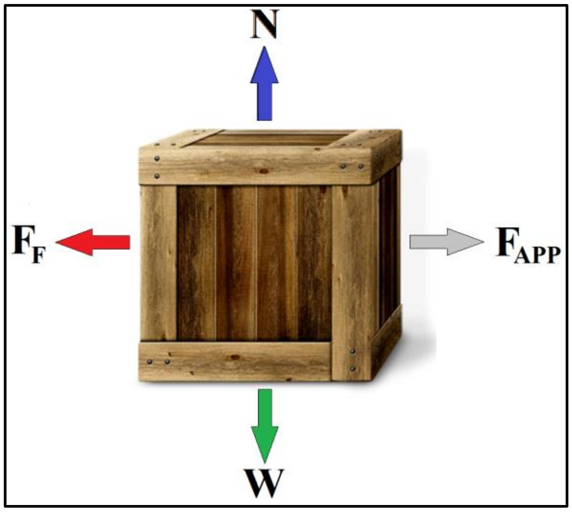
Lecture 11: Friction and Tension

Physics for Engineers & Scientists (Giancoli): Chapters 4 & 5
University Physics V1 (Openstax): Chapters 5 & 6

Example: A block whose weight is 45.0N rests on a horizontal table. The coefficients of static and kinetic friction are 0.650 and 0.420 respectively. A horizontal force of 36.0N is applied to the block. Will the block move under influence of the force, and if so, what will be the blocks acceleration?



Is the crate moving? We don't know.
Assume it's stationary, find F_F, and compare it to $F_{F\text{-}Max}$.

For F_{NET} to be zero, $F_F = F_{APP} = 36.0$ N
$F_{F\text{-}Max} = \mu_S N = (0.650)(45.0\text{ N}) = 29.25$ N

Is the crate moving? Yes! 36.0 N > 29.25 N
We must use kinetic friction.

$F_F = \mu_k N = (0.420)(45.0\text{ N}) = 18.9$ N
$F_{NET\text{-}x} = F_{APP} - F_F = 36.0\text{ N} - 18.9\text{ N} = 17.1$ N

$m = W/g = (45.0\text{ N})/(9.80\text{ m/s}^2) = 4.5918$ kg
$a = a_x = F_{NET\text{-}x}/m = (17.1\text{ N})/(4.5918\text{ kg}) = 3.724\text{ m/s}^2$
$\Rightarrow 3.72\text{ m/s}^2$

Example: A block of mass M = 10.0 kg rests on an incline sloped at an angle θ = 30.0°. Friction's hold on the box is tentative, and the slightest touch will cause it to slide down the incline. Determine the force of friction, the normal force, and the coefficient of static friction.

First, 'sliding at the slightest touch' means $F_F = F_{F\text{-}max}$.
We will solve this problem three different ways.

Method 1: Brute force

Start by making a force diagram and breaking each vector into components.
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)

The angle of N is defined with respect to the y-axis.
You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$\Sigma F_x = 0 \;\Rightarrow\; F_{Fx} = N_x \quad F_F\cdot\text{Cos}(\theta) = N\cdot\text{Sin}(\theta)$

$\mu_S\cdot N\cdot\text{Cos}(\theta) = N\cdot\text{Sin}(\theta)$

$\mu_S = N\cdot\text{Sin}(\theta)/N\cdot\text{Cos}(\theta)$

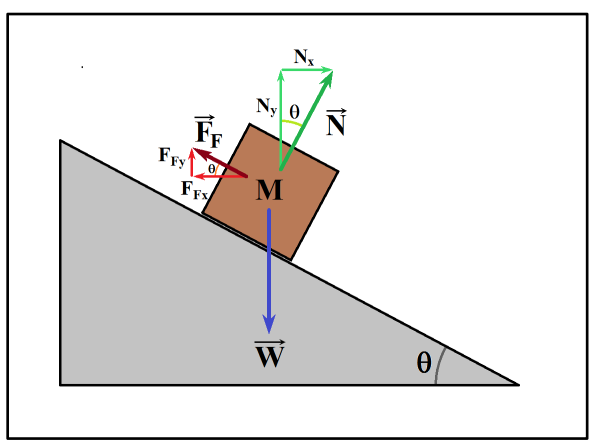
$\mu_S = \text{Tan}(\theta) = \text{Tan}(30.0°) = 0.57735 \;\Rightarrow\; 0.577$

$\Sigma F_y = 0 \;\Rightarrow\; F_{Fy} + N_y = W$

$F_F\cdot\text{Sin}(\theta) + N\cdot\text{Cos}(\theta) = Mg$

$\mu_S\cdot N\cdot\text{Sin}(\theta) + N\cdot\text{Cos}(\theta) = Mg$

$N\cdot[\mu_S\cdot\text{Sin}(\theta) + \text{Cos}(\theta)] = Mg$



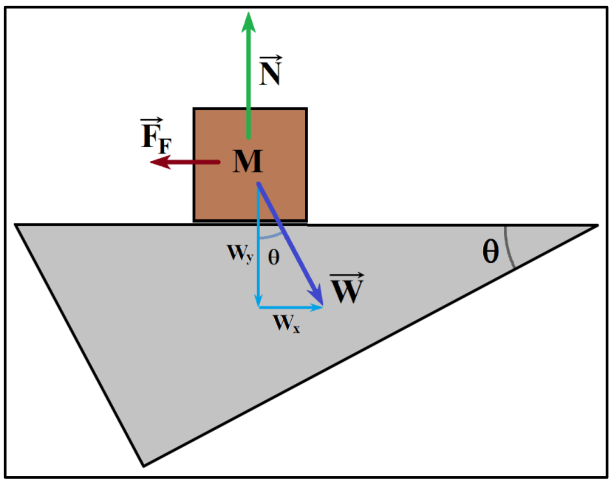
$$N = Mg/[\mu_S \cdot \text{Sin}(\theta) + \text{Cos}(\theta)]$$

$$N = (10.0\text{ kg})(9.80\text{ m/s}^2)[(0.57735)\cdot\text{Sin}(30.0°) + \text{Cos}(30.0°)] = 84.8705\text{ N} \Rightarrow 84.9\text{ N}$$

$$F_F = \mu_S \cdot N = (0.57735)(84.8705\text{ N}) = 49.0\text{ N}$$

<u>Method 2</u>: Use a Different (Rotated) Reference Frame.

Start by making a force diagram and breaking each vector into components.
However, this time let the x-axis run parallel to the slope and the y-axis perpendicular.
This only leaves one vector (W) to break into components instead of two (N and F_F).
Use three equations ($F_F = \mu_S N$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$) to find three unknowns (F_F, μ_S, and N)



$$\Sigma F_x = 0 \Rightarrow F_F = W_x = mg\cdot\text{Sin}(\theta)$$

The angle of W is defined with respect to the y-axis. You must use Sin(θ) or use geometry to find the angle with respect to the positive x-axis.

$$F_F = (10.0\text{ kg})(9.80\text{ m/s}^2)\cdot\text{Sin}(30.0°) = 49.0\text{ N}$$

$$\Sigma F_y = 0 \Rightarrow N = W_y = mg\cdot\text{Cos}(\theta)$$

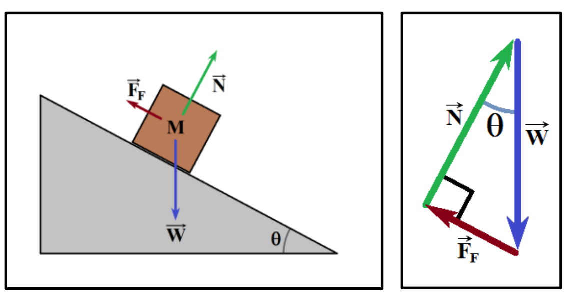
The angle of W is defined with respect to the y-axis. You must use Cos(θ) or use geometry to find the angle with respect to the positive x-axis.

$$N = (10.0\text{ kg})(9.80\text{ m/s}^2)\cdot\text{Cos}(30.0°) = 84.8705\text{ N} \Rightarrow 84.9\text{ N}$$

$$F_F = \mu_S \cdot N \qquad \mu_S = F_F/N = [mg\cdot\text{Sin}(\theta)] / [mg\cdot\text{Cos}(\theta)] = \text{Tan}(\theta) = \text{Tan}(30.0°) = 0.57735 \Rightarrow 0.577$$

<u>Method 3</u>: Trigonometry

Start by making a force diagram. Upon seeing that 3 forces sum to zero, make a triangle out of them. Upon finding N and F_F, use $F_F = \mu_S N$ to get μ_S.



$$N = W\cdot\text{Cos}(\theta) = mg\cdot\text{Cos}(\theta) = (10.0\text{ kg})(9.80\text{ m/s}^2)\cdot\text{Cos}(30.0°) = 84.8705\text{ N} \Rightarrow 84.9\text{ N}$$

$$F_F = W\cdot\text{Sin}(\theta) = mg\cdot\text{Sin}(\theta) = (10.0\text{ kg})(9.80\text{ m/s2})\cdot\text{Sin}(30.0°) = 49.0\text{ N}$$

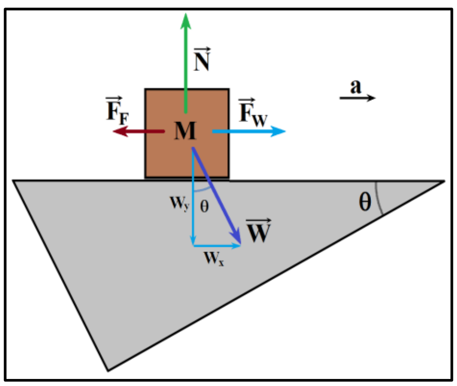
$$\mu_S = F_F/N = \text{Tan}(\theta) = \text{Tan}(30.0°) = 0.57735 \Rightarrow 0.577$$

On Solving Problems

- Solving the problem by making a triangle was the easiest solution, but this doesn't always work.
  - This can only be done when you can make a triangle.
  - It may not work in non-equilibrium or when a 4^{th} force is present.
- Solving the problem by aligning the coordinate axes with the incline was easier.
  - It's only easier when more vectors align with these coordinate axes.
- Brute force is often the most difficult option, but it always works.
- Find the easiest way to solve each problem!

Example 21: A girl is skiing down a slope that is 30.0° with respect to the horizontal. A moderate wind is aiding the motion by providing a steady force of 105N that is parallel to the motion of the skier. The combined mass of the girl and skis is 65.0kg and the coefficient of kinetic friction between the skis and the snow is 0.150. How much time is required for the skier to travel down a 175m slope, starting from rest?

Start with a force diagram. Laying it flat leaves only one vector (W) to be broken into components. Use the force information to determine the acceleration and then do the kinematics.



This is a non-equilibrium situation $\Rightarrow$ $\Sigma F_x = Ma \quad \Sigma F_y = 0$

$\Sigma F_y = 0 \Rightarrow N = W_y = W \cdot Cos(\theta) = Mg \cdot Cos(\theta)$

$F_F = \mu_k N = \mu_k Mg \cdot Cos(\theta)$

$\Sigma F_x = Ma \quad \Rightarrow \quad F_W + W_x - F_F = Ma$

$F_W + Mg \cdot Sin(\theta) - \mu_k Mg \cdot Cos(\theta) = Ma$

$a = F_W/M + g \cdot Sin(\theta) - \mu_k g \cdot Cos(\theta)$

$$a = (105\ N)/(65.0\ kg) + (9.80\ m/s^2) \cdot Sin(30°) - (0.150)(9.80\ m/s^2) \cdot Cos(30°) = 5.24233\ m/s^2$$

Extract kinematic data: Let $x_0 = 0$ $x = 175\ m$ $v_0 = 0$ $v =$ $a = 5.24233\ m/s^2$ $t = ???$

$$x = x_0 + v_0 t + ½ \cdot at^2 = ½ \cdot at^2 \qquad 2x = at^2 \qquad 2x/a = t^2$$

$$t = (2x/a)^{1/2} = [2(175m)/(5.24233)]^{1/2} = 8.17094\ s \Rightarrow 8.17\ s$$

Tension

- Tension is a force that is transmitted along the length of an object, often a flexible object such as a rope, string, or chain.
- Tension always "pulls" and never "pushes".
- The force is typically applied at the connections at each end, pulling along the length of the object.
- The force of tension is felt at each point in the interior of the object (pulled both ways)
- The direction this force is felt can be altered by pulleys. If the pulley has negligible mass and friction the tension is unaltered.