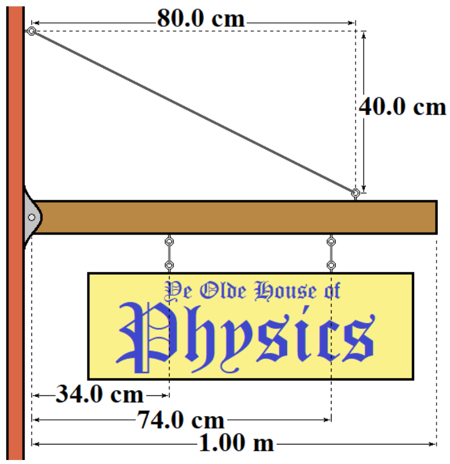
Lecture 15: Static Equilibrium, Deformation, and Fluids

Physics for Engineers & Scientists (Giancoli): Chapters 12 & 13
University Physics V1 (Openstax): Chapters 12 & 14



Example: A 2.50 kg sign is hung from an 11.0 kg beam as shown in the diagram. The sign is balanced so that half its weight is supported by each of the two wires above it. The beam is attached to the wall on the left by a frictionless pin that allows it to rotate and held in place by a guy wire above it. Determine the tension in the guy wire.

$$W_{beam} = m_{beam}g = (11.0\ kg)\left(9.80\frac{m}{s^2}\right) = 107.8\ N$$

$$W_{sign} = m_{sign}g = (2.50\ kg)\left(9.80\frac{m}{s^2}\right) = 24.5\ N$$

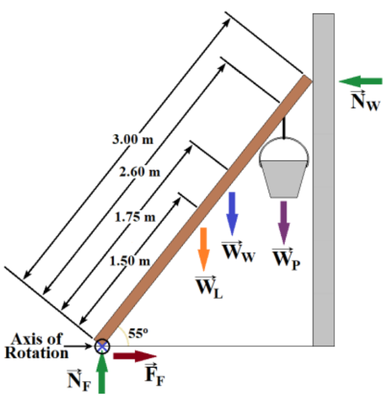
$$\sum \tau = T_y(80.0\ cm) - \frac{1}{2}(24.5\ N)(34.0\ cm) - \frac{1}{2}(24.5\ N)(74.0\ cm) - (107.8\ N)(50.0\ cm)$$

$$\sum \tau = T_y(80.0\ cm) - 416.5\ N\cdot cm - 906.5\ N\cdot cm - 5390.0\ N\cdot cm$$

$$\sum \tau = T_y(80.0\ cm) - 6713\ N\cdot cm = 0 \qquad T_y = \frac{6713\ N\cdot cm}{80.0\ cm} = 83.9125\ N$$

$$\theta = \tan^{-1}\left(\frac{40.0\ cm}{80.0\ cm}\right) = 26.565° \qquad T\sin\theta = T_y \qquad T = \frac{T_y}{\sin\theta} = \frac{83.9125\ N}{\sin 26.565°} = 188\ N$$

Example: A workman (weighing 755 N) leans a ladder (L = 3.00 m long, weighing 155 N) against a smooth (virtually frictionless) wall, making a 55.0° angle with the floor. He hangs a pail holding his work tools with a combined weight of 90.0 N from one of the top rails a distance d_1 = 2.60 m from the bottom of the ladder. The workmen then climbs up to do his work, balancing himself on a rail that is a distance d_2 = 1.75 m up the ladder. Determine the minimum value of the coefficient of static friction needed to keep the ladder from slipping.



When μ_S is at a minimum, then $F_F = F_{F-Max} = \mu_S N$.

$$\sum F_x = F_F - N_W = 0 \qquad F_F = N_W$$

$$\sum F_y = N_F - W_L - W_W - W_P = 0$$

$$N_F = W_L + W_W + W_P = 155\ N + 755\ N + 90.0\ N = 1000\ N$$

To get F_F or N_W we need to use a torque equation.

Place the axis of rotation where it will eliminate the most torques.

$$\sum \tau = N_W L\sin 55° - W_L\left(\frac{1}{2}L\right)\cos 55° - W_W d_2\cos 55° - W_P d_1\cos 55° = 0$$

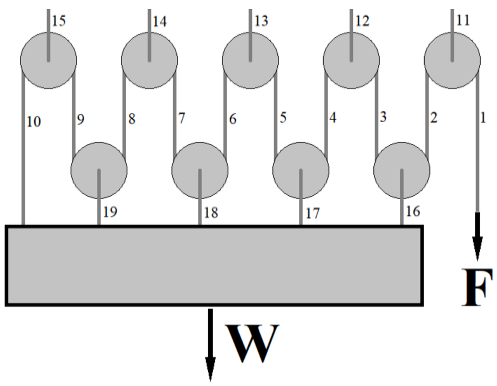
$$N_W L \sin 55° = W_L \left(\frac{1}{2} L\right) \cos 55° + W_W d_2 \cos 55° + W_P d_1 \cos 55°$$

$$N_W = \frac{W_L \cos 55°}{2 \sin 55°} + \frac{W_W d_2 \cos 55°}{L \sin 55°} + \frac{W_P d_1 \cos 55°}{L \sin 55°} \qquad N_W = \frac{W_L}{2 \tan 55°} + \frac{W_W d_2}{L \tan 55°} + \frac{W_P d_1}{L \tan 55°}$$

$$N_W = \frac{155\ N}{2 \tan 55°} + \frac{(755\ N)(1.75\ m)}{(3.00\ m) \tan 55°} + \frac{(90.0\ N)(2.60\ N)}{(3.00\ m) \tan 55°} = 417.2765\ N$$

$$\mu_S = \frac{F_F}{N_F} = \frac{N_W}{N_F} = \frac{417.2765\ N}{1000\ N} = 0.417$$

Example: A system of 9 frictionless massive pulleys is used to lift a heavy object (as shown). The 5 upper pulleys are anchored to the ceiling. The four lower pulleys are anchored to a 900 lb. box. How much force (F) is required to lift the box at a constant velocity?



Let's call T the tension in line 1. $T = F$.

The sum of the torques on the upper right pulley is zero. The tension in line 2 has to equal T, the tension in line 1.

The same logic works for lines 3 through 10. All those tensions are also T.

The sum of the forces on the upper right pulley is zero. The tension in line 11 has to equal 2T, the sum of the tension in lines 1 and 2.

The same logic works for lines 12 through 15, as well as 16 through 19. All those tensions are also 2T.

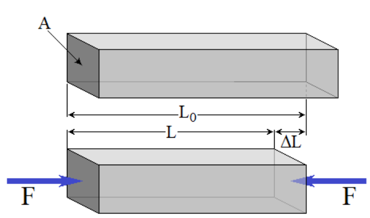
Sum of the forces on the box is zero: $T + 2T + 2T + 2T + 2T - W = 0$

$$9T = W \qquad T = \frac{W}{9} = \frac{900\ lbs}{9} = 100\ lbs$$

Deformation

- When an object changes shape due to applied forces, it is called Deformation.
- When the deformation exceeds The Elastic Limit, the change in shape becomes permanent. Until it reaches this limit, the changes are typically linear and the object will return to its original shape once the forces are removed.
- Elastic Deformation. $F = Y\left(\frac{\Delta L}{L_0}\right) A \qquad Y = \frac{Tensile\ Stress}{Tensile\ Strain} = \frac{F \cdot L_0}{A \cdot \Delta L}$

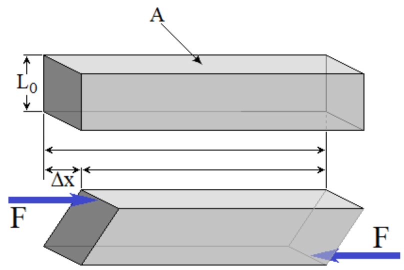
'Tensile' (from tension) refers to stretching. 'Compressive' refers to compression. Apart from that, these work much in the same way.



$$Tensile\ Stress = \frac{F}{A} \qquad Tenstile\ Strain = \frac{\Delta L}{L_o}$$

Young's Modulus (Y) is a property of the material (i.e. a constant that varies from material to material)

- Shear Deformation. $F = S\left(\frac{\Delta x}{L_0}\right)A \qquad S = \frac{Shear\ Stress}{Shear\ Strain} = \frac{F \cdot L_0}{A \cdot \Delta x}$

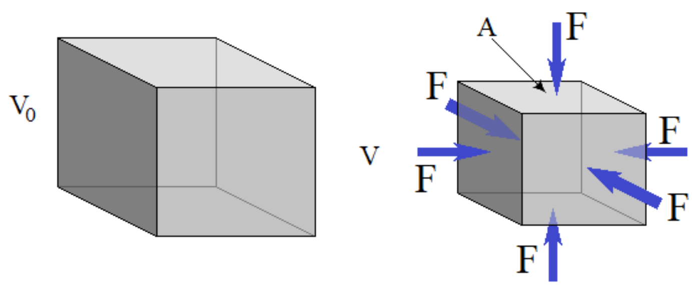


$$Shear\ Stress = \frac{F}{A}$$

$$Shear\ Strain = \frac{\Delta x}{L_o}$$

The Shear Modulus (S) is a property of the material (i.e. a constant that varies from material to material)

- Elastic Deformation. $\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \qquad B = -\frac{Pressure\ Change}{Bulk\ Strain} = -\Delta P\frac{V_0}{\Delta V}$

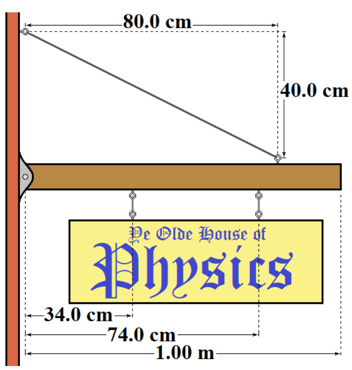


$$Pressure = \frac{F}{A}$$

$$Bulk\ Strain = \frac{\Delta V}{V_o}$$

The Bulk Modulus (B) is a property of fluids/gases (i.e. a constant that varies from material to material)

Example: An 89.4 cm long steel cable is used to hold up a beam and sign under a tension of 188 N. The cable radius is 5.00 mm, and the Young's modulus for steel is 2.00 x 10^{11} N/m^2. How much does the length of the cable change when the tension is added?



$$\Delta L = \frac{F \cdot L_0}{Y \cdot A} = \frac{F L_0}{Y\pi r^2}$$

$$\Delta L = \frac{(188\ N)(0.894\ m)}{\left(2.00 \times 10^{11}\frac{N}{m^2}\right)\pi(0.00500m)^2}$$

$$\Delta L = 10.7\ \mu m$$

Fluids (Static)

- Unlike rigid objects, fluids flow. Often they don't have a fixed shape and sometimes not even a fixed size.
- Rather than using masses and forces, it is more convenient to use mass density (ρ), often referred to as just 'density', and pressure (P).
- The Density of a substance is its mass divided by volume. Units of Density: kg/m^3.

$$\rho = \frac{m}{V}$$

- The Specific Gravity of a substance is the ratio of its density to the density of water at 4°C.

$$\rho_{H_2O(4°C)} = 1000\frac{kg}{m^3}$$

- The molecules in a fluid are in random motion and collide with whatever contains them. The impulse of these collisions impart forces (simultaneously holding the fluid inside and pressing outward on the container). This force per area is defined to be The Pressure (P) of the fluid.

$$P = \frac{F}{A}$$

  - The Units of Pressure are the Pascal. $1\ Pa\ = \frac{kg}{m^3}$.
  - Another common unit is the 'atmosphere' (atm), which is roughly the pressure of the air at Earth's surface.

$$1\ atm = 1.013\ \times\ 10^5\ Pa = 101.3\ kPa$$

- Forces from pressure are always directed perpendicular (normal) to the surface.

Example: The pressure inside a plastic bottle containing a carbonated beverage is 207 kPa. The diameter of the cap is 2.22 cm. Determine the net force of pressure on the cap.

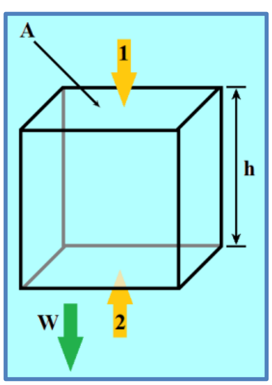
The net force is the force from the inside pressure minus the force from the outside pressure.

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{0.0222\ m}{2}\right)^2 = 3.871\ \times\ 10^{-4}\ m^2$$

$$F_{Net} = F_{Inside} - F_{Outside} = P_{Inside}\cdot A - P_{Outside}\cdot A = (P_{Inside} - P_{Outside})\cdot A$$

$$F_{Net} = [(2.07\ \times\ 10^5\ Pa) - (1.013\ \times\ 10^5\ Pa)]\cdot(3.871\ \times\ 10^{-4}\ m^2) = 40.9\ N$$

Pressure with Depth (Static): $P_2 = P_1 + \rho g h$



Let's take a large mass of a static fluid. We can place an imaginary container (a box) around some of the fluid. Let's examine this object.

The forces on the sides cancel (left/right, front/back) otherwise there would be horizontal acceleration of the fluid enclosed.

As our fluid is stationary, the vertical pressures at top and bottom must cancel out the weight of the fluid enclosed. $F_2 = F_1 + W$

$$P_2 A = P_1 A + mg = P_1 A + \rho V g = P_1 A + \rho A h g \qquad P_2 = P_1 + \rho g h$$

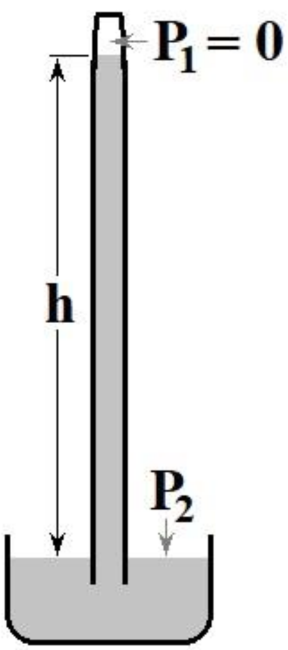
Example: The deepest point in Earth's oceans is in the Mariana Trench at a depth of 10,994 m. If we were to explore this region in a bathysphere, what would be the net force on a 30.0 cm diameter window? You may assume the density of seawater is 1030 kg/m^3 and that the pressure inside the bathysphere is kept at 1.00 atm.

$$F_{Net} = F_{Out} - F_{In} = P_{Out}A - P_{In}A = (P_{Out} - P_{In}) \cdot A = [(P_{atm} + \rho gh) - P_{atm}]\pi r^2$$

$$F_{Net} = \pi \rho g h r^2 = \pi \left(1030 \frac{kg}{m^3}\right)\left(9.80 \frac{m}{s^2}\right)(10{,}994\ m)(0.150\ m)^2 = 7.84 \times 10^6\ N$$

7.84×10^6 N is roughly equivalent to 1.7 million lbs.

Pressure Gauges



- A simple barometer (pressure gauge) can be built by filling a tube with fluid and then turning it over (without allowing air to enter) into a reservoir.
- The difference in pressure between the outside air pressure and the vacuum at the top is sufficient to support the column of fluid of height h.
- The height of the column of fluid is directly proportional to the pressure.

$$P_2 = P_1 + \rho gh = \rho gh \qquad h = \frac{P_2}{\rho g}$$

- To keep the height of the column to a minimum a high density liquid, such as mercury (Hg) must be used.

$$\rho_{Hg} = 13.6 \times 10^3 \frac{kg}{m^3} \qquad h = \frac{P_2}{\rho g} = \frac{\left(1.013 \times 10^5\ Pa\right)}{\left(13.6 \times 10^3 \frac{kg}{m^3}\right)\left(9.80 \frac{m}{s^2}\right)} = 760\ mm$$

The units 'mmHg' (millimeters of mercury) are often used in weather.