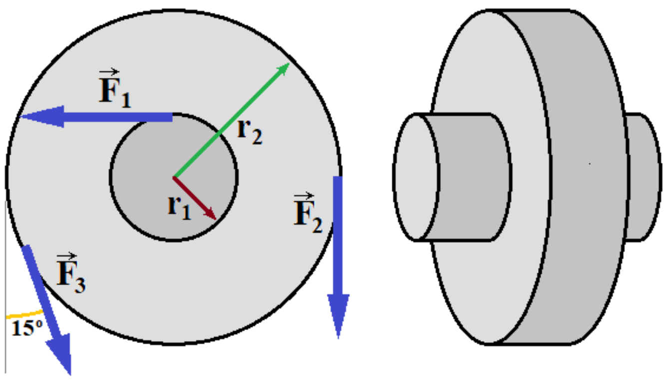
- We gain "Leverage" by increasing the lever arm.

- Both viewpoints give the same result, which is often represented as a vector cross product.

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = Fr \sin\theta$$

There are more advanced methods of calculating vector cross products, but these are rarely used for torque (as we already know the direction along the axis of rotation).

Example: Three forces act on a compound wheel as shown. The forces come from ropes wrapped around the edges of the wheel. The moment of inertia of the wheel is 30.0 kg·m^2 with an inner radius of r_1= 25.0 cm and an outer radius of r_2 = 50.0 cm. Determine the angular acceleration of the wheel in response to the three forces: F_1 = 80.0 N, F_2 = 30.0 N, and F_3 = 20.0 N.



Use Newton's 2nd Law: $\sum \tau = I\alpha$

To find the torque for each force vector:

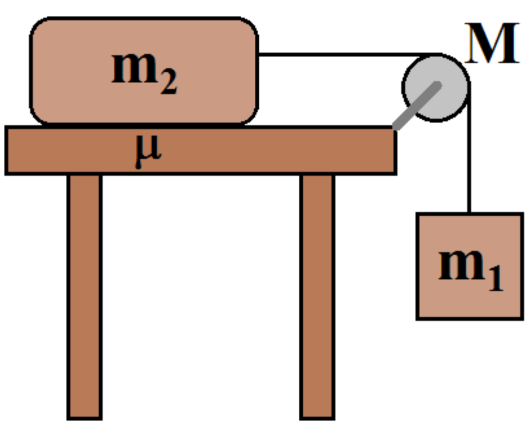
1. *Determine sign on torque: CCW is '+', CW is '-'*
2. *Find the vector $\vec{r}$, note it's magnitude*
3. *Determine the angle between $\vec{r}$ and $\vec{F}$ (that's θ)*
4. $\tau = rF \sin\theta$

$$\sum \tau = r_1F_1 \sin 90° - r_2F_2 \sin 90° + r_2F_3 \sin 90° = r_1F_1 - r_2F_2 + r_2F_3$$

$$\sum \tau = (0.250\ m)(80.0N) - (0.500\ m)(30.0\ N) + \big((0.500\ m)\big)(20.0\ N)$$

$$\sum \tau = 20.0\ N\cdot m - 15.0\ N\cdot m + 10.0\ N\cdot m = 15.0\ N\cdot m$$

$$\alpha = \frac{\tau_{Net}}{I} = \frac{15.0\ N\cdot m}{30.0\ kg\cdot m^2} = 0.500\frac{rad}{s^2}$$



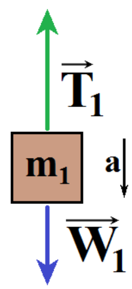
Example: A box of mass m_2 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_1 = 25.0 kg. The cord stretches over a frictionless pulley, a solid disk of uniform density with mass M = 5.00 kg. If the coefficient of kinetic friction between the box and table is 0.300, determine the acceleration of the box on the table.

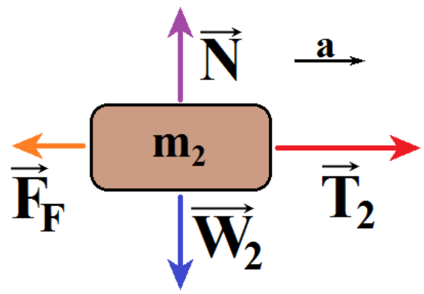
Make 3 force diagrams (one for each object). There are 3 unknowns (2 tensions and acceleration). This means you will need 3 equations, one from each force diagram. Angular acceleration is not another unknown as it is directly related to the acceleration in this problem.

The box on the table (m_2) accelerates to the right, which corresponds to a clockwise (CW) rotation of the pulley (M) and a downward acceleration of the hanging weight (m_1). To match the signs, for this problem we shall let CW rotations and downward accelerations become positive.

$$m_1 g - T_1 = m_1 a$$

$$T_1 = m_1 g - m_1 a$$



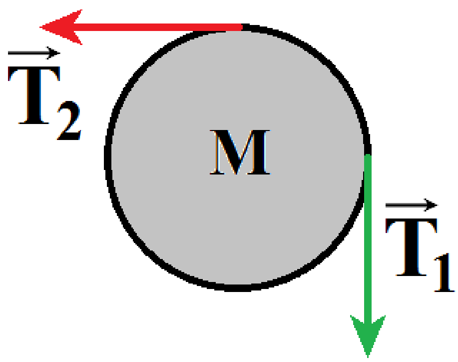


$$N = W_2 = m_2 g$$

$$F_F = \mu_k N = \mu_k m_2 g$$

$$T_2 - F_F = m_2 a$$

$$T_2 = m_2 a + \mu_k m_2 g$$



$$\sum \tau = \tau_1 + \tau_2 = RT_1 - RT_2 = R(T_1 - T_2)$$

$$\sum \tau = R(m_1 g - m_1 a - m_2 a - \mu_k m_2 g)$$

$$I\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right) = \frac{1}{2}MRa \qquad I\alpha = \sum \tau$$

$$\frac{1}{2}MRa = R(m_1 g - m_1 a - m_2 a - \mu_k m_2 g)$$

$$\frac{1}{2}Ma = m_1 g - m_1 a - m_2 a - \mu_k m_2 g \qquad m_1 a + m_2 a + \frac{1}{2}Ma = m_1 g - \mu_k m_2 g$$

$$a = \frac{m_1 g - \mu_k m_2 g}{m_1 + m_2 + \frac{1}{2}M} = \frac{(m_1 - \mu_k m_2) g}{m_1 + m_2 + \frac{1}{2}M} = \frac{[(25.0\,kg) - (0.300)(10.0\,kg)](9.80\,m/s^2)}{25.0\,kg + 10.0\,kg + \frac{1}{2}(5.00\,kg)} = 5.75\,\frac{m}{s^2}$$