Lecture 11: Power, Impulse, and Momentum

***Physics for Engineers & Scientists* (Giancoli): Chapters 8 & 9**
***University Physics V1* (Openstax): Chapters 8 & 9**

Power

- We define **Average Power** as: $P_{avg} = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t} = \frac{E_{final} - E_{init}}{t_{final} - t_{init}}$
- and **Instantaneous Power** as: $P = \frac{dW}{dt} = \frac{dE}{dt}$
- Power is defined using work, but the definition applies to any form of energy.
- The units of power are the Watt (W): 1 W = 1 J/s
- If the power is being delivered by a constant force: $P = Fv \qquad P = \frac{dW}{dt} = \frac{F \cdot dx}{dt} = F\frac{dx}{dt} = Fv$
- Kilowatt·Hour (kwh) is a unit of energy: $1\ kW \cdot Hr = (1000\ W)(3600\ s) = 3.6 \times 10^6\ J$

Example: A low-flying eagle with mass 4.50 kg increases its velocity from 11.3 m/s to 17.2 m/s over a 15.0 second time interval. Over the same time its altitude increases from 1.50 m to 7.75 m. What average power must the eagle's wings deliver to accomplish this?

$$E_{init} = \frac{1}{2}mv_0^2 + mgh_0 \qquad E_{final} = \frac{1}{2}mv^2 + mgh$$

$$P_{avg} = \frac{\Delta E}{\Delta t} = \frac{E_{final} - E_{init}}{\Delta t} = \frac{\frac{1}{2}mv^2 + mgh - \frac{1}{2}mv_0^2 - mgh_0}{\Delta t} = \frac{\frac{1}{2}m(v^2 - v_0^2) + mg(h - h_0)}{\Delta t}$$

$$P_{avg} = \frac{\frac{1}{2}(4.50\ kg)\left\{\left(17.2\frac{m}{s}\right)^2 - \left(11.3\frac{m}{s}\right)^2\right\} + (4.50\ kg)\left(9.80\frac{m}{s^2}\right)(7.75\ m - 1.50\ m)}{15.0\ s} = 43.6\ W$$

Example: The Space-X Falcon 9 rocket has a mass of 1.48×10^6 kg when loaded with payload destined for low-Earth orbit (leo). Its engines generate 22.8 MN (mega-Newtons) of thrust during their initial burn. When the first stage is jettisoned after 157 s, the rocket is going 1,839 m/s at an altitude of 70.4 km. What is the average power output of the engines?

Can we use 'U = mgh' at an altitude of 70.4 km? ...No, but we could use $F = GmM_E/r^2$ and integrate.

Is wind resistance negligible? ...No!

We must use work and not energy.

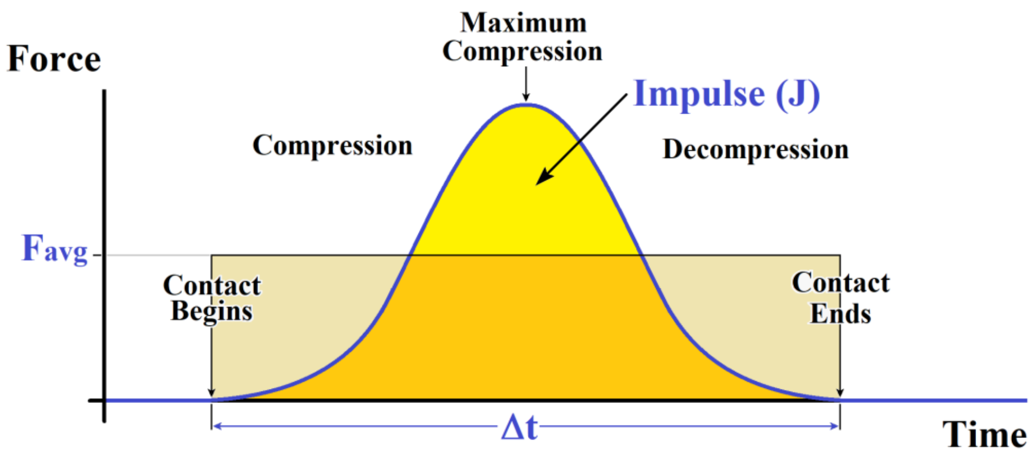
$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{Fd}{\Delta t} = \frac{(22.8 \times 10^6\ N)(70.4 \times 10^3\ m)}{157\ s} = 1.02 \times 10^{10} W$$

Note: The largest nuclear power plant in the US, the Palo Verde Nuclear power plant in Arizona has an output of 3.94×10^9 W and is a major source of electric power for the densely populated parts of Southern Arizona and Southern California, including Phoenix, Tucson, Los Angeles, and San Diego.

The Space-X Falcon 9 rocket delivers 2.5 times more power than the largest nuclear reactor in the country.

Impacts and Impulse

- Collisions generally occur over a short time interval.
- When two objects collide, the surfaces will deform (surfaces compress).
- These surfaces act like springs (albeit with large spring constants). The greater the compression, the greater the force. This allows the contact forces (equivalent to normal forces) to slow the colliding objects and redirect them.
- Some of the collision energy may be released as heat (warming the colliding objects) and sound. Other energy might be consumed in permanently deforming one or both of the objects. As these energies are difficult to account for, conservation of energy can only be used special cases.



- **Impulse** ($\vec{J}$): $\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \vec{F}_{avg} \cdot \Delta t$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \int_{t_1}^{t_2} m\vec{a}(t)dt = m\int_{t_1}^{t_2} \vec{a}(t)dt = m[\vec{v}(t)]_{t_1}^{t_2} = m\vec{v}_2 - m\vec{v}_1$$

- **Momentum** **($\vec{P}$):** $\vec{P} = m\vec{v}$
- The impulse delivered to an object is equal to that object's change in momentum.

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt = \vec{F}_{avg} \cdot \Delta t = m\vec{v}_2 - m\vec{v}_1 = \Delta\vec{P}$$

- Which also means: $\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t}$ $\quad \vec{F} = \frac{d\vec{P}}{dt}$

Example: A golf ball (m_b = 45.0 g) starts at rest on a tee. After the golfer strikes it, it is moving at 38.0 m/s. During impact, the club remains in contact with the ball for 3.00 ms. A) What is the change in momentum of the ball? B) Determine the average force applied to the ball by the club.

A) $\Delta P = P_{Final} - P_{Init} = mv - mv_0 = m(v - v_0) = (0.0450\,kg)\left(38.0\frac{m}{s} - 0\frac{m}{s}\right) = 1.71\,kg \cdot \frac{m}{s}$

B) $F_{Avg} = \frac{\Delta P}{\Delta t} = \frac{1.71\,kg \cdot \frac{m}{s}}{3.00 \times 10^{-3}\,s} = 570\,N$

Example: The Space-X Falcon 9 rocket has a mass of 1.48×10^6 kg when loaded with payload destined for low-Earth orbit (leo). Its engines generate a force given by $F(t) = \beta(1-e^{-\sigma t})$ where $\beta = 2.28 \times 10^7$ N and $\sigma = 0.139\ s^{-1}$. If the rocket is at rest at t = 0, how fast is it moving at t = 5.00 s?

This problem is one dimensional (vertically upward). Vectors can be dispensed with.

$$J = \int_{t_1}^{t_2} F(t)dt = mv$$

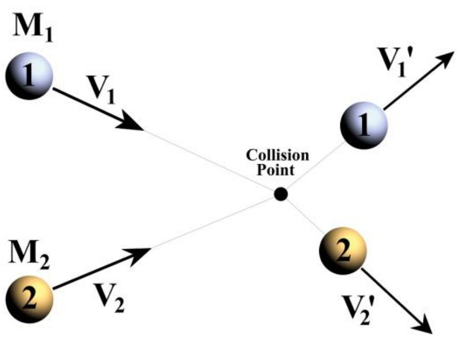
$$J = \int_{t_1}^{t_2} F(t)dt = \int_0^t [\beta(1 - e^{-\sigma t})]dt = \int_0^t [\beta - \beta e^{-\sigma t}]dt = \int_0^t \beta dt + \int_0^t -\beta e^{-\sigma t} dt$$

$$J = [\beta t]_0^t - \left[-\frac{\beta}{\sigma} e^{-\sigma t}\right]_0^t = \beta t - \frac{\beta}{\sigma}(1 - e^{-\sigma t}) = \beta\left[t - \frac{1}{\sigma}(1 - e^{-\sigma t})\right]$$

$$J = (2.28 \times 10^7\ N)\left\{(5.00\ s) - \frac{1}{(0.139\ \mathrm{s}^{-1})}\left[1 - e^{-(0.139\ \mathrm{s}^{-1})(5.00\ s)}\right]\right\} = 3.1834 \times 10^7\ N \cdot s$$

$$v = \frac{J}{m} = \frac{3.1834 \times 10^7\ N \cdot s}{1.48 \times 10^6\ kg} = 21.5 \frac{m}{s}$$

General Collision



Object 1: $\vec{J}_1 = \vec{F}_1 \cdot \Delta t = \Delta\vec{P}_1 = mv_1' - mv_1$

Object 2: $\vec{J}_2 = \vec{F}_2 \cdot \Delta t = \Delta\vec{P}_2 = mv_2' - mv_2$

$$\vec{J}_1 + \vec{J}_2 = \vec{F}_1 \cdot \Delta t + \vec{F}_2 \cdot \Delta t = (\vec{F}_1 + \vec{F}_2) \cdot \Delta t = 0$$

$\vec{F}_1 + \vec{F}_2 = 0$, *because Newton's 3rd law says* $\vec{F}_2 = -\vec{F}_1$

$$\vec{J}_1 + \vec{J}_2 = (mv_1' - mv_1) + (mv_2' - mv_2)$$

$$\vec{J}_1 + \vec{J}_2 = (mv_1' + mv_2') - (mv_1 + mv_2) = \vec{P}_{Final} - \vec{P}_{Init}$$

$$\vec{P}_{Final} - \vec{P}_{Init} = 0 \qquad \vec{P}_{Final} = \vec{P}_{Init}$$

Momentum is conserved!

...As long as no external forces provide outside impulse.

Example: A defensive lineman (m_{DL} = 138 kg) is moving at 8.00 m/s when he tackles a stationary quarterback (m_{QB} = 110 kg). A) What is the velocity of the pair after the collision? B) If the collision takes 0.200 s, what is the average force delivered to the quarterback?

A) $P_{Final} = P_{Init}$ $\qquad (m_{QB} + m_{DL})v = m_{DL}v_0$ $\qquad v = \frac{m_{DL}v_0}{m_{QB}+m_{DL}} = \frac{(138\ kg)\left(8.00\frac{m}{s}\right)}{(110\ kg+138\ kg)} = 4.45\frac{m}{s}$

B) $F_{Avg} = \frac{\Delta P_{QB}}{\Delta t} = \frac{m_{QB}v-0}{\Delta t} = \frac{(110\ kg)\left(4.45\frac{m}{s}\right)}{(0.200\ s)} = 2.45\ kN$ *(approximately 550 lbs of force)*

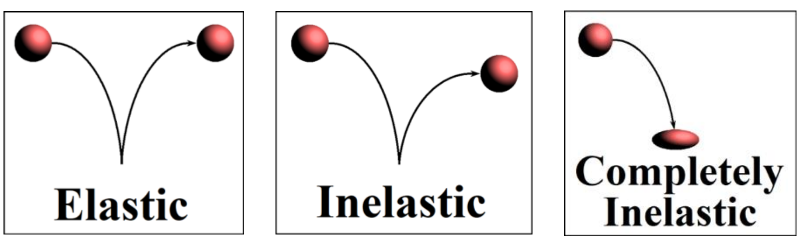
$$\Delta P_{QB} = m_{QB}v = (110\ kg)\left(4.45\frac{m}{s}\right) = 490\ kg\cdot\frac{m}{s}$$

$$\Delta P_{DL} = m_{DL}v - m_{DL}v_0 = m_{DL}(v - v_0) = (138\ kg)\left(4.45\frac{m}{s} - 8.00\frac{m}{s}\right) = -490\ kg\cdot\frac{m}{s}$$

The momentum lost by the defensive lineman is transferred to the quarterback

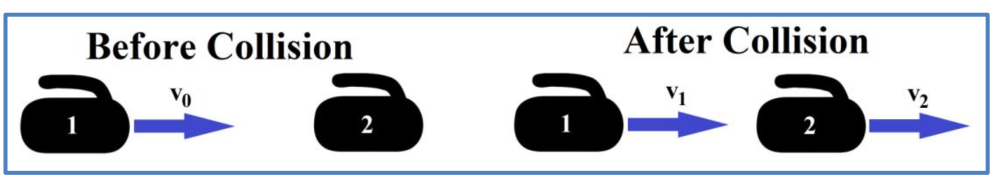
Energy and Momentum in Collisions

- Momentum is conserved in collisions when no net outside forces are present.
- Gravity is typically negligible as it has almost no effect over such a short time interval.
- Energy may or may not be conserved in collisions. For example, energy might be released as heat (lost).
- Three types of collisions:
  - In Elastic Collisions, both energy and momentum are conserved.
  - In Inelastic Collisions, momentum is conserved, but energy is not.
  - In Completely Inelastic Collisions, momentum is conserved, but the maximum possible energy is lost as the objects stick together.



Example: A curler slides a 20.0 kg stone across the ice surface. The stone is moving at 0.750 m/s when it collides head-on with a second, stationary 20.0 kg curling stone. If this is an elastic collision, determine the velocity of both stones after they strike.

Note: The term 'head on' indicates that the center of mass of both objects is in line with the velocity. In other words, this is a 1-dimensional problem. Vectors can be ignored.



Conservation of momentum: $P_{init} = P_{Final}$ $\quad m_1v_0 = m_1v_1 + m_2v_2$

Conservation of energy: $E_{init} = E_{Final}$ $\quad \frac{1}{2}m_1v_0^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ $\quad m_1v_0^2 = m_1v_1^2 + m_2v_2^2$

Here's a mathematical trick that helps with elastic collisions:

Rearrange momentum equation: $m_1(v_0 - v_1) = m_2v_2$

Rearrange and factor energy equation: $m_1(v_0^2 - v_1^2) = m_2v_2^2$ $\quad m_1(v_0 - v_1)(v_0 + v_1) = m_2v_2^2$

Divide equations: $\frac{m_1(v_0-v_1)(v_0+v_1)}{m_1(v_0-v_1)} = \frac{m_2 v_2^2}{m_2 v_2}$ $\quad v_0 + v_1 = v_2$

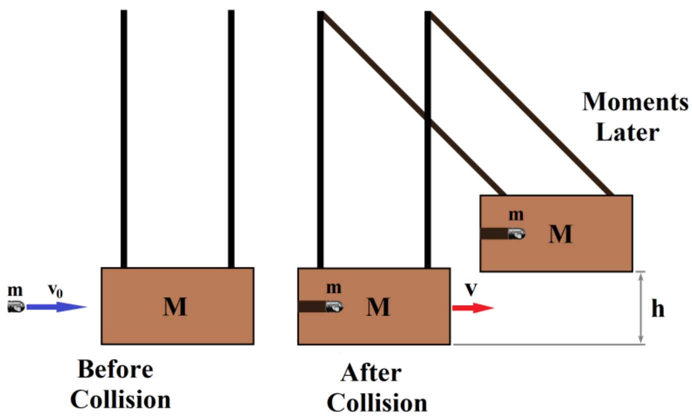
Multiply by m_1 and add the rearranged momentum equation:

$$m_1(v_0+v_1) + m_1(v_0-v_1) = m_1(v_2) + m_2(v_2) \qquad 2m_1v_0 = (m_1+m_2)v_2 \qquad v_2 = \frac{2m_1v_0}{(m_1+m_2)}$$

$$v_1 = v_2 - v_0 = \frac{2m_1v_0}{(m_1+m_2)} - \frac{(m_1+m_2)v_0}{(m_1+m_2)} = \frac{(m_1-m_2)v_0}{(m_1+m_2)}$$

As $m_1 = m_2$ in this case… $\quad v_2 = v_0 = 0.75\frac{m}{s}$ and $v_1 = 0$

Ballistic Pendulum



- The ballistic pendulum is used to measure the velocity of projectiles (such as bullet).
- First, the projectile makes a completely inelastic collision with the much heavier hanging mass of a pendulum bob.
- The velocity of the pair after the collision causes the pendulum bob to swing upwards, and the height is measured. From this height we can produce the velocity of the projectile.

- To start we need to relate the 'Moments Later' image to the 'After Collision' image.
    - Can we use conservation of momentum? No, an external force (gravity) acts on the system. Momentum is not conserved.
    - There is no collision in this interval. Energy is conserved.

$$E_{Init} = E_{Final} \qquad \frac{1}{2}(m+M)v^2 = (m+M)gh \qquad v^2 = 2gh \qquad v = \sqrt{2gh}$$

- Next we need to relate the 'After Collision' image to the 'Before Collision' image.
    - Can we use conservation of energy? No, a collision occurs. Energy is not conserved.
    - There are no (horizontal) external forces in this interval. Momentum is conserved.

$$P_{Init} = P_{Final} \qquad mv_0 = (m+M)v \qquad v_0 = \left(1+\frac{M}{m}\right)v = \left(1+\frac{M}{m}\right)\sqrt{2gh}$$

Collisions in Two Dimensions

- Conservation of momentum is applied by components for each axis of motion.

$$\vec{P}_{final} = \vec{P}_{init} \quad \text{implies} \quad P_{final-x} = P_{init-x} \quad \text{and} \quad P_{final-y} = P_{init-y}$$

- In two dimensions, conservation of motion gives two equations, allowing you to find two unknowns.