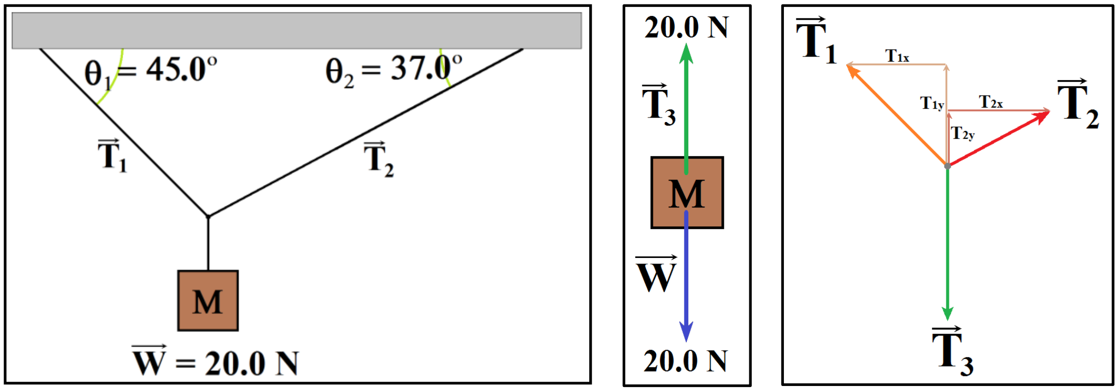
Lecture 12: Tension and Apparent Gravity

***Physics for Engineers & Scientists* (Giancoli): Chapters 4 & 5**
***University Physics V1* (Openstax): Chapters 5 & 6**

Example: A mass weighing 20.0N hangs from a system of strings as shown. Determine the tensions, T_1 and T_2.



Start by making a force diagram for the hanging mass, labeling the tension in the line above it 'T_3'.
Then make a force diagram at the point of intersection of the three tensions.
Break T_1 and T_2 into components.
This is static equilibrium so the forces must cancel. ***$\Sigma F_x = 0$ $\Sigma F_y = 0$***

Hanging Mass: $\Sigma F_y = 0 \Rightarrow T_3 = W = 20.0$ N

Intersection (x-comp): $\Sigma F_x = 0 \Rightarrow T_{1x} = T_{2x}$ $\quad T_2\text{Cos}(37.0°) = T_1\text{Cos}(45.0°)$ $\quad T_2 = 0.885394 \cdot T_1$

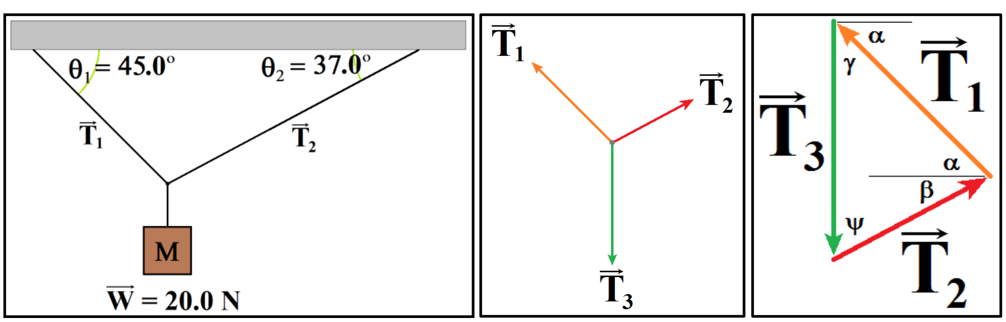
Intersection (y-comp): $\Sigma F_y = 0 \Rightarrow T_3 = T_{1y} + T_{2y}$ $\quad T_3 = T_1\text{Sin}(45.0°) + T_2\text{Sin}(37.0°)$

$$T_3 = T_1\text{Sin}(45.0°) + 0.885394 \cdot T_1\text{Sin}(37.0°) = T_1[\text{Sin}(45.0°) + 0.885394 \cdot \text{Sin}(37.0°)]$$

$$20.0\text{ N} = 1.23995 \cdot T_1 \qquad T_1 = 16.1297\text{ N} \Rightarrow 16.1\text{ N}$$

$$T_2 = 0.885394 \cdot T_1 = 0.885394 \cdot 16.1297\text{ N} = 14.3\text{ N}$$

Alternatively, we could have made a triangle and used the law of sines.



$\alpha = 45.0°$ (Given) $\quad \beta = 37.0°$ (Given) $\quad \gamma = 90° - \alpha = 45.0°$ $\quad \psi = 90° - \beta = 53.0°$

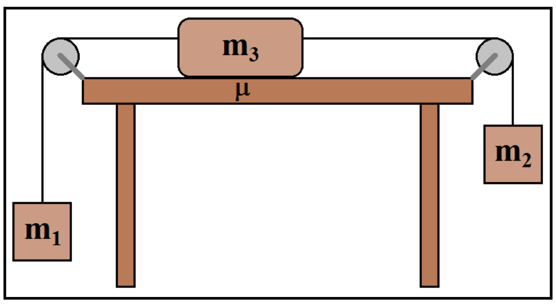
$$\frac{\text{Sin}\,\psi}{T_1} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\psi = T_1 \cdot \text{Sin}\,(\alpha+\beta)$$

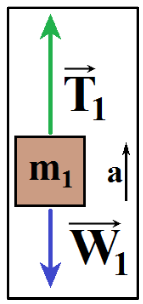
$$T_1 = \frac{T_3 \cdot \text{Sin}\,\psi}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(53.0°)}{\text{Sin}(82.0°)} = 16.1297\text{ N} \Rightarrow 16.1\text{ N}$$

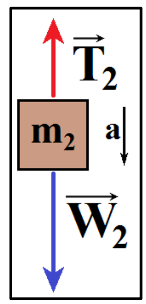
$$\frac{\text{Sin}\,\gamma}{T_2} = \frac{\text{Sin}\,(\alpha+\beta)}{T_3} \qquad T_3 \cdot \text{Sin}\,\gamma = T_2 \cdot \text{Sin}\,(\alpha+\beta)$$

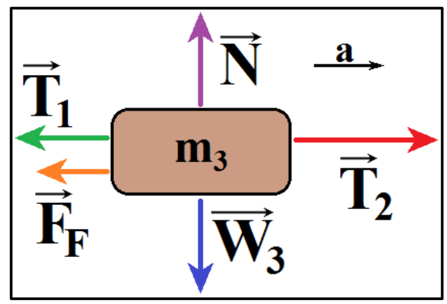
$$T_2 = \frac{T_3 \cdot \text{Sin}\,\gamma}{\text{Sin}\,(\alpha+\beta)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(45.0°+37.0°)} = \frac{(20.0\text{ N})\text{Sin}(45.0°)}{\text{Sin}(82.0°)} = 14.2811\text{ N} \Rightarrow 14.3\text{ N}$$

Example: A box of mass m_3 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_2 = 25.0 kg. On the other side a cord attaches it to a hanging weight of mass m_1 = 5.00 kg. If the coefficient of kinetic friction between the box and table is 0.300, determine the acceleration of the system. Assume the mass and friction of the pulleys is negligible.









Can we go straight to m_3, treating the two tensions as m_1g and m_2g?
No! $a \neq 0 \Rightarrow T \neq W$.

What direction is m_3 accelerating? To the right ($m_2 > m_1$). So, friction points to the left.

Start by making force diagrams for all 3 masses.

There are 4 unknowns (a, F_F, T_1, and T_2).

We need 4 equations (ΣF_y for m_1, ΣF_y for m_2, ΣF_y for m_3, ΣF_x for m_3).

$$\Sigma F_y = m_1 a: \quad T_1 - m_1 g = m_1 a \qquad T_1 = m_1 a + m_1 g$$

$$\Sigma F_y = m_2 a: \quad m_2 g - T_2 = m_2 a \qquad T_2 = m_2 g - m_2 a$$

m_2 accelerates downward (typically negative), but a is being treated as positive.
You either need to make down positive or use '-a' as the acceleration to correct for this.

$$\Sigma F_y = 0 \text{ for } m_3: \quad N = W_3 = m_3 g \qquad F_F = \mu N = \mu m_3 g$$

$$\Sigma F_x = m_3 a: \quad T_2 - T_1 - F_F = m_3 a \qquad m_2 g - m_2 a - m_1 a - m_1 g - \mu m_3 g = m_3 a$$

$$m_2 g - m_1 g - \mu m_3 g = m_1 a + m_2 a + m_3 a$$

$$(m_2 - m_1 - \mu m_3)g = (m_1 + m_2 + m_3)a$$

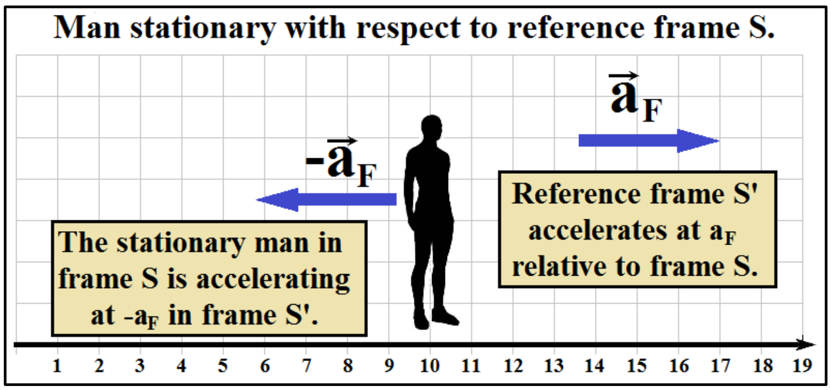
$$a = (m_2 - m_1 - \mu m_3)g / (m_1 + m_2 + m_3)$$

$a = (25.0\text{ kg} – 5.00\text{ kg} – 0.300·10.0\text{ kg})·(9.80\text{ m/s}^2) /(5.00\text{ kg} + 25.0\text{ kg} + 10.0\text{ kg}) = 4.17\text{ m/s}^2$

Apparent Gravity

- One method of dealing with an accelerating reference frame (a non-inertial reference frame) is to ‘package’ the effects created by acceleration in combination with gravity (g) into apparent gravity (g_{app}).
- The acceleration (a_F) of a non-inertial reference frame relative to an inertial reference frame, causes every object in that inertial reference frame to have an additional acceleration ($–a_F$) when viewed in the non inertial frame.

For example, the man below is stationary with respect to his original reference frame. If our coordinate system accelerates to the right at a_F, then he must accelerate in the opposite direction at $–a_F$.



- This additional acceleration ($–a_F$) can be combined (vector addition) to gravity to get apparent gravity.

$$\vec{g}_{APP} = \vec{g} - \vec{a}_F$$

- Problems in the non-inertial frame can then be treated as if they were an inertial frame with a new value (and possibly direction) of gravity.

Example: A person is holding a 5.0 kg package in an elevator that begins accelerating upwards as 1.2 m/s^2. What force must the person exert to hold the package in the same relative position?

You could treat this as a person, elevator and package accelerating in an inertial frame.

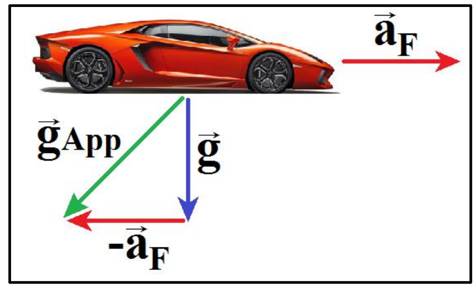
$$F - mg = ma \Rightarrow F = ma + mg = (5.0\text{ kg})(1.2\text{ m/s}^2) + (5.0\text{ kg})(9.8\text{ m/s}^2) = 6N + 49N = 55\text{ N}$$

Or you could treat this as a person and package stationary in an accelerating reference frame.

Note that as a_F points upwards, - a_F points downward (in the same direction as gravity). This indicates that you should add the magnitudes.

$$g_{app} = g – a_F = 9.8\text{ m/s}^2 – (–1.2\text{ m/s}^2) = 11\text{ m/s}^2$$

$$W_{app} = mg_{app} = (5.0\text{ kg})(11\text{ m/s}^2) = 55N$$



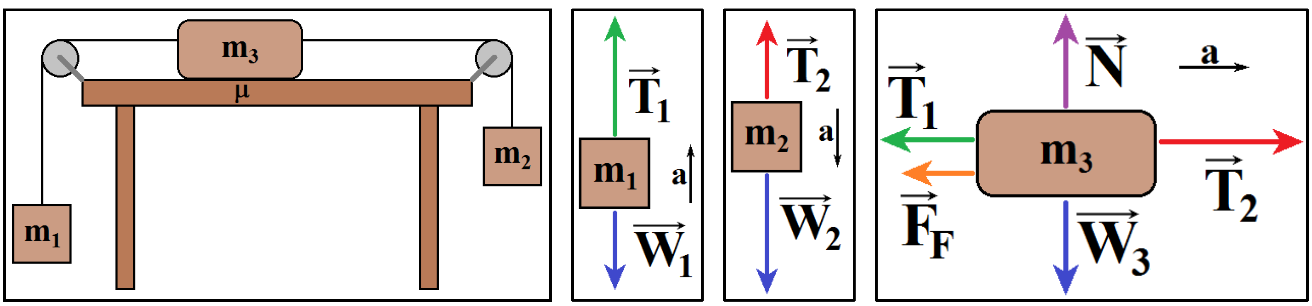
Conceptual Example: When you are in an accelerating car, you feel like you are being pushed back into your seat.

$$|g_{APP}| = \sqrt{g^2 + a_F^2}$$

Please don't measure the acceleration of your car by noting the angle of something hanging from the mirror while driving!

Example: A box of mass m_3 = 10.0 kg sits on a table as shown. On one side a cord connects it to a hanging weight of mass m_2 = 25.0 kg. On the other side a cord attaches it to a hanging weight of mass m_1 = 5.00 kg. The coefficient of kinetic friction between the box and table is 0.300. *If the table is placed in an elevator that is accelerating downward at 1.20 m/s²*, determine the horizontal acceleration of the box on the table. Assume the mass and friction of the pulleys is negligible.

This is identical to the problem we solved earlier with one exception. Now it's in an elevator accelerating downward.



We could solve this without apparent gravity, making modifications in each force diagram.

Decrease the acceleration of m_1 (in $\Sigma F_y = m_1a$): $T_1 - m_1g = m_1(a - 1.20\ \text{m/s}^2)$

Increase the acceleration of m_2 (in $\Sigma F_y = m_2a$): $m_2g - T_2 = m_2(a + 1.20\ \text{m/s}^2)$

Account for the acceleration of m_3 (ΣF_y is not zero): $W_3 - N = m_3(1.20\ \text{m/s}^2)$

Which changes friction to: $F_F = \mu N = \mu m_3g - \mu m_3(1.20\ \text{m/s}^2)$

Then solve for a after plugging T_1, T_2, and F_F into: $T_2 - T_1 - F_F = m_3a$

Or we can solve it as we did before and replace g with $g_{app} = g - a_F$

$$g_{app} = g - a_F = 9.80\ \text{m/s}^2 - 1.20\ \text{m/s}^2 = 8.6\ \text{m/s}^2$$

$$a = (m_2 - m_1 - \mu m_3)g_{app} / (m_1 + m_2 + m_3)$$

$$a = (25.0\ \text{kg} - 5.00\ \text{kg} - 0.300 \cdot 10.0\ \text{kg}) \cdot (8.60\ \text{m/s}^2) / (5.00\ \text{kg} + 25.0\ \text{kg} + 10.0\ \text{kg}) = 3.655\ \text{m/s}^2$$

FYI, one of the major breakthroughs of Albert Einstein was the 'equivalence principle', essentially stating that gravity was no different than any other acceleration. This lead to his prediction that light would bend due to gravity and eventually to general relativity.