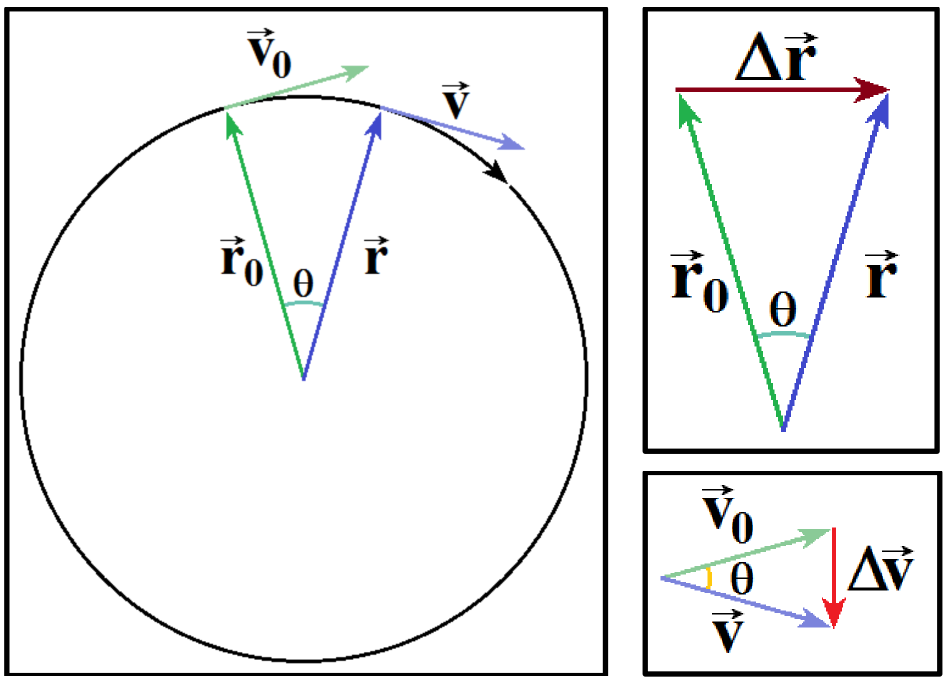
Lecture 13: Uniform Circular Motion

Physics for Engineers & Scientists (Giancoli): Chapter 5
University Physics V1 (Openstax): Chapter 6

Uniform Circular Motion

- **Uniform Circular Motion** is moving in a circular path at a constant speed.
- **The Period (T)** is the time needed to complete one full cycle (once around the circle).
- **The Frequency (f)** is the number of cycles complete divided by the time interval.

$$\mathbf{T} = \frac{\mathbf{1}}{\mathbf{f}} \qquad \mathbf{v} = \frac{\mathbf{2\pi r}}{\mathbf{T}} = \mathbf{2\pi rf}$$



The distance triangle (upper right) and the velocity triangle are similar triangles (identical interior angles). The ratios of corresponding sides of similar triangles are equal.

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

For small time intervals ($\Delta t \rightarrow 0$), the angle theta is very small. This allows us to approximate Δr.

$$\Delta r \approx v \cdot \Delta t$$

Plugging into the previous equation leads to our result.

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v} \qquad \frac{v^2\Delta t}{r} = \Delta v \qquad \frac{v^2}{r} = \frac{\Delta v}{\Delta t}$$

For small time intervals ($\Delta t \rightarrow 0$), $a = \Delta v/\Delta t$. In this case, the acceleration is referred to as "centripetal acceleration (a_c) or radial acceleration (a_r). Centripetal acceleration always points in the direction of Δv, radially inward towards the center of the circle.

$$a_c = \frac{v^2}{r}$$

As the centripetal acceleration a_c is perpendicular to velocity with no component in the direction of the velocity, it only changes the direction and not the magnitude of the velocity.

Example: Computer-controlled display screens provide drivers in the Indianapolis 500 with a variety of information about how their cars are performing. For instance, as a car is going through a turn, a speed of 221 mi/h (98.8m/s) and a centripetal acceleration of 3.00g are displayed. Determine the radius of the turn.

$$3.00g = (3.00)\left(9.80\frac{m}{s^2}\right) = 29.4\ \frac{m}{s^2}$$

$$a_c = \frac{v^2}{r} \qquad a_c \cdot r = v^2 \qquad r = \frac{v^2}{a_c} = \frac{\left(98.8\frac{m}{s}\right)^2}{29.4\frac{m}{s^2}} = 332\ m$$

Centripetal Forces

- The **Centripetal Force** is the force that gives rise to the centripetal acceleration that causes an object to move in a curved path.

$$\mathrm{F_C = ma_C = \frac{mv^2}{r}}$$

- The centripetal force is not a new force. Any of the other forces we've studied (or a combination) may act as the centripetal force.
- The centripetal force is always directed at the center of the circle, perpendicular to the velocity.

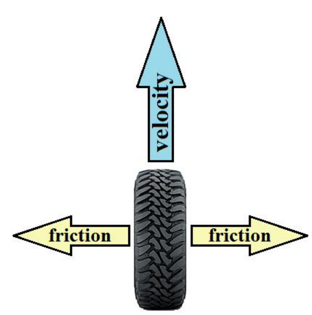
Example: In a skating stunt known as "crack-the-whip", a number of skaters hold hands and form a straight line. They skate so that the line rotates around the skater at one end who acts as a pivot. The skater farthest out has a mass of 80.0kg and is 6.10m from the pivot. He is skating at a speed of 6.80m/s. Determine the magnitude of the centripetal force that acts on him.

$$F_c = \frac{mv^2}{r} = \frac{(80.0\ kg)\left(6.80\frac{m}{s}\right)^2}{6.10\ m} = 606\ \mathrm{N}$$

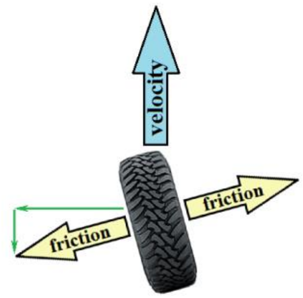
Example: Compare the maximum speeds at which a car can safely negotiate an unbanked turn (radius = 50.0m) in dry weather ($\mu_s = 0.900$) and icy weather ($\mu_s = 0.100$)

Note: Rolling (not sliding) wheels are dependent upon μ_s not μ_k

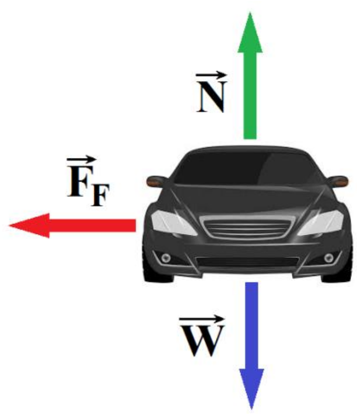
The bottom of a rolling tire is stationary on the road's surface. This is evident when it rolls through water as the tread of the tire is visible in the water trail it leaves behind. This rolling motion prevents any friction forces from being felt in the direction the tire rolls (unless brakes are applied disrupting the rolling motion). Consequently, a rolling tire can only feel friction forces parallel to the tires axle.



When are car turns, the tires can only exert a friction force on one side or the other. One side would have a friction component in the direction of motion (which is not allowed). The friction force is thus felt on the other side with two components. One component is perpendicular to the motion, causing the car to turn. The other is opposite the velocity, causing the car to decelerate.



Note: Anti-lock brakes keep a car from skidding. Skidding cars are affected by the coefficient of kinetic friction, which is typically smaller than the coefficient of static friction. A non-skidding car affected by the larger static coefficient will stop faster.



There are three forces acting on the car: the weight, the normal force, and the friction force.

Step 1: Determine which force is acting as F_C. Remember this force points to the center of the circle the object moves in. In this case, the friction force is acting as the centripetal force.

Step 2: Solve for F_C: $F_C = F_F = \mu_s N = \mu_s mg$

Step 3: Set F_C equal to $\frac{mv^2}{r}$:

$$\frac{mv^2}{r} = \mu_s mg \qquad \frac{v^2}{r} = \mu_s g \qquad v^2 = \mu_s gr \qquad v = \sqrt{\mu_s gr}$$

For dry weather ($\mu_S = 0.900$): $v = \sqrt{\mu_s gr} = \sqrt{(0.900)\left(9.80\frac{m}{s^2}\right)(50.0\ m)} = 21.0\frac{m}{s}$ (47 mph)

For icy weather ($\mu_S = 0.100$): $v = \sqrt{\mu_s gr} = \sqrt{(0.100)\left(9.80\frac{m}{s^2}\right)(50.0\ m)} = 7.00\frac{m}{s}$ (15.7 mph)