$$\Delta E = \frac{h}{4\pi\Delta t} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi(1.0{\times}10^{-10}\ \text{s})} = 5.3 \times 10^{-25}\ \text{J}. \tag{29.50}$$

Now converting to eV yields

$$\Delta E = (5.3 \times 10^{-25}\ \text{J})\left(\frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right) = 3.3 \times 10^{-6}\ \text{eV}. \tag{29.51}$$

Discussion

The lifetime of 10^{-10} s is typical of excited states in atoms—on human time scales, they quickly emit their stored energy. An uncertainty in energy of only a few millionths of an eV results. This uncertainty is small compared with typical excitation energies in atoms, which are on the order of 1 eV. So here the uncertainty principle limits the accuracy with which we can measure the lifetime and energy of such states, but not very significantly.

The uncertainty principle for energy and time can be of great significance if the lifetime of a system is very short. Then Δt is very small, and ΔE is consequently very large. Some nuclei and exotic particles have extremely short lifetimes (as small as 10^{-25} s), causing uncertainties in energy as great as many GeV (10^9 eV). Stored energy appears as increased rest mass, and so this means that there is significant uncertainty in the rest mass of short-lived particles. When measured repeatedly, a spread of masses or decay energies are obtained. The spread is ΔE. You might ask whether this uncertainty in energy could be avoided by not measuring the lifetime. The answer is no. Nature knows the lifetime, and so its brevity affects the energy of the particle. This is so well established experimentally that the uncertainty in decay energy is used to calculate the lifetime of short-lived states. Some nuclei and particles are so short-lived that it is difficult to measure their lifetime. But if their decay energy can be measured, its spread is ΔE, and this is used in the uncertainty principle ($\Delta E \Delta t \geq h/4\pi$) to calculate the lifetime Δt.

There is another consequence of the uncertainty principle for energy and time. If energy is uncertain by ΔE, then conservation of energy can be violated by ΔE for a time Δt. Neither the physicist nor nature can tell that conservation of energy has been violated, if the violation is temporary and smaller than the uncertainty in energy. While this sounds innocuous enough, we shall see in later chapters that it allows the temporary creation of matter from nothing and has implications for how nature transmits forces over very small distances.

Finally, note that in the discussion of particles and waves, we have stated that individual measurements produce precise or particle-like results. A definite position is determined each time we observe an electron, for example. But repeated measurements produce a spread in values consistent with wave characteristics. The great theoretical physicist Richard Feynman (1918–1988) commented, "What there are, are particles." When you observe enough of them, they distribute themselves as you would expect for a wave phenomenon. However, what there are as they travel we cannot tell because, when we do try to measure, we affect the traveling.

29.8 The Particle-Wave Duality Reviewed

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the concept of particle-wave duality, and its scope.

Particle-wave duality—the fact that all particles have wave properties—is one of the cornerstones of quantum mechanics. We first came across it in the treatment of photons, those particles of EM radiation that exhibit both particle and wave properties, but not at the same time. Later it was noted that particles of matter have wave properties as well. The dual properties of particles and waves are found for all particles, whether massless like photons, or having a mass like electrons. (See Figure 29.24.)

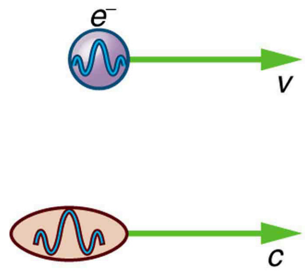

FIGURE 29.24 On a quantum-mechanical scale (i.e., very small), particles with and without mass have wave properties. For example, both electrons and photons have wavelengths but also behave as particles.

There are many submicroscopic particles in nature. Most have mass and are expected to act as particles, or the smallest units of matter. All these masses have wave properties, with wavelengths given by the de Broglie relationship $\lambda = h/p$. So, too, do combinations of these particles, such as nuclei, atoms, and molecules. As a combination of masses becomes large, particularly if it is large enough to be called macroscopic, its wave nature becomes difficult to observe. This is consistent with our common experience with matter.

Some particles in nature are massless. We have only treated the photon so far, but all massless entities travel at the speed of light, have a wavelength, and exhibit particle and wave behaviors. They have momentum given by a rearrangement of the de Broglie relationship, $p = h/\lambda$. In large combinations of these massless particles (such large combinations are common only for photons or EM waves), there is mostly wave behavior upon detection, and the particle nature becomes difficult to observe. This is also consistent with experience. (See Figure 29.25.)

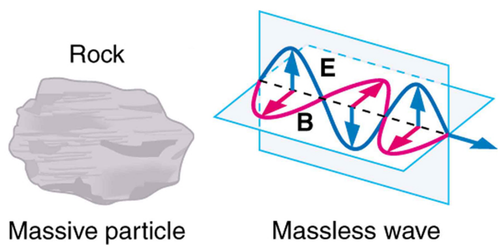

FIGURE 29.25 On a classical scale (macroscopic), particles with mass behave as particles and not as waves. Particles without mass act as waves and not as particles.

The particle-wave duality is a universal attribute. It is another connection between matter and energy. Not only has modern physics been able to describe nature for high speeds and small sizes, it has also discovered new connections and symmetries. There is greater unity and symmetry in nature than was known in the classical era—but they were dreamt of. A beautiful poem written by the English poet William Blake some two centuries ago contains the following four lines:

To see the World in a Grain of Sand

And a Heaven in a Wild Flower

Hold Infinity in the palm of your hand

And Eternity in an hour

Integrated Concepts

The problem set for this section involves concepts from this chapter and several others. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. For example, photons have momentum, hence the relevance of Linear Momentum and Collisions. The following topics are involved in some or all of the problems in this section:

- Dynamics: Newton’s Laws of Motion
- Work, Energy, and Energy Resources
- Linear Momentum and Collisions
- Heat and Heat Transfer Methods

- Electric Potential and Electric Field
- Electric Current, Resistance, and Ohm's Law
- Wave Optics
- Special Relativity

Problem-Solving Strategy

1. Identify which physical principles are involved.
2. Solve the problem using strategies outlined in the text.

Example 29.10 illustrates how these strategies are applied to an integrated-concept problem.

EXAMPLE 29.10

Recoil of a Dust Particle after Absorbing a Photon

The following topics are involved in this integrated concepts worked example:

Photons (quantum mechanics)
Linear Momentum

TABLE 29.2 Topics

A 550-nm photon (visible light) is absorbed by a $1.00\text{-}\mu\text{g}$ particle of dust in outer space. (a) Find the momentum of such a photon. (b) What is the recoil velocity of the particle of dust, assuming it is initially at rest?

Strategy Step 1

To solve an *integrated-concept problem*, such as those following this example, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for the *momentum of a photon*, a topic of the present chapter. Part (b) considers *recoil following a collision*, a topic of Linear Momentum and Collisions.

Strategy Step 2

The following solutions to each part of the example illustrate how specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

The momentum of a photon is related to its wavelength by the equation:

$$p = \frac{h}{\lambda}. \tag{29.52}$$

Entering the known value for Planck's constant h and given the wavelength λ, we obtain

$$\begin{aligned} p &= \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{550\times10^{-9}\ \text{m}} \\ &= 1.21\times10^{-27}\ \text{kg}\cdot\text{m/s}. \end{aligned} \tag{29.53}$$

Discussion for (a)

This momentum is small, as expected from discussions in the text and the fact that photons of visible light carry small amounts of energy and momentum compared with those carried by macroscopic objects.

Solution for (b)

Conservation of momentum in the absorption of this photon by a grain of dust can be analyzed using the equation:

$$p_1 + p_2 = p'_1 + p'_2 (F_{\text{net}} = 0). \tag{29.54}$$

The net external force is zero, since the dust is in outer space. Let 1 represent the photon and 2 the dust particle. Before the collision, the dust is at rest (relative to some observer); after the collision, there is no photon (it is absorbed). So conservation of momentum can be written

$$p_1 = p'_2 = mv, \tag{29.55}$$

where p_1 is the photon momentum before the collision and p'_2 is the dust momentum after the collision. The mass and recoil velocity of the dust are m and v, respectively. Solving this for v, the requested quantity, yields

$$v = \frac{p}{m}, \tag{29.56}$$

where p is the photon momentum found in part (a). Entering known values (noting that a microgram is 10^{-9} kg) gives

$$\begin{aligned} v &= \frac{1.21 \times 10^{-27}\ \text{kg}\cdot\text{m/s}}{1.00 \times 10^{-9}\ \text{kg}} \\ &= 1.21 \times 10^{-18}\ \text{m/s}. \end{aligned} \tag{29.57}$$

Discussion

The recoil velocity of the particle of dust is extremely small. As we have noted, however, there are immense numbers of photons in sunlight and other macroscopic sources. In time, collisions and absorption of many photons could cause a significant recoil of the dust, as observed in comet tails.