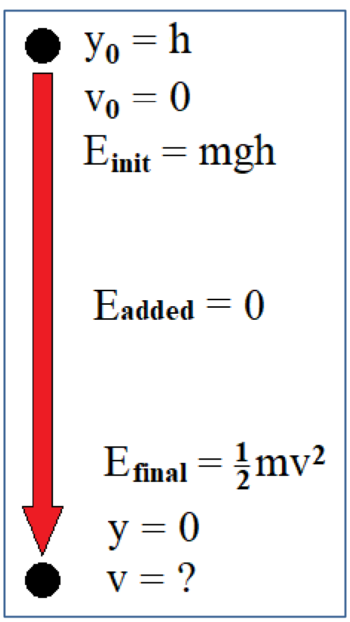
Lecture 16: Conservation of Energy

Physics for Engineers & Scientists (Giancoli): Chapter 8
University Physics V1 (Openstax): Chapter 8

Solving Problems with Conservation of Energy

- You only need to consider the initial and final states. Intermediate states are irrelevant.
- When you determine the initial and final energies, make sure to include every form of energy present.
  - Anything in motion will have kinetic energy.
  - Anything not at ground level (or where you decided to place y=0) will have gravitational potential energy.
- Make sure to traverse the path in between the initial and final positions to include anything that adds or removes energy to find E_{added} (W_{NC}).
  - Any friction forces will remove energy.
  - Other applied forces may add or subtract energy.
  - Gravity is accounted for with potential energy (don't include that with Eadded).

Example: A woman drops a small rock off a balcony 10.2 m above the ground. Assuming wind resistance is negligible, how fast is the rock moving just before it hits the ground?



We will set y = 0 to be at ground level.

Note: Setting y = 0 to occur at ground level is typically the most 'comfortable' thing to do. However, you may find that matching y = 0 to your lowest object is preferable. In this problem, both give the same origin.

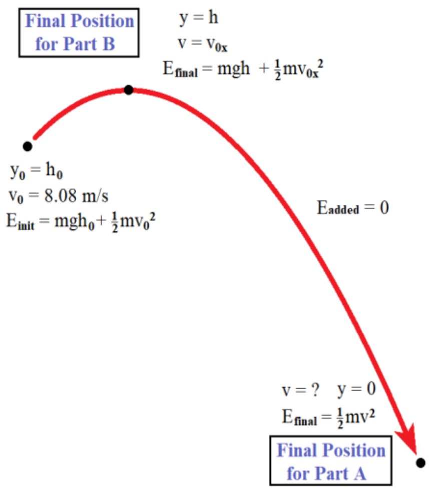
$$E_{init} = mgh \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2 \qquad 2gh = v^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(10.2\ m)} = \sqrt{199.92\frac{m^2}{s^2}} = 14.1\frac{m}{s}$$

Example: A woman throws a small rock off a balcony 10.2 m above the ground. The initial velocity is 8.08 m/s and directed 30.0° above the horizon. Assume wind resistance is negligible. Determine A) the speed of the rock just before it hits the ground, and B) the maximum height of the rock.



We will set y = 0 to be at ground level.

Part A:

$$E_{init} = mgh + \frac{1}{2}mv_0^2 \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad mgh + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$gh + \frac{1}{2}v_0^2 = \frac{1}{2}v^2 \qquad 2gh + v_0^2 = v^2$$

$$v = \sqrt{2gh + v_0^2} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(10.2\,m) + \left(8.08\frac{m}{s}\right)^2}$$

$$v = \sqrt{265.2064\frac{m^2}{s^2}} = 16.3\frac{m}{s}$$

Part B: $E_{init} = mgh_0 + \frac{1}{2}mv_0^2 \qquad E_{final} = mgh + \frac{1}{2}mv^2 \qquad E_{added} = 0$

$$v_y = 0 \qquad v = v_x = v_{0x} = v_0 \cdot Cos(\theta) = \left(8.08\frac{m}{s}\right)\cos(30.0°) = 6.9975\frac{m}{s}$$

$$E_{init} + E_{added} = E_{final} \qquad mgh_0 + \frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2 \qquad gh_0 + \frac{1}{2}v_0^2 = gh + \frac{1}{2}v^2$$

$$gh_0 + \frac{1}{2}v_0^2 - \frac{1}{2}v^2 = gh \qquad h_0 + \frac{v_0^2}{2g} - \frac{v^2}{2g} = h \qquad h = h_0 + \frac{v_0^2 - v^2}{2g}$$

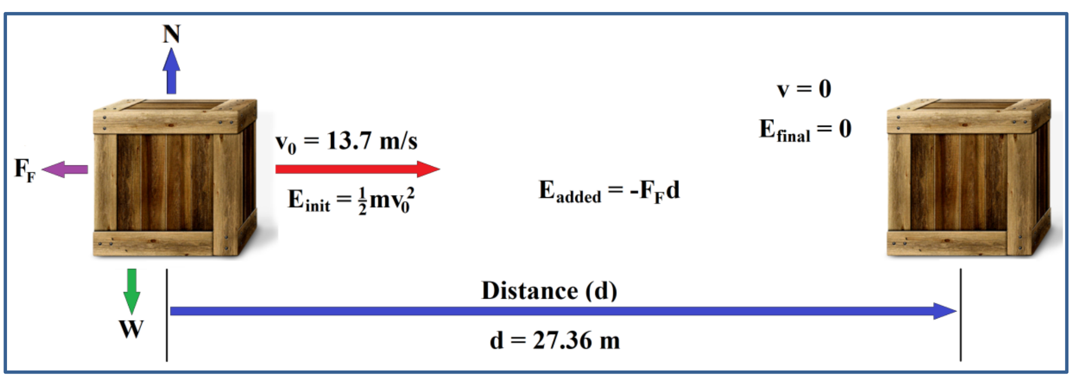
$$h = h_0 + \frac{v_0^2 - v^2}{2g} = 10.2\,m + \frac{\left(8.08\frac{m}{s}\right)^2 - \left(6.9975\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)} = 11.0\,m$$

Alternatively: $v_0^2 - v^2 = \left(v_{0x}^2 + v_{0y}^2\right) - v_{0x}^2 = v_{0y}^2$

$$v_{0y} = v_0 \cdot \sin(\theta) = \left(8.08\frac{m}{s}\right) \cdot \sin(30.0°) = 4.04\frac{m}{s}$$

$$h = h_0 + \frac{v_0^2 - v^2}{2g} = h_0 + \frac{v_{0y}^2}{2g} = 10.2\,m + \frac{\left(4.04\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)} = 11.0\,m$$

Example: Initially a crate is sliding on a horizontal surface at 13.7 m/s. The crate moves a distance of 27.36 m before coming to rest. Determine the coefficient of kinetic friction between the crate and the surface.



$$E_{init} = \frac{1}{2}mv_0^2 \qquad E_{final} = 0 \qquad E_{added} = -F_F d = -\mu_k N d = -\mu_k mgd$$

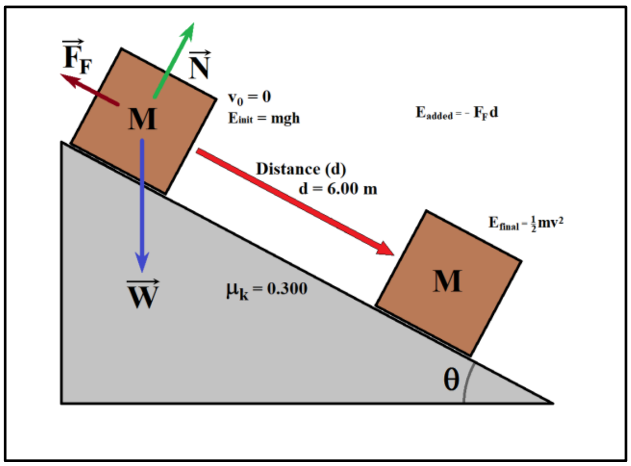
Note: As there are only two vertical forces (N and W) that must cancel, N=W=mg

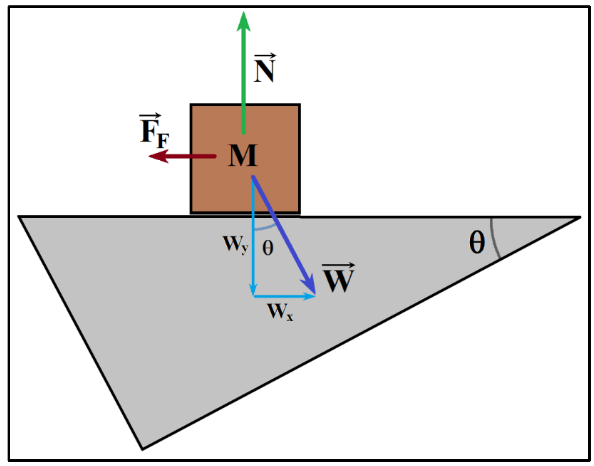
$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}mv_0^2 - \mu_k mgd = 0 \qquad \frac{1}{2}mv_0^2 = \mu_k mgd \qquad \frac{1}{2}v_0^2 = \mu_k gd$$

$$\mu_k = \frac{v_0^2}{2gd} = \frac{\left(13.7\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)(27.36\,m)} = 0.350$$

Example: Initially a crate is at rest at the top of a 30.0° incline. The coefficient of kinetic friction between the crate and the surface is 0.300. How fast is the crate moving after sliding 6.00 m down the incline?

We will set y = 0 to be at ground level.





$$E_{init} = mgh = mgd \cdot \sin(30.0°) = \frac{1}{2}mgd \qquad E_{final} = \frac{1}{2}mv^2$$

The weight (W) does work, but this is not included in E_{added} as it is already covered with potential energy.

The normal force (N) doesn't do any work as it is perpendicular to the direction of motion.

$$E_{added} = -F_F d = -\mu_k N d = -\mu_k W_y d = -\mu_k mgd\cos\theta$$

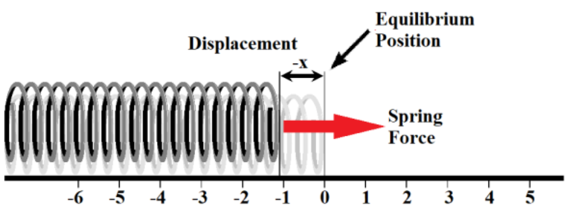
$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}mgd - \mu_k mgd\cos\theta = \frac{1}{2}mv^2$$

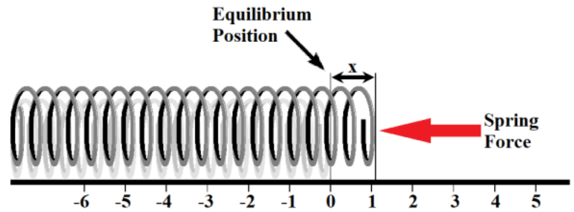
$$mgd - 2\mu_k mgd\cos\theta = mv^2 \qquad gd - 2\mu_k gd\cos\theta = v^2 \qquad v^2 = gd(1 - 2\mu_k\cos\theta)$$

$$v = \sqrt{gd(1 - 2\mu_k\cos\theta)} = \sqrt{\left(9.80\frac{m}{s^2}\right)(6.00\,m)\{1 - 2(0.300)\cos(30.0°)\}} = 5.31\frac{m}{s}$$

Springs and Hooke's Law

- Springs will naturally return to their initial (equilibrium) position after being stretched or compressed. A force that does this can be referred to as a **Restoring Force**.
- There are limits to how far springs can be stretched or compressed before losing their ability to stretch and becoming permanently deformed. This is called the **Elastic Limit** of the spring.
- With springs it is typical to use x as the change in length from equilibrium (equal to the displacement of the end) with it stretching into the positive axis and compressing into the negative axis.





- Hooke's Law: $F_x = -kx$
  - The negative sign indicates that the force points opposite the direction of displacement.
  - The spring constant, k, is only valid for a specific spring (not a universal constant).
  - Hooke's Law is only a first order linear approximation. Many springs will deviate from Hooke's Law before reaching the elastic limit.
  - Solid surfaces typically obey Hooke's law (albeit with very large spring constants). A very slight compression creates a large restoring force. This allows the normal force to take whatever value it needs to be.
  - Hooke's Law also applies to other objects that behave elastically.
- Elastic Potential Energy: $U_{sp} = \frac{1}{2}kx^2$
  - $U_{sp} = -W_{sp} = -\int_0^x F_{sp}(x)dx = -\int_0^x (-kx)dx = k\int_0^x (x)dx = k\left\{\frac{1}{2}x^2\right\}_0^x = \frac{1}{2}kx^2$
  - When using conservation of energy, it is preferable to include the elastic potential energy of springs rather than include it as work from an applied force (as you will just have to do this integral again).

Example: An archer pulls the bowstring back 42.0 cm and fires a 65.0 g arrow at 57.5 m/s. Determine the maximum height the arrow can reach if the archer pulls the bowstring back 50.0 cm.

For objects that obey Hooke's Law, the dynamic characteristics of that object have been reduced to a single quantity, the spring constant. If you don't know the spring constant of the object, you'll need to find that first.

$$E_{init} = \frac{1}{2}kx_1^2 \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx_1^2 = \frac{1}{2}mv^2 \qquad kx_1^2 = mv^2$$

$$k = \frac{mv^2}{x_1^2} = \frac{(0.0650\,kg)\left(57.5\frac{m}{s}\right)^2}{(0.420\,m)^2} = 1218.3\frac{N}{m}$$

Now that we have the spring constant, we can look for the height.

$$E_{init} = \frac{1}{2}kx_2^2 \qquad E_{final} = mgh \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx_2^2 = mgh$$

$$h = \frac{kx_2^2}{2mg} = \frac{\left(1218.3\frac{N}{m}\right)(0.500\,m)^2}{2(0.0650\,kg)\left(9.80\frac{m}{s^2}\right)} = 239\,m$$