One mole of a nuclide ${}^{A}X$ has a mass of $A$ grams, so that one mole of ${}^{137}\text{Cs}$ has a mass of 137 g. A mole has $6.02 \times 10^{23}$ nuclei. Thus the mass of ${}^{137}\text{Cs}$ released was

$$\begin{aligned} m &= \left(\frac{137\text{ g}}{6.02\times10^{23}}\right)(3.1\times10^{26}) = 70\times10^{3}\text{ g} \\ &= 70\text{ kg.} \end{aligned} \tag{31.58}$$

**Discussion**

While 70 kg of material may not be a very large mass compared to the amount of fuel in a power plant, it is extremely radioactive, since it only has a 30-year half-life. Six megacuries (6.0 MCi) is an extraordinary amount of activity but is only a fraction of what is produced in nuclear reactors. Similar amounts of the other isotopes were also released at Chernobyl. Although the chances of such a disaster may have seemed small, the consequences were extremely severe, requiring greater caution than was used. More will be said about safe reactor design in the next chapter, but it should be noted that more recent reactors have a fundamentally safer design.

Activity $R$ decreases in time, going to half its original value in one half-life, then to one-fourth its original value in the next half-life, and so on. Since $R = \frac{0.693N}{t_{1/2}}$, the activity decreases as the number of radioactive nuclei decreases. The equation for $R$ as a function of time is found by combining the equations $N = N_0 e^{-\lambda t}$ and $R = \frac{0.693N}{t_{1/2}}$, yielding

$$R = R_0 e^{-\lambda t}, \tag{31.59}$$

where $R_0$ is the activity at $t = 0$. This equation shows exponential decay of radioactive nuclei. For example, if a source originally has a 1.00-mCi activity, it declines to 0.500 mCi in one half-life, to 0.250 mCi in two half-lives, to 0.125 mCi in three half-lives, and so on. For times other than whole half-lives, the equation $R = R_0 e^{-\lambda t}$ must be used to find $R$.

### PHET EXPLORATIONS

**Alpha Decay**

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half life.

Click to view content (https://openstax.org/l/02alphadecay).

## 31.6 Binding Energy

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define and discuss binding energy.
- Calculate the binding energy per nucleon of a particle.

The more tightly bound a system is, the stronger the forces that hold it together and the greater the energy required to pull it apart. We can therefore learn about nuclear forces by examining how tightly bound the nuclei are. We define the **binding energy** (BE) of a nucleus to be *the energy required to completely disassemble it into separate protons and neutrons*. We can determine the BE of a nucleus from its rest mass. The two are connected through Einstein's famous relationship $E = (\Delta m)c^2$. A bound system has a *smaller* mass than its separate constituents; the more tightly the nucleons are bound together, the smaller the mass of the nucleus.

Imagine pulling a nuclide apart as illustrated in Figure 31.22. Work done to overcome the nuclear forces holding the nucleus together puts energy into the system. By definition, the energy input equals the binding energy BE. The pieces are at rest when separated, and so the energy put into them increases their total rest mass compared with what it was when they were glued together as a nucleus. That mass increase is thus $\Delta m = \text{BE}/c^2$. This difference in mass is known as *mass defect*. It implies that the mass of the nucleus is less than the sum of the masses of its constituent protons and neutrons. A nuclide $^{A}\text{X}$ has $Z$ protons and $N$ neutrons, so that the difference in mass is

$$\Delta m = (Zm_p + Nm_n) - m_{\text{tot}}. \quad (31.60)$$

Thus,

$$\text{BE} = (\Delta m)c^2 = [(Zm_p + Nm_n) - m_{\text{tot}}]c^2, \quad (31.61)$$

where $m_{\text{tot}}$ is the mass of the nuclide $^{A}\text{X}$, $m_p$ is the mass of a proton, and $m_n$ is the mass of a neutron. Traditionally, we deal with the masses of neutral atoms. To get atomic masses into the last equation, we first add $Z$ electrons to $m_{\text{tot}}$, which gives $m\left(^{A}\text{X}\right)$, the atomic mass of the nuclide. We then add $Z$ electrons to the $Z$ protons, which gives $Zm\left(^{1}\text{H}\right)$, or $Z$ times the mass of a hydrogen atom. Thus the binding energy of a nuclide $^{A}\text{X}$ is

$$\text{BE} = \left\{[Zm(^{1}\text{H}) + Nm_n] - m(^{A}X)\right\}c^2. \quad (31.62)$$

The atomic masses can be found in Appendix A, most conveniently expressed in unified atomic mass units u $(1 \text{ u} = 931.5 \text{ MeV}/c^2)$. BE is thus calculated from known atomic masses.

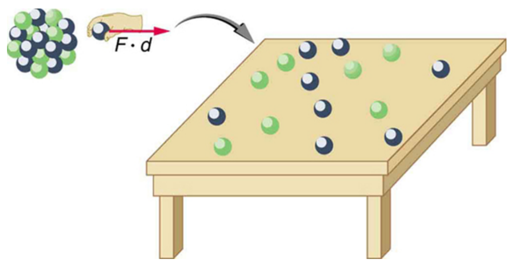


FIGURE 31.22 Work done to pull a nucleus apart into its constituent protons and neutrons increases the mass of the system. The work to disassemble the nucleus equals its binding energy BE. A bound system has less mass than the sum of its parts, especially noticeable in the nuclei, where forces and energies are very large.

## Things Great and Small

**Nuclear Decay Helps Explain Earth's Hot Interior**

A puzzle created by radioactive dating of rocks is resolved by radioactive heating of Earth's interior. This intriguing story is another example of how small-scale physics can explain large-scale phenomena.

Radioactive dating plays a role in determining the approximate age of the Earth. The oldest rocks on Earth solidified about $3.5 \times 10^9$ years ago—a number determined by uranium-238 dating. These rocks could only have solidified once the surface of the Earth had cooled sufficiently. The temperature of the Earth at formation can be estimated based on gravitational potential energy of the assemblage of pieces being converted to thermal energy. Using heat transfer concepts discussed in Thermodynamics it is then possible to calculate how long it would take for the surface to cool to rock-formation temperatures. The result is about $10^9$ years. The first rocks formed have been solid for $3.5 \times 10^9$ years, so that the age of the Earth is approximately $4.5 \times 10^9$ years. There is a large body of other types of evidence (both Earth-bound and solar system characteristics are used) that supports this age. The puzzle is that, given its age and initial temperature, the center of the Earth should be much cooler than it is today (see Figure 31.23).

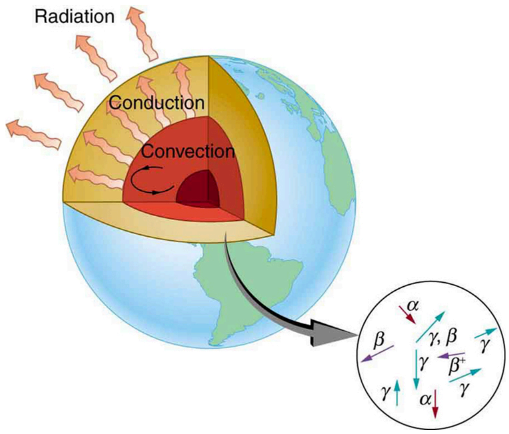


FIGURE 31.23 The center of the Earth cools by well-known heat transfer methods. Convection in the liquid regions and conduction move thermal energy to the surface, where it radiates into cold, dark space. Given the age of the Earth and its initial temperature, it should have cooled to a lower temperature by now. The blowup shows that nuclear decay releases energy in the Earth's interior. This energy has slowed the cooling process and is responsible for the interior still being molten.

Danish geophysicist Inge Lehmann was the first to properly interpret seismological data from earthquakes as evidence that the Earth had a solid inner core surrounded by a liquid outer core. Shear or transverse waves cannot travel through a liquid and are not transmitted through the Earth's core. Yet compression or longitudinal waves can pass through a liquid and do go through the core. From this information, the temperature of the interior can be estimated. As noticed, the interior should have cooled more from its initial temperature in the $4.5 \times 10^9$ years since its formation. In fact, it should have taken no more than about $10^9$ years to cool to its present temperature. What is keeping it hot? The answer seems to be radioactive decay of primordial elements that were part of the material that formed the Earth (see the blowup in Figure 31.23).

Nuclides such as $^{238}\mathrm{U}$ and $^{40}\mathrm{K}$ have half-lives similar to or longer than the age of the Earth, and their decay still contributes energy to the interior. Some of the primordial radioactive nuclides have unstable decay products that also release energy— $^{238}\mathrm{U}$ has a long decay chain of these. Further, there were more of these primordial radioactive nuclides early in the life of the Earth, and thus the activity and energy contributed were greater then (perhaps by an order of magnitude). The amount of power created by these decays per cubic meter is very small. However, since a huge volume of material lies deep below the surface, this relatively small amount of energy cannot escape quickly. The power produced near the surface has much less distance to go to escape and has a negligible effect on surface temperatures.

A final effect of this trapped radiation merits mention. Alpha decay produces helium nuclei, which form helium atoms when they are stopped and capture electrons. Most of the helium on Earth is obtained from wells and is produced in this manner. Any helium in the atmosphere will escape in geologically short times because of its high thermal velocity.

What patterns and insights are gained from an examination of the binding energy of various nuclides? First, we find that BE is approximately proportional to the number of nucleons $A$ in any nucleus. About twice as much energy is needed to pull apart a nucleus like $^{24}\mathrm{Mg}$ compared with pulling apart $^{12}\mathrm{C}$, for example. To help us look at other effects, we divide BE by $A$ and consider the **binding energy per nucleon**, $\mathrm{BE}/A$. The graph of $\mathrm{BE}/A$ in Figure 31.24 reveals some very interesting aspects of nuclei. We see that the binding energy per nucleon averages about 8 MeV, but is lower for both the lightest and heaviest nuclei. This overall trend, in which nuclei with $A$ equal to about 60 have the greatest $\mathrm{BE}/A$ and are thus the most tightly bound, is due to the combined characteristics of the attractive nuclear forces and the repulsive Coulomb force. It is especially important to note two things—the strong nuclear force is about 100 times stronger than the Coulomb force, *and* the nuclear forces are shorter in range compared to the Coulomb force. So, for low-mass nuclei, the nuclear attraction dominates and each added nucleon forms bonds with all others, causing progressively heavier nuclei to have progressively greater values of $\mathrm{BE}/A$. This continues up to $A \approx 60$, roughly corresponding to the mass number of iron. Beyond that, new nucleons added to a nucleus will be too far from some others to feel their nuclear attraction. Added protons, however, feel the repulsion of all other protons, since the Coulomb force is longer in range. Coulomb repulsion grows for progressively heavier nuclei, but nuclear attraction remains about the same, and so $\mathrm{BE}/A$ becomes smaller. This is why stable nuclei heavier than $A \approx 40$ have more neutrons than protons. Coulomb repulsion is reduced by having more neutrons to keep the protons farther apart (see Figure 31.25).

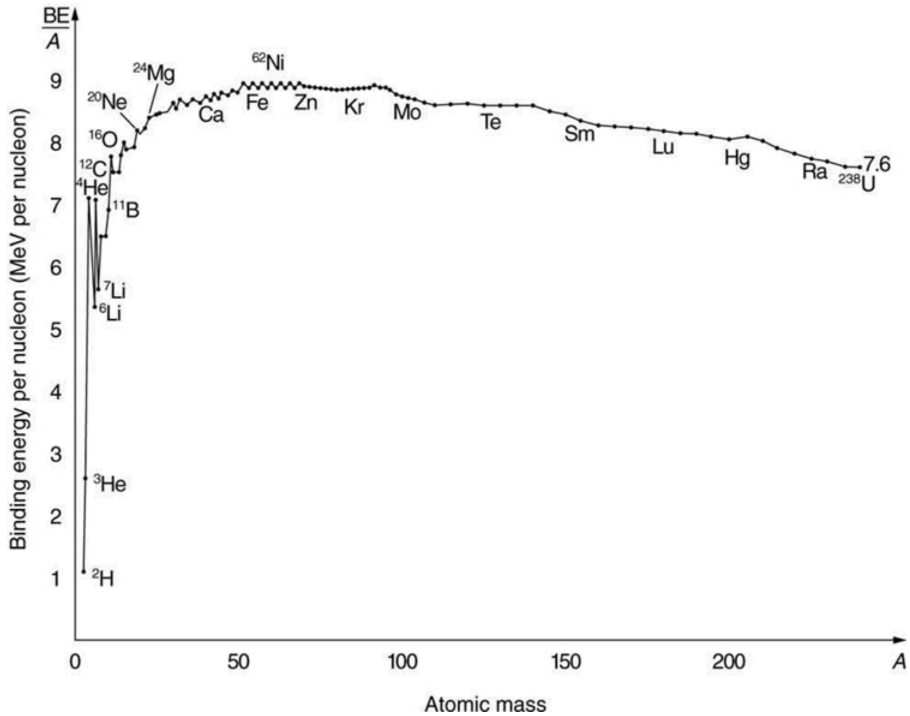


FIGURE 31.24 A graph of average binding energy per nucleon, $\mathrm{BE}/A$, for stable nuclei. The most tightly bound nuclei are those with $A$ near 60, where the attractive nuclear force has its greatest effect. At higher $A$ s, the Coulomb repulsion progressively reduces the binding energy per nucleon, because the nuclear force is short ranged. The spikes on the curve are very tightly bound nuclides and indicate shell closures.

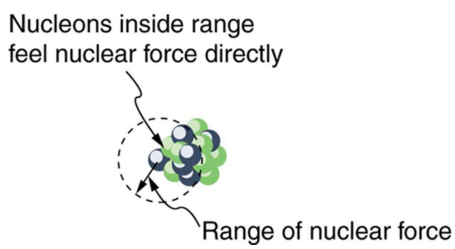


FIGURE 31.25 The nuclear force is attractive and stronger than the Coulomb force, but it is short ranged. In low-mass nuclei, each nucleon feels the nuclear attraction of all others. In larger nuclei, the range of the nuclear force, shown for a single nucleon, is smaller than the size of the nucleus, but the Coulomb repulsion from all protons reaches all others. If the nucleus is large enough, the Coulomb repulsion can add to overcome the nuclear attraction.

There are some noticeable spikes on the $\mathrm{BE}/A$ graph, which represent particularly tightly bound nuclei. These spikes reveal further details of nuclear forces, such as confirming that closed-shell nuclei (those with magic numbers of protons or neutrons or both) are more tightly bound. The spikes also indicate that some nuclei with even numbers for $Z$ and $N$, and with $Z = N$, are exceptionally tightly bound. This finding can be correlated with some of the cosmic abundances of the elements. The most common elements in the universe, as determined by observations of atomic spectra from outer space, are hydrogen, followed by $^4\mathrm{He}$, with much smaller amounts of $^{12}\mathrm{C}$ and other elements. It should be noted that the heavier elements are created in supernova explosions, while the lighter ones are produced by nuclear fusion during the normal life cycles of stars, as will be discussed in subsequent chapters. The most common elements have the most tightly bound nuclei. It is also no accident that one of the most tightly bound light nuclei is $^4\mathrm{He}$, emitted in $\alpha$ decay.

## EXAMPLE 31.7

### What Is $\mathrm{BE}/A$ for an Alpha Particle?

Calculate the binding energy per nucleon of $^4\mathrm{He}$, the $\alpha$ particle.

#### Strategy

To find $\mathrm{BE}/A$, we first find BE using the Equation $\mathrm{BE} = \{[Zm(^1\mathrm{H}) + Nm_n] - m(^A\mathrm{X})\}\,c^2$ and then divide by $A$. This is straightforward once we have looked up the appropriate atomic masses in Appendix A.

#### Solution

The binding energy for a nucleus is given by the equation

$$\mathrm{BE} = \{[Zm(^1\mathrm{H}) + Nm_n] - m(^A\mathrm{X})\}\,c^2. \tag{31.63}$$

For $^4\mathrm{He}$, we have $Z = N = 2$; thus,

$$\mathrm{BE} = \{[2m(^1\mathrm{H}) + 2m_n] - m(^4\mathrm{He})\}\,c^2. \tag{31.64}$$

Appendix A gives these masses as $m(^4\mathrm{He}) = 4.002602\ \mathrm{u}$, $m(^1\mathrm{H}) = 1.007825\ \mathrm{u}$, and $m_n = 1.008665\ \mathrm{u}$. Thus,

$$\mathrm{BE} = (0.030378\ \mathrm{u})c^2. \tag{31.65}$$

Noting that $1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$, we find

$$\mathrm{BE} = (0.030378)(931.5\ \mathrm{MeV}/c^2)c^2 = 28.3\ \mathrm{MeV}. \tag{31.66}$$

Since $A = 4$, we see that $\mathrm{BE}/A$ is this number divided by 4, or

$$\mathrm{BE}/A = 7.07\ \mathrm{MeV/nucleon}. \tag{31.67}$$

#### Discussion

This is a large binding energy per nucleon compared with those for other low-mass nuclei, which have $\mathrm{BE}/A \approx 3\ \mathrm{MeV/nucleon}$. This indicates that $^4\mathrm{He}$ is tightly bound compared with its neighbors on the chart of the nuclides. You can see the spike representing this value of $\mathrm{BE}/A$ for $^4\mathrm{He}$ on the graph in Figure 31.24. This is why $^4\mathrm{He}$ is stable. Since $^4\mathrm{He}$ is tightly bound, it has less mass than other $A = 4$ nuclei and, therefore, cannot spontaneously decay into them. The large binding energy also helps to explain why some nuclei undergo $\alpha$ decay. Smaller mass in the decay products can mean energy release, and such decays can be spontaneous. Further, it can happen that two protons and two neutrons in a nucleus can randomly find themselves together, experience the exceptionally large nuclear force that binds this combination, and act as a $^4\mathrm{He}$ unit within the nucleus, at least for a while. In some cases, the $^4\mathrm{He}$ escapes, and $\alpha$ decay has then taken place.

There is more to be learned from nuclear binding energies. The general trend in $\mathrm{BE}/A$ is fundamental to energy production in stars, and to fusion and fission energy sources on Earth, for example. This is one of the applications of nuclear physics covered in Medical Applications of Nuclear Physics. The abundance of elements on Earth, in stars, and in the universe as a whole is related to the binding energy of nuclei and has implications for the continued expansion of the universe.

## Problem-Solving Strategies

### For Reaction And Binding Energies and Activity Calculations in Nuclear Physics

1. *Identify exactly what needs to be determined in the problem (identify the unknowns).* This will allow you to decide whether the energy of a decay or nuclear reaction is involved, for example, or whether the problem is primarily concerned with activity (rate of decay).
2. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
3. *For reaction and binding-energy problems, we use atomic rather than nuclear masses.* Since the masses of neutral atoms are used, you must count the number of electrons involved. If these do not balance (such as in $\beta^+$ decay), then an energy adjustment of 0.511 MeV per electron must be made. Also note that atomic masses

may not be given in a problem; they can be found in tables.

4. *For problems involving activity, the relationship of activity to half-life, and the number of nuclei given in the equation* $R = \frac{0.693N}{t_{1/2}}$ *can be very useful.* Owing to the fact that number of nuclei is involved, you will also need to be familiar with moles and Avogadro's number.
5. *Perform the desired calculation; keep careful track of plus and minus signs as well as powers of 10.*
6. *Check the answer to see if it is reasonable: Does it make sense?* Compare your results with worked examples and other information in the text. (Heeding the advice in Step 5 will also help you to be certain of your result.) You must understand the problem conceptually to be able to determine whether the numerical result is reasonable.

**PHET EXPLORATIONS**

**Nuclear Fission**

Start a chain reaction, or introduce non-radioactive isotopes to prevent one. Control energy production in a nuclear reactor!

Click to view content (https://openstax.org/l/16fission).

## 31.7 Tunneling

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define and discuss tunneling.
- Define potential barrier.
- Explain quantum tunneling.

Protons and neutrons are *bound* inside nuclei, that means energy must be supplied to break them away. The situation is analogous to a marble in a bowl that can roll around but lacks the energy to get over the rim. It is bound inside the bowl (see Figure 31.26). If the marble could get over the rim, it would gain kinetic energy by rolling down outside. However classically, if the marble does not have enough kinetic energy to get over the rim, it remains forever trapped in its well.

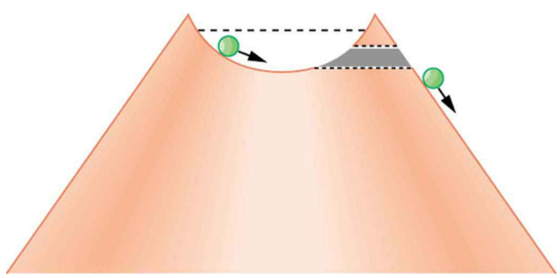

FIGURE 31.26 The marble in this semicircular bowl at the top of a volcano has enough kinetic energy to get to the altitude of the dashed line, but not enough to get over the rim, so that it is trapped forever. If it could find a tunnel through the barrier, it would escape, roll downhill, and gain kinetic energy.

In a nucleus, the attractive nuclear potential is analogous to the bowl at the top of a volcano (where the "volcano" refers only to the shape). Protons and neutrons have kinetic energy, but it is about 8 MeV less than that needed to get out (see Figure 31.27). That is, they are bound by an average of 8 MeV per nucleon. The slope of the hill outside the bowl is analogous to the repulsive Coulomb potential for a nucleus, such as for an $\alpha$ particle outside a positive nucleus. In $\alpha$ decay, two protons and two neutrons spontaneously break away as a $^4\text{He}$ unit. Yet the protons and neutrons do not have enough kinetic energy to get over the rim. So how does the $\alpha$ particle get out?