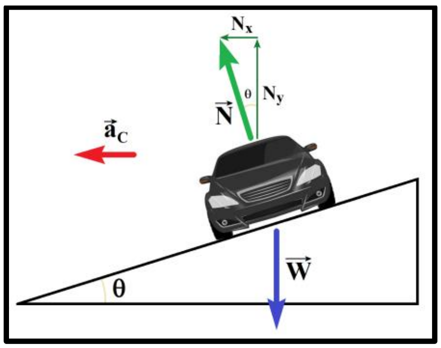
Lecture 14: Banked Curves and Gravity

Physics for Engineers & Scientists (Giancoli): Chapters 5 & 6
University Physics V1 (Openstax): Chapters 6 & 13

Frictionless Banked Curve

There are only two forces on a frictionless banked curve (weight and the normal force). When combined these forces result in an acceleration that is inward.



Step 1: Determine which force is acting as F_C.

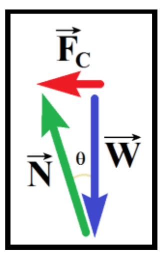
What force points to the center? $N_x = F_C$.

Step 2: Solve for F_C: $\quad \mathrm{Tan}\,\theta = \frac{N_x}{N_y} = \frac{F_C}{W} = \frac{F_C}{mg}$

$$F_C = mg\mathrm{Tan}\,\theta$$

Step 3: Set $F_C = mv^2/r$: $\quad F_C = mg\mathrm{Tan}\,\theta = \frac{mv^2}{r}$

$$g\mathrm{Tan}\,\theta = \frac{v^2}{r} \qquad v^2 = rg\mathrm{Tan}\,\theta \qquad v = \sqrt{rg\mathrm{Tan}\,\theta}$$



Alternatively, we could have made a triangle from a simple vector equation:

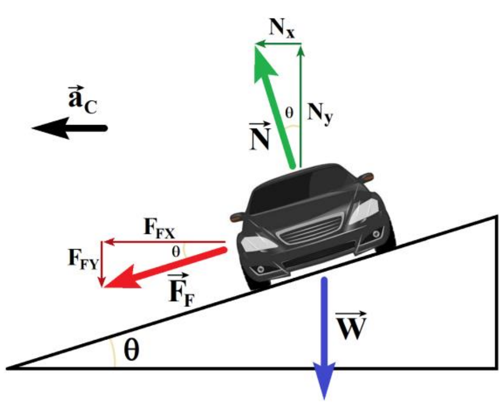
$$\vec{W} + \vec{N} = \vec{F}_C$$

$$\mathrm{Tan}\,\theta = \frac{F_C}{W} = \frac{F_C}{mg}$$

Banked Curve with Friction

There are three forces on a frictionless banked curve (weight, the normal force, and friction). When combined these forces result in an acceleration that is inward.

The direction of the friction force is dependent upon circumstances. A slow moving car would tend to slide down the incline (opposing this, the friction would point up the incline). Alternatively, a fast moving car would tend to skid outward and up the incline. In this latter case the friction would point down the incline. As we will be considering maximum velocities, we will have the friction pointing down the slope.



Step 1: Determine which force(s) is acting as F_C.

What force(s) points to the center? $F_C = N_x + F_{Fx}$.

Step 2: Solve for F_C: start with x-components

$$F_c = N_x + F_{Fx} = N\mathrm{Sin}\,\theta + F_F\mathrm{Cos}\,\theta$$

$$F_c = N\mathrm{Sin}\,\theta + \mu_S N\mathrm{Cos}\,\theta = (\mathrm{Sin}\,\theta + \mu_S\mathrm{Cos}\,\theta)N$$

Need N from y-components: $N_y = W + F_{Fy}$

$$N\mathrm{Cos}\,\theta = mg + F_F\mathrm{Sin}\,\theta = mg + \mu_S N\mathrm{Sin}\,\theta$$

$$N\cos\theta - \mu_S N\sin\theta = mg \qquad (\cos\theta - \mu_S\sin\theta)N = mg \qquad N = \frac{mg}{\cos\theta - \mu_S\sin\theta}$$

Plug N (from y-components) back in to the equation for F_C (from the x-components):

$$F_c = (\sin\theta + \mu_S\cos\theta)N = (\sin\theta + \mu_S\cos\theta)\left(\frac{mg}{\cos\theta - \mu_S\sin\theta}\right)$$

$$F_c = mg\left(\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta}\right)$$

Step 3: Set $F_C = mv^2/r$: $\quad F_c = mg\left(\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta}\right) = \frac{mv^2}{r} \qquad g\left(\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta}\right) = \frac{v^2}{r}$

$$v^2 = rg\left(\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta}\right) \qquad v = \sqrt{rg\left(\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta}\right)}$$

Earlier we solved an unbanked turn with friction and got the maximum velocity $v = \sqrt{\mu_s gr}$.

This should be equivalent to the banked turn with friction if we let the banking angle be zero.

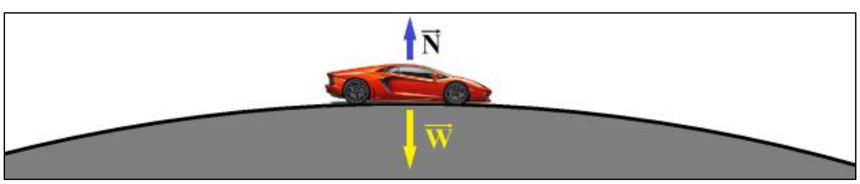
$$v = \sqrt{rg\left(\frac{\sin 0 + \mu_S\cos 0}{\cos 0 - \mu_S\sin 0}\right)} = \sqrt{rg\left(\frac{0 + \mu_S\cdot 1}{1 - \mu_S\cdot 0}\right)} = \sqrt{\mu_s gr}$$

Earlier we solved a banked turn without friction and got the velocity $v = \sqrt{rgTan\,\theta}$

This should be equivalent to the banked turn with friction if we let μ_S *be zero.*

$$v = \sqrt{rg\left(\frac{\sin\theta + 0\cdot\cos\theta}{\cos\theta - 0\cdot\sin\theta}\right)} = \sqrt{rg\left(\frac{\sin\theta}{\cos\theta}\right)} = \sqrt{rg\tan\theta}$$

<u>Example</u>: A car drives over a hill. At the crest, the radius of curvature is 50.0 m. What is the maximum speed the car can have and still keep its tires on the road?



Step 1: Which force(s) is acting as F_C?

The centripetal force must be directed vertically downward (towards the center of the circle). As there are two vertical forces acting on the car, these must combine to act as the centripetal force.

$$F_C = W - N$$

Step 2: Solve for F_C: $\quad F_C = W - N = mg - N$

To determine the maximum velocity, we need the maximum centripetal force. W=mg is fixed in value. The normal force can take any positive value. The maximum value of F_C *comes when N=0.*

$$F_C = mg$$

Step 3: Set $F_C = mv^2/r$: $F_C = mg = \frac{mv^2}{r}$ $g = \frac{v^2}{r}$ $v^2 = gr$

$$v = \sqrt{gr} = \sqrt{\left(9.80\frac{m}{s^2}\right)(50.0\ m)} = 22.1\frac{m}{s}$$

The Forces of Nature

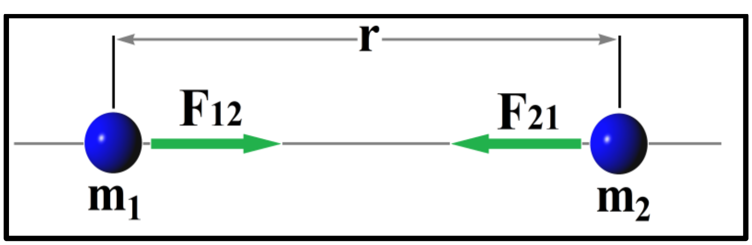
- The **Gravitational Force** creates forces between massive objects. It is responsible for pulling objects to the surface of the Earth, causing planets to orbit the sun, and causing stars to orbit the centers of galaxies.
- The **Electromagnetic Force** creates forces between electric charges. It allows us to generate and use electrical power, transmit signals across vast distances, and use a multitude of convenient devices.
- The **Weak Nuclear Force** is most noticeable in radioactive decays, but also plays a role in nuclear fission (as used in nuclear power plants and nuclear bombs).
- The **Strong Nuclear Force** binds quarks into hadrons (such as protons and neutrons). In addition, it holds atomic nuclei together.

Newton's Law of Gravity

- Newton's Law of Gravity determines the magnitude of the gravitational force (F) between two objects.

$$F = G\frac{m_1 m_2}{r^2}$$

  - F is the force on both massive objects (equal and opposite)
  - G is the gravitational constant $\left(G = 6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right)$
  - m_1 and m_2 are the masses of the two objects
  - r is the distance separating the two object's centers of mass.
- This force is attractive, and is directed along the line connecting the two object's centers of mass.



- The gravitational acceleration (g) is caused by the Earth's gravitational pull and can be directly calculated from Newton's law of gravity, given the mass of the Earth ($M_E = 5.98 \times 10^{24}$ kg), the radius of the earth ($R_E = 6.38 \times 10^6$ m) and the gravitational constant ($G = 6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$).

$$W = G\frac{M_E m}{R_E^2} = m\left[\frac{GM_E}{R_E^2}\right] = mg$$

$$g = \frac{GM_E}{R_E^2} = \frac{\left(6.673 \times 10^{-11}\,\frac{N\cdot m^2}{kg^2}\right)(5.98 \times 10^{24}\,kg)}{(6.38 \times 10^6\ \mathrm{m})^2} = 9.8035\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

Satellites in Circular Orbits

- For orbiting satellites, gravitational attraction acts as the centripetal force. Let's let the mass of a satellite be m_s and the mass of the Earth be M_E.

$$F_C = F_g \qquad \frac{m_s v^2}{r} = G\frac{m_s M_E}{r^2} \qquad \frac{v^2}{r} = G\frac{M_E}{r^2} \qquad v^2 = G\frac{M_E}{r} \qquad v = \sqrt{G\frac{M_E}{r}}$$

- Each orbital radius can only be held by an object with a specific velocity, and that velocity is independent of that object's mass:

$$v = \sqrt{G\frac{M_E}{r}}$$

Example: The moon orbits the Earth at a distance of 3.85 x 10^8 m. The Earth's mass is 5.98 x 10^{24} kg. Find the period for the moon's motion around the Earth (in days).

$$v = \sqrt{G\frac{M_E}{r}} = \sqrt{\left(6.673 \times 10^{-11}\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\right)\frac{(5.98 \times 10^{24}\ \mathrm{kg})}{(3.85 \times 10^8\ \mathrm{m})}} = 1018.077\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$v = \frac{2\pi r}{T} \qquad T = \frac{2\pi r}{v} = \frac{2\pi(3.85 \times 10^8\ \mathrm{m})}{1018.077\frac{\mathrm{m}}{\mathrm{s}}} = 2{,}376{,}073\ \mathrm{s}\left(\frac{1\ \mathrm{hour}}{3600\ \mathrm{s}}\right)\left(\frac{1\ \mathrm{day}}{24\ \mathrm{hours}}\right) = 27.5\ \mathrm{days}$$

Example: Objects orbiting the center of galaxy M87 have been found to have an orbiting speed of 7.5 x 10^5 m/s for matter orbiting at a distance of 5.7 x 10^{17} m from the center. Find the mass (M) of the object located at the center.

$$v = \sqrt{G\frac{M_{obj}}{r}} \qquad v^2 = G\frac{M_{obj}}{r} \qquad v^2 r = GM_{obj}$$

$$M_{obj} = \frac{v^2 r}{G} = \frac{\left(7.5 \times 10^5\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 (5.7 \times 10^{17}\ \mathrm{m})}{6.673 \times 10^{-11}\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}} = 4.8 \times 10^{39}\ \mathrm{kg}$$

As the mass of our sun is 2.0 $\times 10^{30}$ kg, the object at the center of M87 is 2.4 billion solar masses. Due to the limited volume available for this amount of mass it was concluded that the object at the center of M87 was a black hole.

Note: Because of the equivalence of inertial mass and gravitational mass, Einstein concluded that there should be no difference between gravity and an accelerating reference frame. This is known as "the equivalence principle", and it led him to conclude that light should bend in a gravitational field even though it has no mass.