Lecture 15: Artificial Gravity, Work, and Energy

Physics for Engineers & Scientists (Giancoli): Chapters 6 & 7
University Physics V1 (Openstax): Chapters 7 & 13

Artificial Gravity

- For accelerating reference frames we can account for the acceleration by adding the effects in with gravity, creating "apparent gravity". We used a_F as the acceleration of the reference frame.

$$\vec{g}_{app} = \vec{g} - \vec{a}_F$$

- For an object in orbit around the Earth, the gravitational force acts as the centripetal acceleration. Under these conditions the apparent gravity is zero.

$$\vec{a}_F = \vec{g} \qquad \vec{g}_{app} = \vec{g} - \vec{g} = 0$$

- Astronauts spending long periods in space suffer debilitating conditions due to the lack of gravity, including loss of bone and muscle mass. Artificial gravity provides a means of eliminating these effects, allowing the life forms of Earth to spend an indefinite amount of time in space.
- By making the object in orbit rotate, we can introduce an additional centripetal force that can be used to create artificial gravity. This gravity is equal to a_C, but pointing radially outward.

$$\vec{a}_F = \vec{g} + \vec{a}_C \qquad \vec{g}_{app} = \vec{g} - \vec{a}_F = \vec{g} - (\vec{g} + \vec{a}_C) = -\vec{a}_C$$

Example: The outer wall of a toroidal (donut-shaped) space station is 50.0 m from its central axis. If the toroid rotates uniformly to create a gravity equivalent to Earth's surface, what is the speed of the outer wall?

$$a_c = g = \frac{v^2}{r} \qquad \mathrm{v} = \sqrt{\mathrm{gr}} = \sqrt{\left(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(50.0\ \mathrm{m})} = 22.1\,\frac{\mathrm{m}}{\mathrm{s}}$$

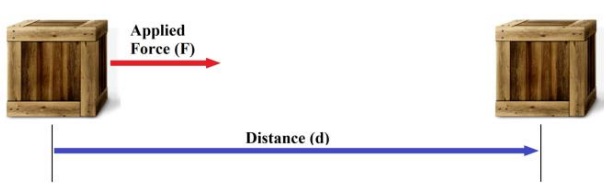
Work and Energy

- All motion and interactions can be understood in terms of energy and the exchange of energy.
- Work is derived from force (a vector), but work and energy are both scalar quantities (not vectors!)
- Many of the problems you have been working can be solved using an energy-based approach.
- In most cases, an energy-based approach to solving problems is preferable to other means.

In other words, don't go running back to kinematics on your homework!

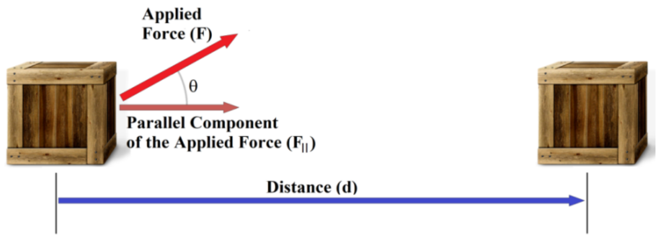
Work $\quad \boldsymbol{W} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$

- When an object moves a distance d in the direction of an applied force, the work done by that force is the product of the force and the distance.



$$\mathrm{W} = \mathrm{F} \cdot \mathrm{d}$$

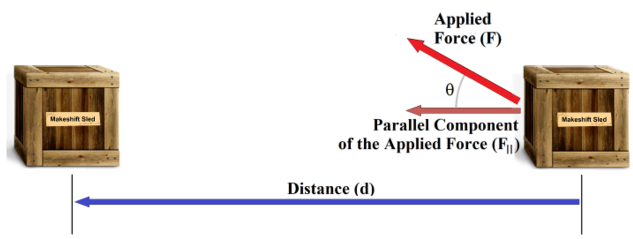
- The standard unit of work and energy the Joule. $1\ J = 1\ N{\cdot}m$
- When an object moves a distance d NOT in the direction of an applied force, the work done is the product of the parallel component of the force and the distance.



$$W = F_{||}{\cdot}d = F{\cdot}d{\cdot}Cos(\theta) = \vec{F}{\cdot}\vec{d}$$

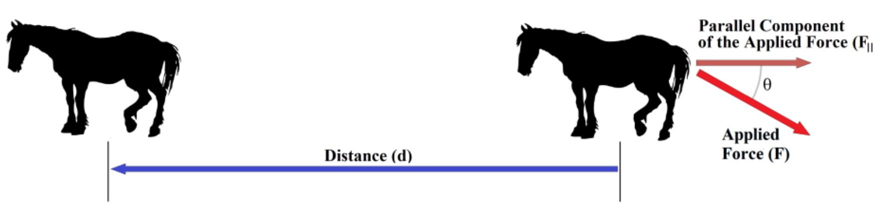
- This is also equivalent to multiply the applied force in full by the component of the distance in the direction of that force.

Example: Two horses pull a man on a makeshift sled. The man and the sled have a combined mass of 204 kg, and the force of friction between the sled and the ground is 700 N. When the horses pull the sled, each of the three chains has a tension of 396 N and makes an angle of 30.0° with respect to the horizontal as they pull the man a distance of 20.2 m. Determine A) the work done on the sled by one of the chains, B) the work done on the horses by one of the chains, and C) the work done on the sled by friction.



A) $W = F_{||}{\cdot}d = F{\cdot}d{\cdot}Cos(\theta) = (396\ N)(20.2\ m)Cos(30.0°) = 6.93\ kJ$

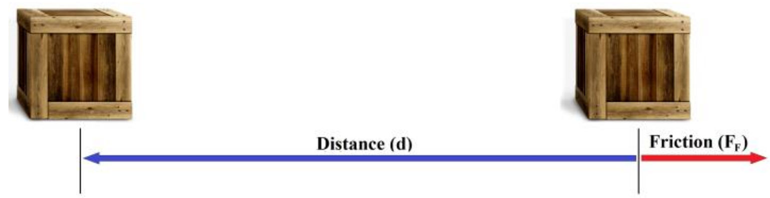
The positive sign on W indicates that the sled is gaining energy.



B) The angle between the force and the distance is now $180° \pm \theta$

$$W = F_{||}{\cdot}d = F{\cdot}d{\cdot}Cos(180° \pm \theta) = (396\ N)(20.2\ m)Cos(210.0°) = -6.93\ kJ$$

The negative sign on W indicates that the horse is losing energy.
The energy lost by the horse is being given to the sled.



A) The angle between the force and the distance is now 180°

$$W = F_{\parallel} \cdot d = F \cdot d \cdot Cos(180°) = (700\ N)(20.2\ m)Cos(180°) = -14.14\ kJ$$

The remaining energy absorbed by the sled is converted to motion of the sled.
Three chains each deliver 6.93 kJ and friction removes 14.14 kJ
Change in Energy = 3(6.93 kJ) – 14.14 kJ = 6.65 kJ

As friction always opposes motion, any work done by friction to a moving object will always be negative.

Example: How much work is done by a constant force, F = (3.26 N)$\hat{\imath}$ + (5.67 N)$\hat{\jmath}$, as it acts on an object that moves from point P_1 = (1.23 m, 4.15 m) to point P_2 = (2.71 m, 3.85 m)?

$$\vec{d} = \Delta x\hat{\boldsymbol{\imath}} + \Delta y\hat{\boldsymbol{\jmath}} = (x_2 - x_1)\hat{\boldsymbol{\imath}} + (y_2 - y_1)\hat{\boldsymbol{\jmath}}$$

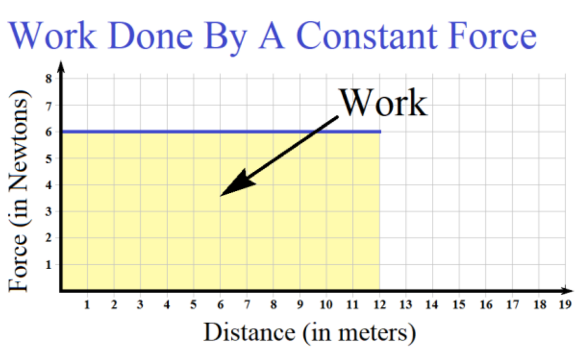
$$\vec{d} = (2.71\ m - 1.23m)\hat{\boldsymbol{\imath}} + (3.85\ m - 4.15m)\hat{\boldsymbol{\jmath}} = (1.48\ m)\hat{\boldsymbol{\imath}} + (-0.30\ m)\hat{\boldsymbol{\jmath}}$$

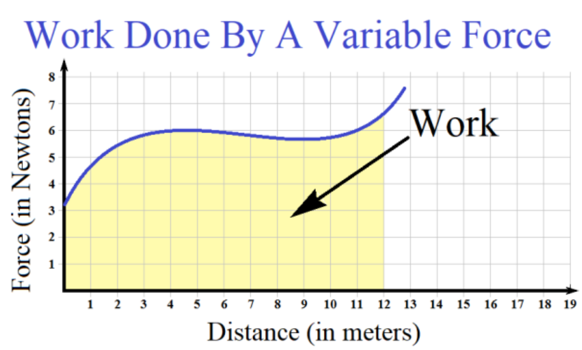
$$W = \vec{F} \cdot \vec{d} = \{(3.26\ N)\hat{\boldsymbol{\imath}} + (5.67\ N)\hat{\boldsymbol{\jmath}}\} \cdot \{(1.48\ m)\hat{\boldsymbol{\imath}} + (-0.30m)\hat{\boldsymbol{\jmath}}\}$$

$$W = (3.26\ N)(1.48\ m) + (5.67\ N)(-0.30m) = 3.12\ J$$

Work for Variable Forces

- For a constant force, the work may be considered to be the area created on a plot of force versus distance.
- The same holds true for a variable force. The work is the area under the curve when force is plotted against distance.





- In one dimensional motion: $W = \int_{x_1}^{x_2} F(x)dx$

- In higher dimensional motion: $W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$
  - W is the work done
  - P_1 is the starting point
  - P_2 is the ending point
  - $\vec{F}$ is the force vector, which is a function of position.
  - $d\vec{r}$ is an infinitesimal displacement vector
  - $\vec{F} \cdot d\vec{r}$ is a vector dot-product

Example: An object moves from the origin to x = 1.53 m under the influence of a single force given by $F(x) = (2.54\ N/m^2)x^2 + (7.95\ N/m)x + (16.95\ N)$. Determine the work done by the force.

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{0}^{1.53\ m} (\alpha x^2 + \beta x + \gamma)dx = \left\{\frac{\alpha}{3}x^3 + \frac{\beta}{2}x^2 + \gamma x\right\}_0^{1.53\ m}$$

$$W = \frac{\left(2.54\ \frac{N}{m^2}\right)}{3}(1.53\ m)^3 + \frac{\left(7.95\frac{N}{m}\right)}{2}(1.53\ m)^2 + (16.95\ N)(1.53\ m) = 38.3\ J$$

Example: An object moves from the origin to point P = (1.12 m, 1.74 m) under the influence of a single force $F(x,y) = \{(3.17\ N/m)x\}\hat{\imath} + \{(1.15\ N/m^2)y^2 + (9.41\ N)\}\hat{\jmath}$. Determine the work done by the force.

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_{P_1}^{P_2} (F_x\hat{\imath} + F_y\hat{\jmath}) \cdot (dx\hat{\imath} + dy\hat{\jmath}) = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$$

$$W = \int_0^{1.12m} \alpha x dx + \int_0^{1.74\ m} \{\beta y^2 + \gamma\}dy = \left\{\frac{\alpha}{2}x^2\right\}_0^{1.12\ m} + \left\{\frac{\beta}{3}y^3 + \gamma y\right\}_0^{1.74\ m}$$

$$W = \frac{\left(3.17\frac{N}{m}\right)}{2}(1.12\ m)^2 + \frac{\left(1.15\frac{N}{m^2}\right)}{3}(1.74\ m)^3 + (9.41\ N)(1.74\ m) = 20.4\ J$$

Kinetic Energy $KE = \frac{1}{2}mv^2$

- **Kinetic energy** is the energy an object possesses due to its motion.

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} ma \cdot dx = \int_{x_1}^{x_2} m \cdot \frac{dv}{dt} \cdot dx = \int_{x_1}^{x_2} m \cdot \frac{dx}{dt} \cdot dv$$

Remember, dv and dx are just numbers and can be swapped, infinitesimally small, but numbers nonetheless.

$$W = \int_{v_1}^{v_2} mv \cdot dv = \left\{\frac{1}{2}mv^2\right\}_{v_1}^{v_2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- We define kinetic energy to be: $KE = \frac{1}{2}mv^2$
- This derivation suggests that any work done to an object results in a change in kinetic energy. This is valid when all the forces acting on an object are accounted for.

$$W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- While this view is conceptually clearer, it is mathematically equivalent to the kinematics done previously (simply multiply an equation by ½m).

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + ma(x - x_0)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0) = Fd = W$$

Example: A 5.00 kg bald eagle is initially gliding horizontally at a speed of 11.3 m/s. It begins flapping its wings, generating a horizontal force of 19.6 N. How fast is the Eagle flying when it stops flapping its wings after a distance of 21.3 m??

$$W = \Delta KE \qquad Fd = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 \qquad 2Fd = mv^2 - mv_0^2 \qquad \frac{2Fd}{m} = v^2 - v_0^2$$

$$v^2 = v_0^2 + \frac{2Fd}{m} \qquad v = \sqrt{v_0^2 + \frac{2Fd}{m}} = \sqrt{\left(11.3\,\frac{m}{s}\right)^2 + \frac{2(19.6N)(21.3m)}{5.00\,kg}} = 17.2\,\frac{m}{s}$$

Gravitational Potential Energy U_G = mgh

- When an object moves against the force of gravity, the work done lifting it becomes stored in the objects position (gravitational potential energy).
- When that object is released, the entirety of the stored energy is converted back into kinetic energy as it falls (assuming friction/wind resistance is not present).
- By definition, the potential energy of a force (U) differs by a negative sign from the work done moving against that force.

$$U = -W$$

- On the surface of the Earth, the gravitational force is effectively constant and equal to an object's weight (F = mg). Moving upward, the distance it moves would be the change in height. The force of gravity is directed opposite to the motion.

$$U_g = -W_g = -F_g \cdot d \cdot Cos(\theta) = -mg \cdot h \cdot Cos(180°) = mgh$$

- As we can choose to place our coordinate axis anywhere we desire, the gravitational potential energy of an object may vary with our choice of origin. Consequently, gravitational potential energy doesn't have an absolute value. Only the change in potential energy is relevant.

Conservation of Energy

- Energy is neither created nor destroyed. It only changes from one form to another.
- If work is done to an object due to external forces, it must be converted into either kinetic energy or potential energy (or some of both).

$$W_{NC} = \Delta KE + \Delta U$$

$$\Delta KE = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 \qquad \Delta U = mgy - mgy_0$$

- The work done by non-conservative forces (W_{NC}) includes any energy added by applied forces as well as energy lost due to friction. Gravity is excluded as it is a conservative force and is covered under potential energy.
- Conservative forces are those where the work done moving from one point to another does not depend on the path. Any work done by these forces is stored as potential energy (which might be returned later).

- It can be useful to rearrange this equation to find: $\boldsymbol{E_{init} + E_{added} = E_{final}}$

$$W_{NC} = \Delta KE + \Delta U = \left(KE_{final} - KE_{init}\right) + \left(U_{final} - U_{init}\right)$$
$$W_{NC} = \left(KE_{final} + U_{final}\right) - \left(KE_{init} + U_{init}\right) = E_{final} - E_{init}$$
$$E_{init} + W_{NC} = E_{final} \qquad E_{init} + E_{added} = E_{final}$$

- The variable time does not show up directly in any form of energy. Any problem that includes motion without a collision and makes no mention of time is a good candidate to solve using conservation of energy.
- Energy lost due to friction is not actually lost, but rather it is transformed in heat (molecular vibrations).