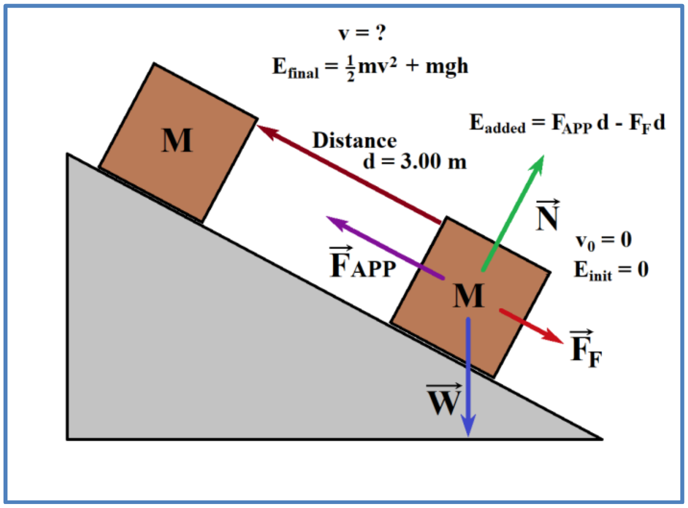
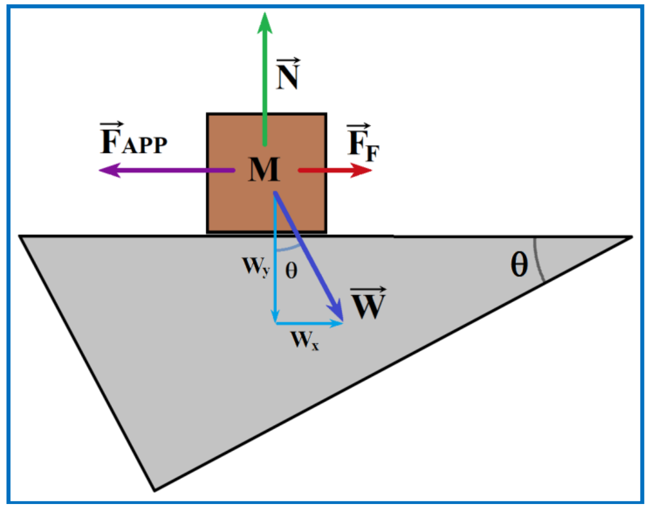
Lecture 17: Power, Impulse, and Momentum

Physics for Engineers & Scientists (Giancoli): Chapters 8 & 9
University Physics V1 (Openstax): Chapters 8 & 9

Example: A worker pushes a crate up a 30.0° incline by delivering a 10.0 N force. The coefficient of kinetic friction between the crate and incline is 0.200. If the crate starts from rest and weighs 10.0 N, how fast is the crate moving after it traverses a distance of 3.00 m along the incline?





$$E_{init} = 0 \qquad E_{final} = \frac{1}{2}mv^2 + mgh$$

The weight (W) does work, but this is not included in E_{added} as it is already covered with potential energy.

The normal force (N) doesn't do any work as it is perpendicular to the direction of motion.

$$E_{added} = F_{App}d - F_F d \qquad F_F d = \mu_k N d = \mu_k W_y d = \mu_k mgd\cos\theta$$

$$E_{init} + E_{added} = E_{final} \qquad F_{App}d - \mu_k mgd\cos\theta = \frac{1}{2}mv^2 + mgh$$

$$F_{App}d - \mu_k mgd\cos\theta = \frac{1}{2}mv^2 + mgd\sin\theta \qquad F_{App}d - \mu_k mgd\cos\theta - mgd\sin\theta = \frac{1}{2}mv^2$$

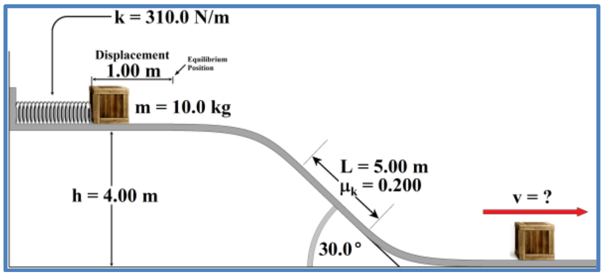
$$2F_{App}d - 2\mu_k mgd\cos\theta - 2mgd\sin\theta = mv^2$$

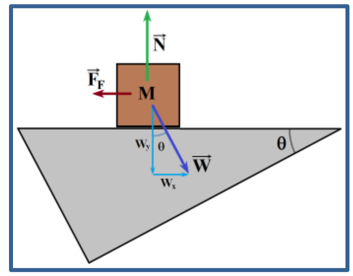
$$\frac{2F_{App}d}{m} - 2\mu_k gd\cos\theta - 2gd\sin\theta = v^2 \qquad \frac{2F_{App}gd}{W} - 2\mu_k gd\cos\theta - 2gd\sin\theta = v^2$$

$$2gd\left(\frac{F_{App}}{W} - \mu_k\cos\theta - \sin\theta\right) = v^2 \qquad v = \sqrt{2gd\left(\frac{F_{App}}{W} - \mu_k\cos\theta - \sin\theta\right)}$$

$$v = \sqrt{2\left(9.80\frac{m}{s^2}\right)(3.00\,m)\left(\frac{10.0\,N}{10.0\,N} - (0.200)\cos(30.0°) - \sin(30.0°)\right)} = 4.38\frac{m}{s}$$

Example: A distribution center has an interesting system for moving crates. A mechanism compresses a spring (k = 310.0 N/m) by 1.00 m. A 10.0 kg crate is then placed in front of the spring, which is then triggered sending the crate on its way. It falls a height of 4.00 m as it moves down a 30.0° incline before leveling off. The entire surface is frictionless except for a 5.00 m long stretch down the incline where the coefficient of friction is 0.200. How fast is the crate moving at the bottom of the incline?





$$E_{init} = \frac{1}{2}kx^2 + mgh \qquad E_{final} = \frac{1}{2}mv^2$$

$$E_{added} = -F_F d = -\mu_k N d = -\mu_k W_y d = -\mu_k mgd\cos\theta$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx^2 + mgh - \mu_k mgd\cos\theta = \frac{1}{2}mv^2$$

$$kx^2 + 2mgh - 2\mu_k mgd\cos\theta = mv^2 \qquad \frac{k}{m}x^2 + 2gh - 2\mu_k gd\cos\theta = v^2$$

$$v = \sqrt{\frac{k}{m}x^2 + 2gh - 2\mu_k gd\cos\theta}$$

$$v = \sqrt{\frac{310\frac{N}{m}}{10.0\,kg}(1.00m)^2 + 2\left(9.80\frac{m}{s^2}\right)(4.00\,m) - 2(0.200)\left(9.80\frac{m}{s^2}\right)(5.00\,m)\cos(30°)} = 9.61 \text{ m/s}$$

Power

- We define **Average Power** as: $P_{avg} = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t} = \frac{E_{final}-E_{init}}{t_{final}-t_{init}}$
- and **Instantaneous Power** as: $P = \frac{dW}{dt} = \frac{dE}{dt}$
- Power is defined using work, but the definition applies to any form of energy.
- The units of power are the Watt (W): 1 W = 1 J/s
- If the power is being delivered by a constant force: $P = Fv \qquad P = \frac{dW}{dt} = \frac{F\cdot dx}{dt} = F\frac{dx}{dt} = Fv$
- Kilowatt·Hour (kwh) is a unit of energy: $1\,kW\cdot Hr = (1000\,W)(3600\,s) = 3.6 \times 10^6\,J$

Example: A low-flying eagle with mass 4.50 kg increases its velocity from 11.3 m/s to 17.2 m/s over a 15.0 second time interval. Over the same time its altitude increases from 1.50 m to 7.75 m. What average power must the eagle's wings deliver to accomplish this?

$$E_{init} = \frac{1}{2}mv_0^2 + mgh_0 \qquad E_{final} = \frac{1}{2}mv^2 + mgh$$

$$P_{avg} = \frac{\Delta E}{\Delta t} = \frac{E_{final} - E_{init}}{\Delta t} = \frac{\frac{1}{2}mv^2 + mgh - \frac{1}{2}mv_0^2 - mgh_0}{\Delta t} = \frac{\frac{1}{2}m(v^2 - v_0^2) + mg(h - h_0)}{\Delta t}$$

$$P_{avg} = \frac{\frac{1}{2}(4.50\,kg)\left\{\left(17.2\frac{m}{s}\right)^2 - \left(11.3\frac{m}{s}\right)^2\right\} + (4.50\,kg)\left(9.80\frac{m}{s^2}\right)(7.75\,m - 1.50\,m)}{15.0\,s} = 43.6\,W$$

Example: The Space-X Falcon 9 rocket has a mass of 1.48×10^6 kg when loaded with payload destined for low-Earth orbit (leo). Its engines generate 22.8 MN (mega-Newtons) of thrust during their initial burn. When the first stage is jettisoned after 157 s, the rocket is going 1,839 m/s at an altitude of 70.4 km. What is the average power output of the engines?

Can we use 'U = mgh' at an altitude of 70.4 km? ...No, but we could use $F = GmM_E/r^2$ and integrate.

Is wind resistance negligible? ...No!

We must use work and not energy.

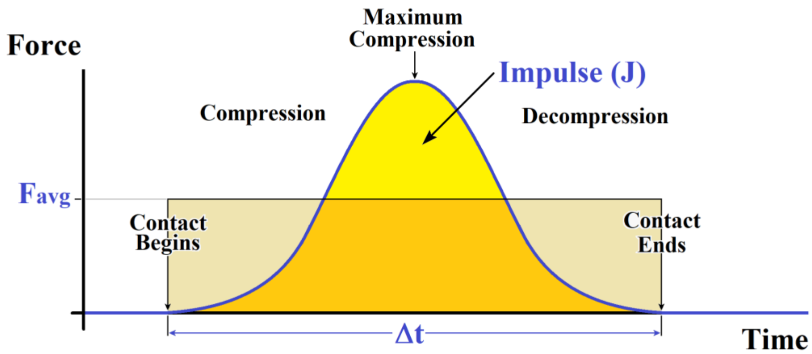
$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{Fd}{\Delta t} = \frac{(22.8 \times 10^6\, N)(70.4 \times 10^3\, m)}{157\, s} = 1.02 \times 10^{10} W$$

Note: The largest nuclear power plant in the US, the Palo Verde Nuclear power plant in Arizona has an output of 3.94×10^9 W and is a major source of electric power for the densely populated parts of Southern Arizona and Southern California, including Phoenix, Tucson, Los Angeles, and San Diego.

The Space-X Falcon 9 rocket delivers 2.5 times more power than the largest nuclear reactor in the country.

Impacts and Impulse

- Collisions generally occur over a short time interval.
- When two objects collide, the surfaces will deform (surfaces compress).
- These surfaces act like springs (albeit with large spring constants). The greater the compression, the greater the force. This allows the contact forces (equivalent to normal forces) to slow the colliding objects and redirect them.
- Some of the collision energy may be released as heat (warming the colliding objects) and sound. Other energy might be consumed in permanently deforming one or both of the objects. As these energies are difficult to account for, conservation of energy can only be used special cases.



- **Impulse** ($\vec{J}$): $\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \vec{F}_{avg} \cdot \Delta t$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \int_{t_1}^{t_2} m\vec{a}(t)dt = m\int_{t_1}^{t_2} \vec{a}(t)dt = m[\vec{v}(t)]_{t_1}^{t_2} = m\vec{v}_2 - m\vec{v}_1$$

- **Momentum** ($\vec{P}$): $\vec{P} = m\vec{v}$
- The impulse delivered to an object is equal to that object's change in momentum.

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \vec{F}_{avg} \cdot \Delta t = m\vec{v}_2 - m\vec{v}_1 = \Delta\vec{P}$$

- Which also means: $\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t}$ $\quad \vec{F} = \frac{d\vec{P}}{dt}$