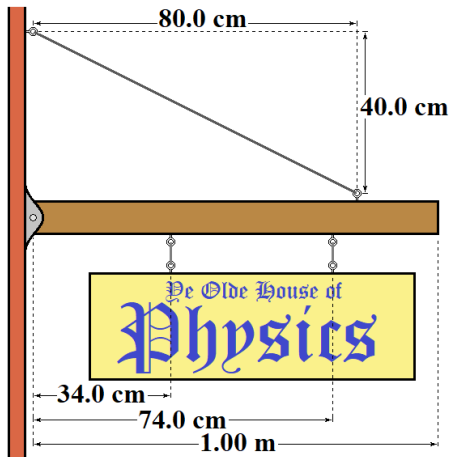


Lecture 15: Static Equilibrium, Deformation, and Fluids

Physics (Giancoli): Chapters 9 & 10

College Physics (Openstax): Chapters 5, 9, & 11



Example: A 2.50 kg sign is hung from an 11.0 kg beam as shown in the diagram. The sign is balanced so that half its weight is supported by each of the two wires above it. The beam is attached to the wall on the left by a frictionless pin that allows it to rotate and held in place by a guy wire above it. Determine the tension in the guy wire.

$$W_{beam} = m_{beam}g = (11.0 \text{ kg}) \left(9.80 \frac{\text{m}}{\text{s}^2} \right) = 107.8 \text{ N}$$

$$W_{sign} = m_{sign}g = (2.50 \text{ kg}) \left(9.80 \frac{\text{m}}{\text{s}^2} \right) = 24.5 \text{ N}$$

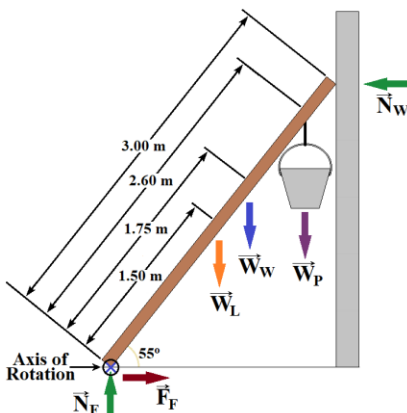
$$\sum \tau = T_y(80.0 \text{ cm}) - \frac{1}{2}(24.5 \text{ N})(34.0 \text{ cm}) - \frac{1}{2}(24.5 \text{ N})(74.0 \text{ cm}) - (107.8 \text{ N})(50.0 \text{ cm})$$

$$\sum \tau = T_y(80.0 \text{ cm}) - 416.5 \text{ N} \cdot \text{cm} - 906.5 \text{ N} \cdot \text{cm} - 5390.0 \text{ N} \cdot \text{cm}$$

$$\sum \tau = T_y(80.0 \text{ cm}) - 6713 \text{ N} \cdot \text{cm} = 0 \quad T_y = \frac{6713 \text{ N} \cdot \text{cm}}{80.0 \text{ cm}} = 83.9125 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{40.0 \text{ cm}}{80.0 \text{ cm}} \right) = 26.565^\circ \quad T \sin \theta = T_y \quad T = \frac{T_y}{\sin \theta} = \frac{83.9125 \text{ N}}{\sin 26.565^\circ} = 188 \text{ N}$$

Example: A workman (weighing 755 N) leans a ladder (L = 3.00 m long, weighing 155 N) against a smooth (virtually frictionless) wall, making a 55.0° angle with the floor. He hangs a pail holding his work tools with a combined weight of 90.0 N from one of the top rails a distance $d_1 = 2.60$ m from the bottom of the ladder. The workman then climbs up to do his work, balancing himself on a rail that is a distance $d_2 = 1.75$ m up the ladder. Determine the minimum value of the coefficient of static friction needed to keep the ladder from slipping.



When μ_S is at a minimum, then $F_F = F_{F-Max} = \mu_S N$.

$$\sum F_x = F_F - N_W = 0 \quad F_F = N_W$$

$$\sum F_y = N_F - W_L - W_W - W_P = 0$$

$$N_F = W_L + W_W + W_P = 155 \text{ N} + 755 \text{ N} + 90.0 \text{ N} = 1000 \text{ N}$$

To get F_F or N_W we need to use a torque equation.

Place the axis of rotation where it will eliminate the most torques.

$$\sum \tau = N_W L \sin 55^\circ - W_L \left(\frac{1}{2}L\right) \cos 55^\circ - W_W d_2 \cos 55^\circ - W_P d_1 \cos 55^\circ = 0$$

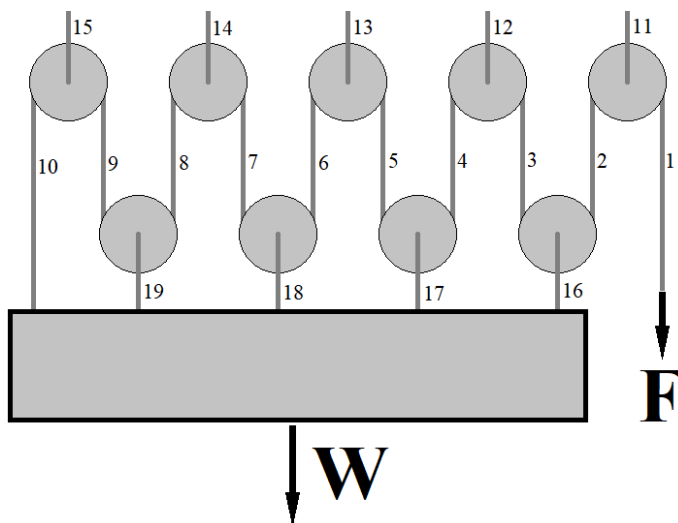
$$N_W L \sin 55^\circ = W_L \left(\frac{1}{2}L\right) \cos 55^\circ + W_W d_2 \cos 55^\circ + W_P d_1 \cos 55^\circ$$

$$N_W = \frac{W_L \cos 55^\circ}{2 \sin 55^\circ} + \frac{W_W d_2 \cos 55^\circ}{L \sin 55^\circ} + \frac{W_P d_1 \cos 55^\circ}{L \sin 55^\circ} \quad N_W = \frac{W_L}{2 \tan 55^\circ} + \frac{W_W d_2}{L \tan 55^\circ} + \frac{W_P d_1}{L \tan 55^\circ}$$

$$N_W = \frac{155 \text{ N}}{2 \tan 55^\circ} + \frac{(755 \text{ N})(1.75 \text{ m})}{(3.00 \text{ m}) \tan 55^\circ} + \frac{(90.0 \text{ N})(2.60 \text{ m})}{(3.00 \text{ m}) \tan 55^\circ} = 417.2765 \text{ N}$$

$$\mu_s = \frac{F_F}{N_F} = \frac{N_W}{N_F} = \frac{417.2765 \text{ N}}{1000 \text{ N}} = 0.417$$

Example: A system of 9 frictionless massive pulleys is used to lift a heavy object (as shown). The 5 upper pulleys are anchored to the ceiling. The four lower pulleys are anchored to a 900 lb. box. How much force (F) is required to lift the box at a constant velocity?



Let's call T the tension in line 1. $T = F$.

The sum of the torques on the upper right pulley is zero.

The tension in line 2 has to equal T , the tension in line 1.

The same logic works for lines 3 through 10. All those tensions are also T .

The sum of the forces on the upper right pulley is zero.

The tension in line 11 has to equal $2T$, the sum of the tension in lines 1 and 2.

The same logic works for lines 12 through 15, as well as 16 through 19. All those tensions are also $2T$.

Sum of the forces on the box is zero: $T + 2T + 2T + 2T + 2T - W = 0$

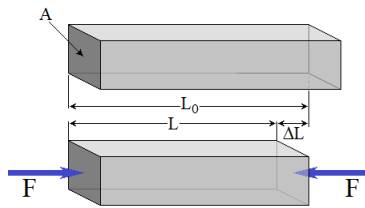
$$9T = W \quad T = \frac{W}{9} = \frac{900 \text{ lbs}}{9} = 100 \text{ lbs}$$

Deformation

- When an object changes shape due to applied forces, it is called Deformation.
- When the deformation exceeds The Elastic Limit, the change in shape becomes permanent. Until it reaches this limit, the changes are typically linear and the object will return to its original shape once the forces are removed.

- Elastic Deformation. $F = Y \left(\frac{\Delta L}{L_0} \right) A$ $Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F \cdot L_0}{A \cdot \Delta L}$

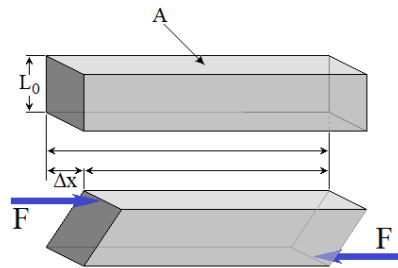
'Tensile' (from tension) refers to stretching. 'Compressive' refers to compression. Apart from that, these work much in the same way.



$$\text{Tensile Stress} = \frac{F}{A} \quad \text{Tensile Strain} = \frac{\Delta L}{L_0}$$

Young's Modulus (Y) is a property of the material (i.e. a constant that varies from material to material)

- Shear Deformation. $F = S \left(\frac{\Delta x}{L_0} \right) A$ $S = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F \cdot L_0}{A \cdot \Delta x}$

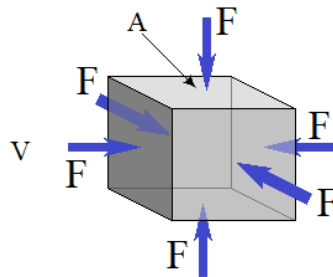
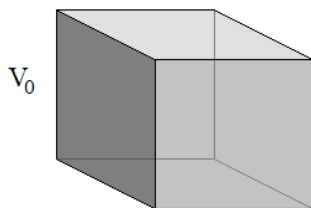


$$\text{Shear Stress} = \frac{F}{A}$$

$$\text{Shear Strain} = \frac{\Delta x}{L_0}$$

The Shear Modulus (S) is a property of the material (i.e. a constant that varies from material to material)

- Elastic Deformation. $\Delta P = -B \left(\frac{\Delta V}{V_0} \right)$ $B = -\frac{\text{Pressure Change}}{\text{Bulk Strain}} = -\Delta P \frac{V_0}{\Delta V}$

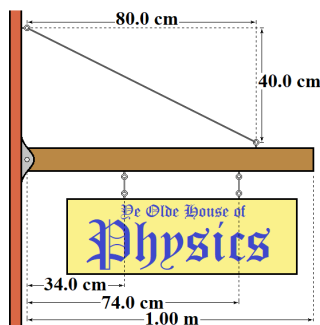


$$\text{Pressure} = \frac{F}{A}$$

$$\text{Bulk Strain} = \frac{\Delta V}{V_0}$$

The Bulk Modulus (B) is a property of fluids/gases (i.e. a constant that varies from material to material)

Example: An 89.4 cm long steel cable is used to hold up a beam and sign under a tension of 188 N. The cable radius is 5.00 mm, and the Young's modulus for steel is $2.00 \times 10^{11} \text{ N/m}^2$. How much does the length of the cable change when the tension is added?



$$\Delta L = \frac{F \cdot L_0}{Y \cdot A} = \frac{F L_0}{Y \pi r^2}$$

$$\Delta L = \frac{(188 \text{ N})(0.894 \text{ m})}{\left(2.00 \times 10^{11} \frac{\text{N}}{\text{m}^2}\right) \pi (0.00500 \text{ m})^2}$$

$$\Delta L = 10.7 \mu\text{m}$$

Fluids (Static)

- Unlike rigid objects, fluids flow. Often they don't have a fixed shape and sometimes not even a fixed size.
- Rather than using masses and forces, it is more convenient to use mass density (ρ), often referred to as just 'density', and pressure (P).
- The Density of a substance is its mass divided by volume. Units of Density: kg/m^3 .

$$\rho = \frac{m}{V}$$

- The Specific Gravity of a substance is the ratio of its density to the density of water at 4°C.

$$\rho_{H_2O(4^\circ C)} = 1000 \frac{\text{kg}}{\text{m}^3}$$

- The molecules in a fluid are in random motion and collide with whatever contains them. The impulse of these collisions imparts forces (simultaneously holding the fluid inside and pressing outward on the container). This force per area is defined to be The Pressure (P) of the fluid.

$$P = \frac{F}{A}$$

- The Units of Pressure are the Pascal. $1 \text{ Pa} = \frac{\text{kg}}{\text{m}^3}$.
- Another common unit is the 'atmosphere' (atm), which is roughly the pressure of the air at Earth's surface.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1013 \text{ kPa}$$

- Forces from pressure are always directed perpendicular (normal) to the surface.

Example: The pressure inside a plastic bottle containing a carbonated beverage is 207 kPa. The diameter of the cap is 2.22 cm. Determine the net force of pressure on the cap.

The net force is the force from the inside pressure minus the force from the outside pressure.

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{0.0222 \text{ m}}{2}\right)^2 = 3.871 \times 10^{-4} \text{ m}^2$$

$$F_{Net} = F_{Inside} - F_{Outside} = P_{Inside} \cdot A - P_{Outside} \cdot A = (P_{Inside} - P_{Outside}) \cdot A$$

$$F_{Net} = [(2.07 \times 10^5 \text{ Pa}) - (1.013 \times 10^5 \text{ Pa})] \cdot (3.871 \times 10^{-4} \text{ m}^2) = 40.9 \text{ N}$$