Solution for (a)

The atomic mass of deuterium (^2H) is 2.014102 u, while that of tritium (^3H) is 3.016049 u, for a total of 5.032151 u per reaction. So a mole of reactants has a mass of 5.03 g, and in 1.00 kg there are $(1000\text{ g})/(5.03\text{ g/mol}){=}198.8\text{ mol of reactants}$. The number of reactions that take place is therefore

$$(198.8\text{ mol})\left(6.02\times10^{23}\text{ mol}^{-1}\right)=1.20\times10^{26}\text{ reactions.} \tag{32.23}$$

The total energy output is the number of reactions times the energy per reaction:

$$\begin{aligned}E &= \left(1.20\times10^{26}\text{ reactions}\right)(17.59\text{ MeV/reaction})\left(1.602\times10^{-13}\text{ J/MeV}\right)\\ &= 3.37\times10^{14}\text{ J.}\end{aligned} \tag{32.24}$$

Solution for (b)

Power is energy per unit time. One year has $3.16\times10^{7}\text{ s}$, so

$$\begin{aligned}P &= \frac{E}{t}=\frac{3.37{\times}10^{14}\text{ J}}{3.16{\times}10^{7}\text{ s}}\\ &= 1.07\times10^{7}\text{ W}=10.7\text{ MW.}\end{aligned} \tag{32.25}$$

Discussion

By now we expect nuclear processes to yield large amounts of energy, and we are not disappointed here. The energy output of $3.37\times10^{14}\text{ J}$ from fusing 1.00 kg of deuterium and tritium is equivalent to 2.6 million gallons of gasoline and about eight times the energy output of the bomb that destroyed Hiroshima. Yet the average backyard swimming pool has about 6 kg of deuterium in it, so that fuel is plentiful if it can be utilized in a controlled manner. The average power output over a year is more than 10 MW, impressive but a bit small for a commercial power plant. About 32 times this power output would allow generation of 100 MW of electricity, assuming an efficiency of one-third in converting the fusion energy to electrical energy.

32.6 Fission

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define nuclear fission.
- Discuss how fission fuel reacts and describe what it produces.
- Describe controlled and uncontrolled chain reactions.

Nuclear fission is a reaction in which a nucleus is split (or *fissured*). Controlled fission is a reality, whereas controlled fusion is a hope for the future. Hundreds of nuclear fission power plants around the world attest to the fact that controlled fission is practical and, at least in the short term, economical, as seen in Figure 32.20. Whereas nuclear power was of little interest for decades following TMI and Chernobyl (and now Fukushima Daiichi), growing concerns over global warming has brought nuclear power back on the table as a viable energy alternative. By the end of 2009, there were 442 reactors operating in 30 countries, providing 15% of the world's electricity. France provides over 75% of its electricity with nuclear power, while the US has 104 operating reactors providing 20% of its electricity. Australia and New Zealand have none. China is building nuclear power plants at the rate of one start every month.

FIGURE 32.20 The people living near this nuclear power plant have no measurable exposure to radiation that is traceable to the plant. About 16% of the world's electrical power is generated by controlled nuclear fission in such plants. The cooling towers are the most prominent features but are not unique to nuclear power. The reactor is in the small domed building to the left of the towers. (credit: Kalmthouts)

Fission is the opposite of fusion and releases energy only when heavy nuclei are split. As noted in Fusion, energy is released if the products of a nuclear reaction have a greater binding energy per nucleon (BE/A) than the parent nuclei. Figure 32.21 shows that BE/A is greater for medium-mass nuclei than heavy nuclei, implying that when a heavy nucleus is split, the products have less mass per nucleon, so that mass is destroyed and energy is released in the reaction. The amount of energy per fission reaction can be large, even by nuclear standards. The graph in Figure 32.21 shows BE/A to be about 7.6 MeV/nucleon for the heaviest nuclei (A about 240), while BE/A is about 8.6 MeV/nucleon for nuclei having A about 120. Thus, if a heavy nucleus splits in half, then about 1 MeV per nucleon, or approximately 240 MeV per fission, is released. This is about 10 times the energy per fusion reaction, and about 100 times the energy of the average α, β, or γ decay.

EXAMPLE 32.3

Calculating Energy Released by Fission

Calculate the energy released in the following spontaneous fission reaction:

$$^{238}\mathrm{U} \rightarrow {}^{95}\mathrm{Sr} + {}^{140}\mathrm{Xe} + 3n \tag{32.26}$$

given the atomic masses to be $m(^{238}\mathrm{U}) = 238.050784\ \mathrm{u}$, $m(^{95}\mathrm{Sr}) = 94.919388\ \mathrm{u}$, $m(^{140}\mathrm{Xe}) = 139.921610\ \mathrm{u}$, and $m(n) = 1.008665\ \mathrm{u}$.

Strategy

As always, the energy released is equal to the mass destroyed times c^2 , so we must find the difference in mass between the parent $^{238}\mathrm{U}$ and the fission products.

Solution

The products have a total mass of

$$\begin{aligned} m_{\text{products}} &= 94.919388\ \mathrm{u} + 139.921610\ \mathrm{u} + 3(1.008665\ \mathrm{u}) \\ &= 237.866993\ \mathrm{u}. \end{aligned} \tag{32.27}$$

The mass lost is the mass of $^{238}\mathrm{U}$ minus m_{products}, or

$$\Delta m = 238.050784\ \mathrm{u} - 237.8669933\ \mathrm{u} = 0.183791\ \mathrm{u}, \tag{32.28}$$

so the energy released is

$$\begin{aligned} E &= (\Delta m)c^2 \\ &= (0.183791\ \mathrm{u})\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}c^2 = 171.2\ \mathrm{MeV}. \end{aligned} \tag{32.29}$$

Discussion

A number of important things arise in this example. The 171-MeV energy released is large, but a little less than the earlier estimated 240 MeV. This is because this fission reaction produces neutrons and does not split the nucleus into two equal parts. Fission of a given nuclide, such as ^{238}U , does not always produce the same products. Fission is a statistical process in which an entire range of products are produced with various probabilities. Most fission produces neutrons, although the number varies with each fission. This is an extremely important aspect of fission, because *neutrons can induce more fission*, enabling self-sustaining chain reactions.

Spontaneous fission can occur, but this is usually not the most common decay mode for a given nuclide. For example, ^{238}U can spontaneously fission, but it decays mostly by α emission. Neutron-induced fission is crucial as seen in Figure 32.21. Being chargeless, even low-energy neutrons can strike a nucleus and be absorbed once they feel the attractive nuclear force. Large nuclei are described by a **liquid drop model** with surface tension and oscillation modes, because the large number of nucleons act like atoms in a drop. The neutron is attracted and thus, deposits energy, causing the nucleus to deform as a liquid drop. If stretched enough, the nucleus narrows in the middle. The number of nucleons in contact and the strength of the nuclear force binding the nucleus together are reduced. Coulomb repulsion between the two ends then succeeds in fissioning the nucleus, which pops like a water drop into two large pieces and a few neutrons. **Neutron-induced fission** can be written as

$$n + {}^{A}\text{X} \rightarrow \text{FF}_1 + \text{FF}_2 + xn, \tag{32.30}$$

where FF_1 and FF_2 are the two daughter nuclei, called **fission fragments**, and x is the number of neutrons produced. Most often, the masses of the fission fragments are not the same. Most of the released energy goes into the kinetic energy of the fission fragments, with the remainder going into the neutrons and excited states of the fragments. Since neutrons can induce fission, a self-sustaining chain reaction is possible, provided more than one neutron is produced on average — that is, if $x > 1$ in $n + {}^{A}\text{X} \rightarrow \text{FF}_1 + \text{FF}_2 + xn$. This can also be seen in Figure 32.22.

An example of a typical neutron-induced fission reaction is

$$n + {}^{235}_{92}\text{U} \rightarrow {}^{142}_{56}\text{Ba} + {}^{91}_{36}\text{Kr} + 3n. \tag{32.31}$$

Note that in this equation, the total charge remains the same (is conserved): $92 + 0 = 56 + 36$. Also, as far as whole numbers are concerned, the mass is constant: $1 + 235 = 142 + 91 + 3$. This is not true when we consider the masses out to 6 or 7 significant places, as in the previous example.

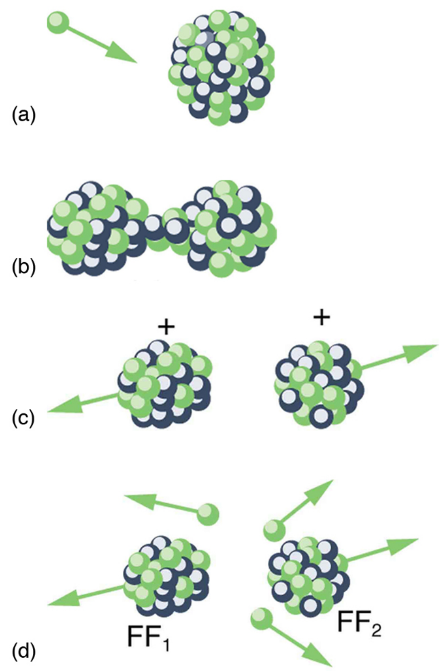

FIGURE 32.21 Neutron-induced fission is shown. First, energy is put into this large nucleus when it absorbs a neutron. Acting like a struck liquid drop, the nucleus deforms and begins to narrow in the middle. Since fewer nucleons are in contact, the repulsive Coulomb force is able to break the nucleus into two parts with some neutrons also flying away.

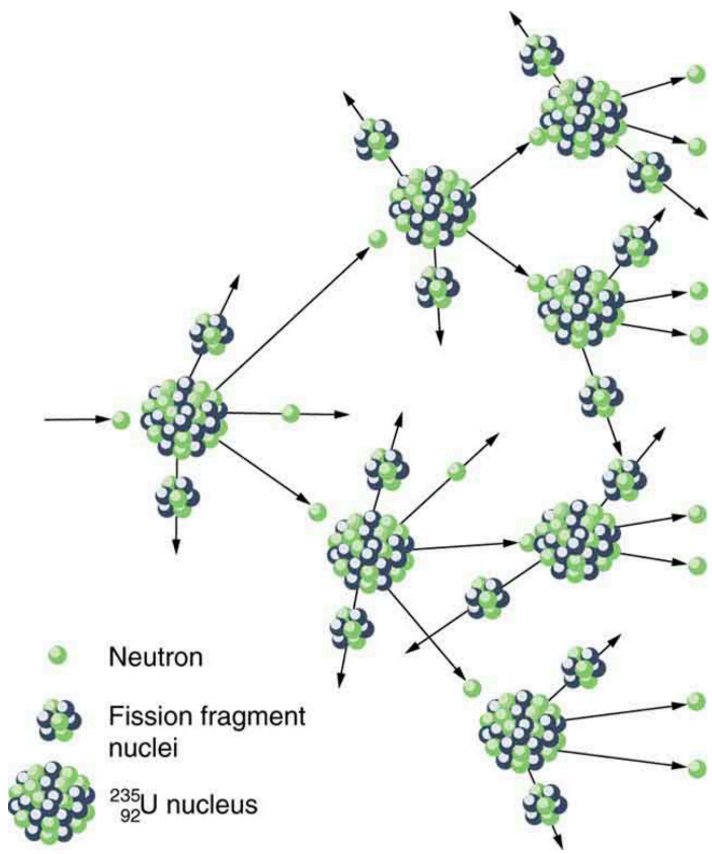

FIGURE 32.22 A chain reaction can produce self-sustained fission if each fission produces enough neutrons to induce at least one more fission. This depends on several factors, including how many neutrons are produced in an average fission and how easy it is to make a particular type of nuclide fission.

Not every neutron produced by fission induces fission. Some neutrons escape the fissionable material, while others interact with a nucleus without making it fission. We can enhance the number of fissions produced by neutrons by having a large amount of fissionable material. The minimum amount necessary for self-sustained fission of a given nuclide is called its **critical mass**. Some nuclides, such as ^{239}Pu , produce more neutrons per fission than others, such as ^{235}U . Additionally, some nuclides are easier to make fission than others. In particular, ^{235}U and ^{239}Pu are easier to fission than the much more abundant ^{238}U . Both factors affect critical mass, which is smallest for ^{239}Pu .

The reason ^{235}U and ^{239}Pu are easier to fission than ^{238}U is that the nuclear force is more attractive for an even number of neutrons in a nucleus than for an odd number. Consider that $^{235}_{92}\text{U}_{143}$ has 143 neutrons, and $^{239}_{94}\text{P}_{145}$ has 145 neutrons, whereas $^{238}_{92}\text{U}_{146}$ has 146. When a neutron encounters a nucleus with an odd number of neutrons, the nuclear force is more attractive, because the additional neutron will make the number even. About 2-MeV more energy is deposited in the resulting nucleus than would be the case if the number of neutrons was already even. This extra energy produces greater deformation, making fission more likely. Thus, ^{235}U and ^{239}Pu are superior fission fuels. The isotope ^{235}U is only 0.72 % of natural uranium, while ^{238}U is 99.27%, and ^{239}Pu does not exist in nature. Australia has the largest deposits of uranium in the world, standing at 28% of the total. This is followed by Kazakhstan and Canada. The US has only 3% of global reserves.

Most fission reactors utilize ^{235}U , which is separated from ^{238}U at some expense. This is called enrichment. The most common separation method is gaseous diffusion of uranium hexafluoride (UF_6) through membranes. Since ^{235}U has less mass than ^{238}U , its UF_6 molecules have higher average velocity at the same temperature and diffuse faster. Another interesting characteristic of ^{235}U is that it preferentially absorbs very slow moving neutrons (with energies a fraction of an eV), whereas fission reactions produce fast neutrons with energies in the order of an MeV. To make a self-sustained fission reactor with ^{235}U , it is thus necessary to slow down ("thermalize") the neutrons. Water is very effective, since neutrons collide with protons in water molecules and lose energy. Figure 32.23 shows a schematic of a reactor design, called the pressurized water reactor.

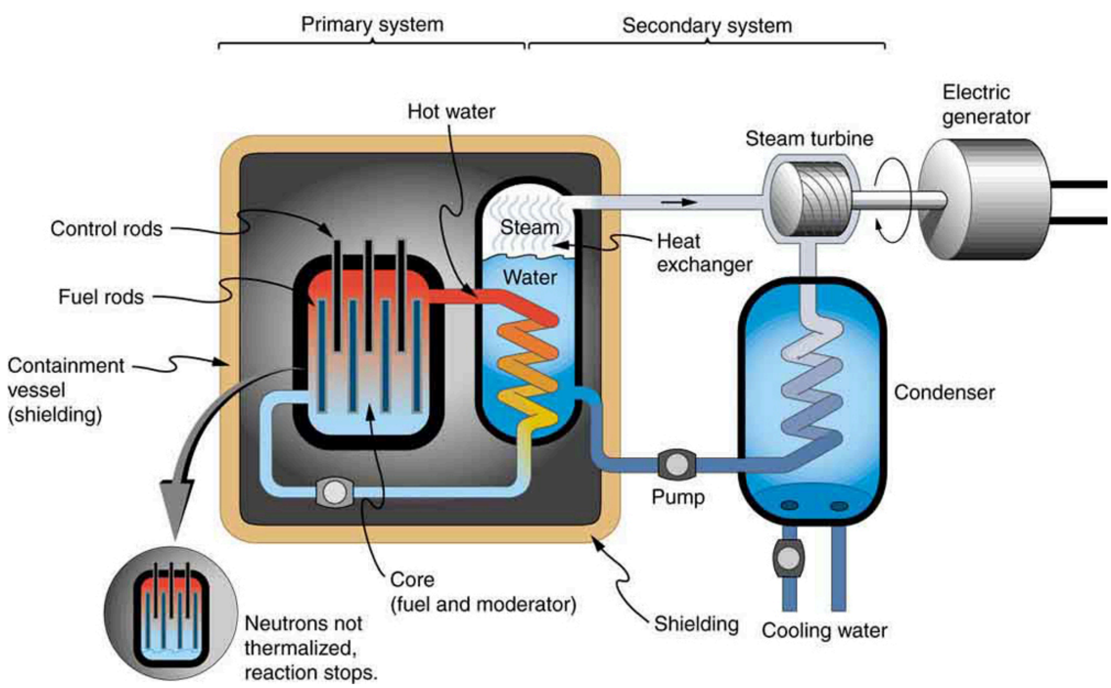

FIGURE 32.23 A pressurized water reactor is cleverly designed to control the fission of large amounts of ^{235}U , while using the heat produced in the fission reaction to create steam for generating electrical energy. Control rods adjust neutron flux so that criticality is obtained, but not exceeded. In case the reactor overheats and boils the water away, the chain reaction terminates, because water is needed to thermalize the neutrons. This inherent safety feature can be overwhelmed in extreme circumstances.

Control rods containing nuclides that very strongly absorb neutrons are used to adjust neutron flux. To produce large power, reactors contain hundreds to thousands of critical masses, and the chain reaction easily becomes self-sustaining, a condition called **criticality**. Neutron flux should be carefully regulated to avoid an exponential increase in fissions, a condition called **supercriticality**. Control rods help prevent overheating, perhaps even a meltdown or explosive disassembly. The water that is used to thermalize neutrons, necessary to get them to induce fission in ^{235}U , and achieve criticality, provides a negative feedback for temperature increases. In case the reactor overheats and boils the water to steam or is breached, the absence of water kills the chain reaction. Considerable heat, however, can still be generated by the reactor's radioactive fission products. Other safety features, thus, need to be incorporated in the event of a *loss of coolant* accident, including auxiliary cooling water and pumps.

EXAMPLE 32.4

Calculating Energy from a Kilogram of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U, given the average fission reaction of ^{235}U produces 200 MeV.

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1000\text{ g})/(235.04\text{ g/mol}) = 4.25\text{ mol}$. The number of ^{235}U atoms is therefore,

$$(4.25\text{ mol})\left(6.02\times10^{23}\ {}^{235}\text{U/mol}\right) = 2.56\times10^{24}\ {}^{235}\text{U}. \tag{32.32}$$

So the total energy released is

$$\begin{aligned} E &= \left(2.56\times10^{24}\ {}^{235}\text{U}\right)\left(\frac{200\text{ MeV}}{{}^{235}\text{U}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{\text{MeV}}\right) \\ &= 8.21\times10^{13}\text{ J}. \end{aligned} \tag{32.33}$$

Discussion

This is another impressively large amount of energy, equivalent to about 14,000 barrels of crude oil or 600,000 gallons of gasoline. But, it is only one-fourth the energy produced by the fusion of a kilogram mixture of deuterium and tritium as seen in Example 32.2. Even though each fission reaction yields about ten times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides. Fission fuel is also much more scarce than fusion fuel, and less than 1% of uranium (the ^{235}U) is readily usable.

One nuclide already mentioned is ^{239}Pu, which has a 24,120-y half-life and does not exist in nature. Plutonium-239 is manufactured from ^{238}U in reactors, and it provides an opportunity to utilize the other 99% of natural uranium as an energy source. The following reaction sequence, called **breeding**, produces ^{239}Pu. Breeding begins with neutron capture by ^{238}U:

$$^{238}\text{U} + n \rightarrow {}^{239}\text{U} + \gamma. \tag{32.34}$$

Uranium-239 then β^- decays:

$$^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- + \upsilon_e(\text{t}_{1/2} = 23\text{ min}). \tag{32.35}$$

Neptunium-239 also β^- decays:

$$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^- + \upsilon_e(\text{t}_{1/2} = 2.4\text{ d}). \tag{32.36}$$

Plutonium-239 builds up in reactor fuel at a rate that depends on the probability of neutron capture by ^{238}U (all reactor fuel contains more ^{238}U than ^{235}U). Reactors designed specifically to make plutonium are called **breeder reactors**. They seem to be inherently more hazardous than conventional reactors, but it remains unknown whether their hazards can be made economically acceptable. The four reactors at Chernobyl, including the one that was destroyed, were built to breed plutonium and produce electricity. These reactors had a design that was significantly different from the pressurized water reactor illustrated above.

Plutonium-239 has advantages over ^{235}U as a reactor fuel — it produces more neutrons per fission on average, and it is easier for a thermal neutron to cause it to fission. It is also chemically different from uranium, so it is inherently easier to separate from uranium ore. This means ^{239}Pu has a particularly small critical mass, an advantage for nuclear weapons.

PHET EXPLORATIONS

Nuclear Fission

Start a chain reaction, or introduce non-radioactive isotopes to prevent one. Control energy production in a nuclear reactor!

Click to view content (https://openstax.org/l/16fission).

32.7 Nuclear Weapons

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss different types of fission and thermonuclear bombs.
- Explain the ill effects of nuclear explosion.

The world was in turmoil when fission was discovered in 1938. The discovery of fission, made by two German physicists, Otto Hahn and Fritz Strassman, was quickly verified by two Jewish refugees from Nazi Germany, Lise Meitner and her nephew Otto Frisch. Fermi, among others, soon found that not only did neutrons induce fission; more neutrons were produced during fission. The possibility of a self-sustained chain reaction was immediately recognized by leading scientists the world over. The enormous energy known to be in nuclei, but considered inaccessible, now seemed to be available on a large scale.

Within months after the announcement of the discovery of fission, Adolf Hitler banned the export of uranium from newly occupied Czechoslovakia. It seemed that the military value of uranium had been recognized in Nazi Germany, and that a serious effort to build a nuclear bomb had begun.

Alarmed scientists, many of them who fled Nazi Germany, decided to take action. None was more famous or revered than Einstein. It was felt that his help was needed to get the American government to make a serious effort at nuclear weapons as a matter of survival. Leo Szilard, an escaped Hungarian physicist, took a draft of a letter to Einstein, who, although pacifistic, signed the final version. The letter was for President Franklin Roosevelt, warning of the German potential to build extremely powerful bombs of a new type. It was sent in August of 1939, just before the German invasion of Poland that marked the start of World War II.

It was not until December 6, 1941, the day before the Japanese attack on Pearl Harbor, that the United States made a massive commitment to building a nuclear bomb. The top secret Manhattan Project was a crash program aimed at beating the Germans. It was carried out in remote locations, such as Los Alamos, New Mexico, whenever possible, and eventually came to cost billions of dollars and employ the efforts of more than 100,000 people. J. Robert Oppenheimer (1904–1967), whose talent and ambitions made him ideal, was chosen to head the project. The first major step was made by Enrico Fermi and his group in December 1942, when they achieved the first self-sustained nuclear reactor. This first “atomic pile”, built in a squash court at the University of Chicago, used carbon blocks to thermalize neutrons. It not only proved that the chain reaction was possible, it began the era of nuclear reactors. Glenn Seaborg, an American chemist and physicist, received the Nobel Prize in physics in 1951 for discovery of several transuranic elements, including plutonium. Carbon-moderated reactors are relatively inexpensive and simple in design and are still used for breeding plutonium, such as at Chernobyl, where two such reactors remain in operation.

Plutonium was recognized as easier to fission with neutrons and, hence, a superior fission material very early in the Manhattan Project. Plutonium availability was uncertain, and so a uranium bomb was developed simultaneously. Figure 32.24 shows a gun-type bomb, which takes two subcritical uranium masses and blows them together. To get an appreciable yield, the critical mass must be held together by the explosive charges inside the cannon barrel for a