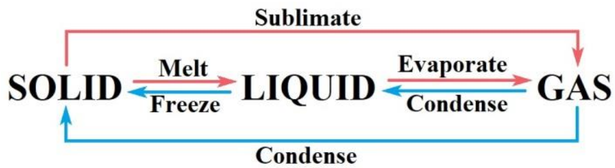
Lecture 19: Phase Change, Heat Transfer & Ideal Gas Law

Physics for Engineers & Scientists (Giancoli): Chapters 17 & 19
University Physics V2 (Openstax): Chapters 1 & 2

Heat and Phase Change

- Traditionally there are 3 states ("phases") of matter: solids, liquids, and gases

Today some would include plasmas, Bose-Einstein condensates, quark-gluon plasmas, etc.



- When a substance changes from one phase to another an amount of heat must be added (or removed) for the atoms/molecules rearrange themselves.

$$Q = mL$$

- 'Q' is the heat added to the substance, and 'm' is the mass of the substance.
- The Latent Heat (L) is a property of the material and depends upon which transition is occurring.
  - Solid/liquid transitions use the Latent Heat of Fusion (L_F)
  - Liquid/gas transitions use the Latent Heat of Vaporization (L_v)
  - Solid/gas transitions use the Latent Heat of Sublimation (L_s)

Example: 3.78 kg of water (a gallon) has been frozen into ice. How much heat is required to convert it from ice at -15.0°C into steam at 130°C? The latent heat of fusion for water/ice is 334 J/g. The latent heat of vaporization for water/steam is 2230 J/g. The specific heat of water is 4187 J/(kg·K). The specific heat for ice is 2108 J/(kg·K). The specific heat for steam is 1996 J/(kg·K).

Ice from -15°C to 0°C: $Q_1 = cm\Delta T = \left(2108\,\frac{\text{J}}{\text{kg·K}}\right)(3.78\,kg)(15.0°\text{C}) = 119{,}523.6\,J$

Ice to water at 0°C: $Q_2 = mL_F = \left(334\,\frac{\text{J}}{\text{g}}\right)(3780\,g) = 1{,}262{,}520\,J$

Water from 0°C to 100°C: $Q_3 = cm\Delta T = \left(4187\,\frac{\text{J}}{\text{kg·K}}\right)(3.78\,kg)(100°\text{C}) = 1{,}582{,}686\,J$

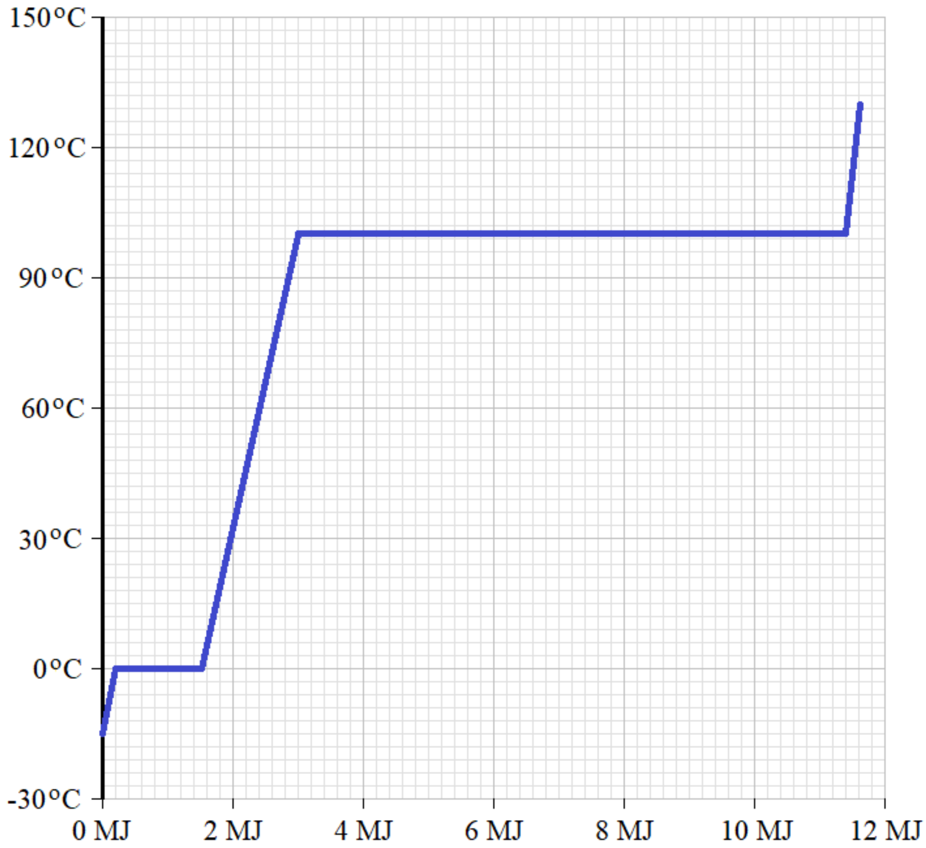
Water to steam at 100°C: $Q_4 = mL_V = \left(2230\,\frac{\text{J}}{\text{g}}\right)(3780\,g) = 8{,}429{,}400\,J$

Steam from 100°C to 130°C: $Q_5 = cm\Delta T = \left(1996\,\frac{\text{J}}{\text{kg·K}}\right)(3.78\,kg)(30°\text{C}) = 226{,}346.4\,J$

$$Q_{Tot} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_{Tot} = 119{,}523.6\,J + 1{,}262{,}520\,J + 1{,}582{,}686\,J + 8{,}429{,}400\,J + 226{,}346.4\,J$$

$$Q_{Tot} = 11.6\,MJ$$

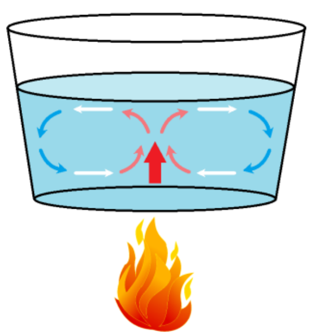


As heat is added the temperature of the ice rises until it hits 0°C.
Once the ice hits 0°C (the melting point of ice) additional energy only changes the state.
Once it has been converted to a liquid (water) the temperature rises with added heat.
At 100°C (the boiling point), additional energy changes the state into a gas (steam).
Once the water has all been vaporized, additional heat can once again raise the temperature.

Heat Transfer

- There are 3 methods in which heat is transferred: convection, conduction, and radiation.
- In **Convection**, heat is transferred by the bulk movement of a gas or liquid.

For example, the hot air coming out of a blow dryer carries heat with it.



Water expands as it is heated, becoming less dense. This causes it to rise to the surface.

The rising water pushes warmer water from the center to the outsides, cooling as it moves.

Cooler water on the outer edges falls replacing the water that has risen in the center.

The flow of liquid or gas responsible for transporting heat is called a **Convection Current**.

- In **Conduction**, heat is transferred directly through a material without bulk movement of that material.

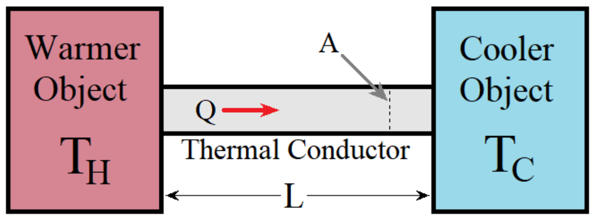
For example, if you were to grab the handle of a hot iron skillet, heat would flow through the metal and into your hand.

  - **Thermal Conductors** readily transport heat.

Iron skillets are thermal conductors.

  - **Thermal insulators** transport heat poorly.

Oven mitts are thermal insulators.



$$Q = \left(\frac{kA\Delta T}{L}\right)t \qquad P = \frac{Q}{t} = \frac{kA\Delta T}{L}$$

Q is the heat transferred.

k is the thermal conductivity of the conducting material.

A is the cross-sectional area of the conducting material (the area through which heat can flow).

ΔT is the temperature difference between the ends. $\Delta T = T_H - T_C$

L is the length of the thermal conductor (how far the heat must travel).

t is the time interval.

P is the conductive power.

Example: The temperature of an oven is 375°F when baking bread. The temperature of the kitchen outside the oven door is 78.0°F. The surface of the oven door is 0.300 m^2 and is coated with a 2.00 mm thick calcium silicate insulator with a thermal conductivity of 0.0500 J/(s·m·°C). (A) How much energy is used to operate the pre-heated oven for 5.00 hours if losses through the other surfaces are negligible compared to losses through the door, and (B) How much does this cost at 12¢ per kilo-Watt-hour for energy?

$$\Delta T = T_H - T_C = 375°\text{F} - 78.0°\text{F} = 297°\text{F} \qquad \Delta T(°\text{C}) = \frac{5}{9}\Delta T(°\text{F}) = \frac{5}{9}(297°\text{F}) = 165°\text{C}$$

$$5.00\ hours\left(\frac{3600\ s}{1\ hour}\right) = 18000\ s$$

$$Q = \left(\frac{kA\Delta T}{L}\right)t = \left(\frac{\left(0.0500\ \frac{J}{s\cdot m\cdot °C}\right)(0.300\ \text{m}^2)(165°\text{C})}{2.00\times 10^{-3}\ m}\right)(18000\ s) = 2.2275\times 10^7\ J$$

$$2.2275\times 10^7\ J\left(\frac{\$\ 0.12}{1\ kW\cdot hr}\right)\left(\frac{1\ kW\cdot hr}{3.60\times 10^6\ J}\right) = \$\ 0.7425$$

- In **Radiation**, heat is transferred by electromagnetic waves (typically in the infra-red range).
  - Objects appear black because they absorb most of the light and reflect little. Objects that readily absorb radiation are also good emitters.
  - A **Black Body** is an ideal emitter of radiation.
  - **Stefan-Boltzmann Law of Radiation** $Q = e\sigma T^4 At$ $P = \frac{Q}{t} = e\sigma T^4 A$
    - Q is the heat transferred.
    - e is the emissivity (0 to 1), a measure of how close the object is to an ideal black body (e = 1)
    - Stefan-Boltzmann constant (σ): $\sigma = 5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}$
    - T is the temperature in **Kelvin**.
    - A is the surface area of the object (the area emitting the EM waves).
    - t is the time interval.
    - P is the radiant power.

Example: Betelgeuse is a red supergiant star in the constellation Orion. It has a surface temperature of 3590 K and emits a radiant power of 4.64×10^{31} W. Determine the radius of Betelgeuse assuming it is approximately spherical and a perfect emitter (e = 1).

$$P = \frac{Q}{t} = e\sigma T^4 A = \sigma T^4 A = \sigma T^4 (4\pi r^2) = 4\pi\sigma T^4 r^2$$

$$r^2 = \frac{P}{4\pi\sigma T^4}$$

$$r = \sqrt{\frac{P}{4\pi\sigma T^4}} = \sqrt{\frac{4.64 \times 10^{31}\ \text{W}}{4\pi\left(5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}\right)(3590\ \text{K})^4}} = 6.26 \times 10^{11}\ m$$

For comparison, the average separation between the sun and Jupiter is 7.78×10^{11} m.

The Atomic Mass Scale

- The **Atomic Mass Unit** (u) is the mass of a carbon-12 atom (^{12}C) divided by 12. The atomic mass unit is roughly the mass of a proton or neutron.

$$u = 1.66 \times 10^{-27}\ kg$$

- The mass of an atom in atomic mass units is the same as the number of nucleons in the atom.

^{4}He has 4 nucleons (2 protons and 2 neutrons). The mass of a ^{4}He atom is 4u.

- The mass of a molecule is the sum of the masses of its atoms.
- One **Mole** (mol) of a substance contains Avogadro's number (6.022×10^{23}) of particles (could be atoms or molecules), which is the number of atoms in 12.0 g of carbon-12.

$$N_A = 6.022 \times 10^{23}$$

This creates a correspondence between the masses of atoms (atomic mass scale) and masses at the macroscopic scale (the masses of moles in grams).

For example, the mass of 1 mol of a substance with atomic mass 28 u (silicon) is going to be 28 g.

$$N = \#\ of\ particles\ in\ a\ substance$$

$$n = \#\ of\ moles\ of\ a\ substance$$

$$n = \frac{N}{N_A}$$

Ideal Gas Law $PV = nRT$ $PV = NkT$

- The ideal gas law can be written in two ways. One uses 'n', the number of moles, and the other uses 'N' the number of particles.
- P is the pressure of the gas.
- V is the container volume.
- T is the gas temperature.
- R is the **Universal Gas Constant**: $R = 8.314\ \frac{J}{mol{\cdot}K}$
- K is the **Boltzmann Constant**: $k = 1.38 \times 10^{-23}\ \frac{J}{K}$
- For the two equations to be equal, $R = N_A k$ $nR = Nk$

The units of both sides of the ideal gas law are Joules. This is an energy equation!

Example: The Airlander 10 is the largest operational aircraft in the world (as of 2016). It typically contains 38,000 cubic meters of helium at 1.01 atm. If the temperature is 20.0°C, how many moles of helium are needed to fill the craft?

$$P = (1.01\ atm)\left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right) = 102{,}313\ Pa \qquad T(K) = 20.0°\text{C} + 273.15 = 293.15\ K$$

$$PV = nRT \qquad n = \frac{PV}{RT} = \frac{(102{,}313\ Pa)(38000\ m^3)}{\left(R{=}8.314\ \frac{J}{mol{\cdot}K}\right)(293.15\ K)} = 1.60 \times 10^6\ moles$$

- **Using the Ideal Gas Law**
    - Absolute pressure must be used in the ideal gas law (not gauge pressure). Normally this must also be in Pascal (Pa) except as noted below.
    - Temperature must be in Kelvin.
    - Normally the volume must be in cubic meters (m^3) except as noted below.
    - Often we are comparing two different states of the same gas at two different time periods. If so, we can use that PV/nT (or PV/NT) is a constant.

      $$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \quad \text{...Or if n is constant:} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

        - When comparing two different states of the same gas, multiplicative conversion factors will cancel. This allows most units to be used for volume (liters, cm^3, etc) and pressure (atm).
        - Conversions that involve adding/subtracting cannot be used. Temperature must be in Kelvin, and absolute pressure (not gauge pressure) must be used.

- If the temperature is held constant, you get **Boyle's Law**: $P_1V_1 = P_2V_2$
- If the pressure is held constant, you get **Charles' Law**: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
- If the volume is held constant, you get **Gay-Lussac's Law**: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Example: A driver in Texas carefully prepares her car for a trip. This includes making sure the tires are inflated to a gauge pressure of 30.0 lbs per square inch (207 kPa). The tires have a volume of 1.64×10^{-2} m^3 and were filled at 95.0°F. Three days later the driver is in Canada at a temperature of 23.0°F. Assuming no air was lost from the tires since they were filled, what is the gauge pressure of the tires in Canada?

$$P_1 = P_{Absolute} = P_{Gauge} + P_{atm} = 207\ kPa + 101.3\ kPa = 308.3\ kPa$$

$$T_1(°C) = \frac{5}{9}[T_1(°F) - 32] = \frac{5}{9}[95.0°F - 32] = 35.0°C \qquad T_1(K) = 35.0°C + 273.15 = 308.15\ K$$

$$T_2(°C) = \frac{5}{9}[T_2(°F) - 32] = \frac{5}{9}[23.0°F - 32] = -5.0°C \qquad T_2(K) = -5.0°C + 273.15 = 268.15\ K$$

As no air was lost from the tires, n is constant.
The volume of the tires is given. We can assume that is fixed. So V is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad P_2 = \frac{T_2}{T_1}P_1 = \frac{268.15\ K}{308.15\ k}(308.3\ kPa) = 268.3\ kPa$$

$$P_{Gauge} = P_{Absolute} - P_{atm} = 268.3\ kPa - 101.3\ kPa = 167.0\ kPa$$ (roughly 24.2 lbs/in^2)

- **Standard Temperature and Pressure** (STP)
  - Standard Temperature is 0°C (or 273.15 K)
  - Standard Pressure is 1 atm (1.013×10^5 Pa)
  - We can determine the volume of 1 mole of an ideal gas at STP.

$$V = \frac{nRT}{P} = \frac{(1\ mol)\left(8.314\ \frac{J}{mol \cdot K}\right)(273.15\ K)}{(1.013 \times 10^5\ Pa)} = 0.02242\ m\ (22.4\ liters)$$