Lecture 18: Impulse and Momentum

Physics for Engineers & Scientists (Giancoli): Chapter 9
University Physics V1 (Openstax): Chapter 9

Example: A golf ball (m_b = 45.0 g) starts at rest on a tee. After the golfer strikes it, it is moving at 38.0 m/s. During impact, the club remains in contact with the ball for 3.00 ms. A) What is the change in momentum of the ball? B) Determine the average force applied to the ball by the club.

A) $\Delta P = P_{Final} - P_{Init} = mv - mv_0 = m(v - v_0) = (0.0450\ kg)\left(38.0\frac{m}{s} - 0\frac{m}{s}\right) = 1.71\ kg \cdot \frac{m}{s}$

B) $F_{Avg} = \frac{\Delta P}{\Delta t} = \frac{1.71\ kg \cdot \frac{m}{s}}{3.00 \times 10^{-3}\ s} = 570\ N$

Example: The Space-X Falcon 9 rocket has a mass of 1.48×10^6 kg when loaded with payload destined for low-Earth orbit (leo). Its engines generate a force given by $F(t) = \beta(1-e^{-\sigma t})$ where $\beta = 2.28 \times 10^7$ N and $\sigma = 0.139\ s^{-1}$. If the rocket is at rest at t = 0, how fast is it moving at t = 5.00 s?

This problem is one dimensional (vertically upward). Vectors can be dispensed with.

$$J = \int_{t_1}^{t_2} F(t)dt = mv$$

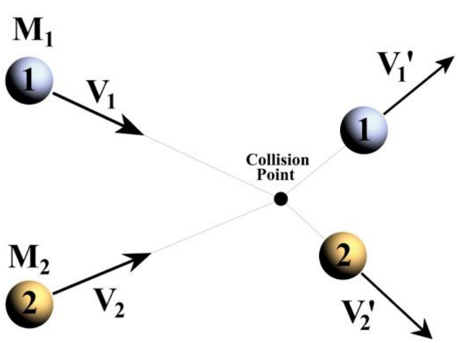
$$J = \int_{t_1}^{t_2} F(t)dt = \int_0^t [\beta(1 - e^{-\sigma t})]dt = \int_0^t [\beta - \beta e^{-\sigma t}]dt = \int_0^t \beta dt + \int_0^t -\beta e^{-\sigma t} dt$$

$$J = [\beta t]_0^t - \left[-\frac{\beta}{\sigma} e^{-\sigma t}\right]_0^t = \beta t - \frac{\beta}{\sigma}(1 - e^{-\sigma t}) = \beta\left[t - \frac{1}{\sigma}(1 - e^{-\sigma t})\right]$$

$$J = (2.28 \times 10^7\ N)\left\{(5.00\ s) - \frac{1}{(0.139\ s^{-1})}\left[1 - e^{-(0.139\ s^{-1})(5.00\ s)}\right]\right\} = 3.1834 \times 10^7\ N \cdot s$$

$$v = \frac{J}{m} = \frac{3.1834 \times 10^7\ N \cdot s}{1.48 \times 10^6\ kg} = 21.5\frac{m}{s}$$

General Collision



Object 1: $\vec{J}_1 = \vec{F}_1 \cdot \Delta t = \Delta\vec{P}_1 = mv_1' - mv_1$

Object 2: $\vec{J}_2 = \vec{F}_2 \cdot \Delta t = \Delta\vec{P}_2 = mv_2' - mv_2$

$$\vec{J}_1 + \vec{J}_2 = \vec{F}_1 \cdot \Delta t + \vec{F}_2 \cdot \Delta t = \left(\vec{F}_1 + \vec{F}_2\right) \cdot \Delta t = 0$$

$\vec{F}_1 + \vec{F}_2 = 0$, *because Newton's 3rd law says* $\vec{F}_2 = -\vec{F}_1$

$$\vec{J}_1 + \vec{J}_2 = (mv_1' - mv_1) + (mv_2' - mv_2)$$

$$\vec{J}_1 + \vec{J}_2 = (mv_1' + mv_2') - (mv_1 + mv_2) = \vec{P}_{Final} - \vec{P}_{Init}$$

$$\vec{P}_{Final} - \vec{P}_{Init} = 0 \qquad \vec{P}_{Final} = \vec{P}_{Init}$$

Momentum is conserved!

...As long as no external forces provide outside impulse.

Example: A defensive lineman (m_{DL} = 138 kg) is moving at 8.00 m/s when he tackles a stationary quarterback (m_{QB} = 110 kg). A) What is the velocity of the pair after the collision? B) If the collision takes 0.200 s, what is the average force delivered to the quarterback?

A) $P_{Final} = P_{Init}$ $\quad (m_{QB} + m_{DL})v = m_{DL}v_0 \quad v = \frac{m_{DL}v_0}{m_{QB}+m_{DL}} = \frac{(138\,kg)\left(8.00\frac{m}{s}\right)}{(110\,kg+138\,kg)} = 4.45\,\frac{m}{s}$

B) $F_{Avg} = \frac{\Delta P_{QB}}{\Delta t} = \frac{m_{QB}v - 0}{\Delta t} = \frac{(110\,kg)\left(4.45\frac{m}{s}\right)}{(0.200\,s)} = 2.45\,kN$ $\quad$ *(approximately 550 lbs of force)*

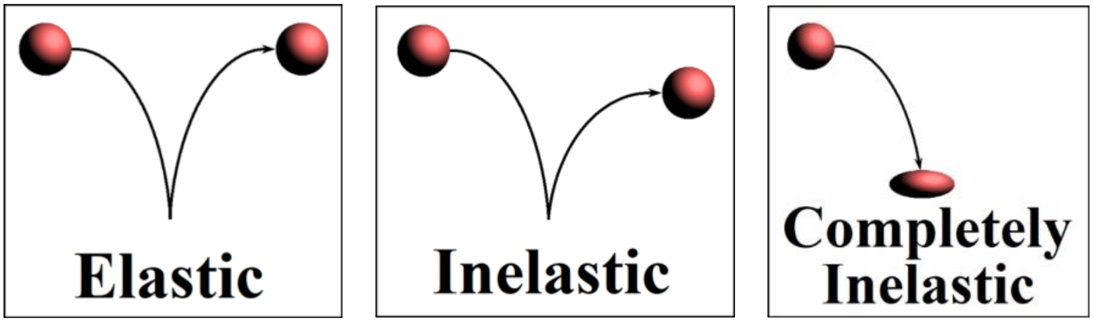
$$\Delta P_{QB} = m_{QB}v = (110\,kg)\left(4.45\frac{m}{s}\right) = 490\,kg \cdot \frac{m}{s}$$

$$\Delta P_{DL} = m_{DL}v - m_{DL}v_0 = m_{DL}(v - v_0) = (138\,kg)\left(4.45\frac{m}{s} - 8.00\frac{m}{s}\right) = -490\,kg \cdot \frac{m}{s}$$

The momentum lost by the defensive lineman is transferred to the quarterback

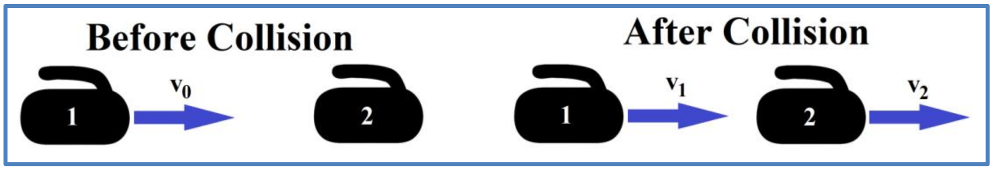
Energy and Momentum in Collisions

- Momentum is conserved in collisions when no net outside forces are present.
- Gravity is typically negligible as it has almost no effect over such a short time interval.
- Energy may or may not be conserved in collisions. For example, energy might be released as heat (lost).
- Three types of collisions:
  - In Elastic Collisions, both energy and momentum are conserved.
  - In Inelastic Collisions, momentum is conserved, but energy is not.
  - In Completely Inelastic Collisions, momentum is conserved, but the maximum possible energy is lost as the objects stick together.



Example: A curler slides a 20.0 kg stone across the ice surface. The stone is moving at 0.750 m/s when it collides head-on with a second, stationary 20.0 kg curling stone. If this is an elastic collision, determine the velocity of both stones after they strike.

Note: The term 'head on' indicates that the center of mass of both objects is in line with the velocity. In other words, this is a 1-dimensional problem. Vectors can be ignored.



Conservation of momentum: $P_{init} = P_{Final}$ $\quad m_1 v_0 = m_1 v_1 + m_2 v_2$

Conservation of energy: $E_{init} = E_{Final}$ $\quad \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad m_1 v_0^2 = m_1 v_1^2 + m_2 v_2^2$

Here's a mathematical trick that helps with elastic collisions:

Rearrange momentum equation: $m_1(v_0 - v_1) = m_2 v_2$

Rearrange and factor energy equation: $m_1(v_0^2 - v_1^2) = m_2 v_2^2 \quad m_1(v_0 - v_1)(v_0 + v_1) = m_2 v_2^2$

Divide equations: $\frac{m_1(v_0 - v_1)(v_0 + v_1)}{m_1(v_0 - v_1)} = \frac{m_2 v_2^2}{m_2 v_2} \quad v_0 + v_1 = v_2$

Multiply by m_1 and add the rearranged momentum equation:

$$m_1(v_0 + v_1) + m_1(v_0 - v_1) = m_1(v_2) + m_2(v_2) \qquad 2m_1 v_0 = (m_1 + m_2)v_2 \qquad v_2 = \frac{2m_1 v_0}{(m_1 + m_2)}$$

$$v_1 = v_2 - v_0 = \frac{2m_1 v_0}{(m_1 + m_2)} - \frac{(m_1 + m_2)v_0}{(m_1 + m_2)} = \frac{(m_1 - m_2)v_0}{(m_1 + m_2)}$$

As $m_1 = m_2$ in this case… $v_2 = v_0 = 0.75\frac{m}{s}$ and $v_1 = 0$