$$\begin{aligned}\lambda_{\text{obs}} &= \lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \\ &= (0.525\text{ m})\sqrt{\frac{1+\frac{0.825c}{c}}{1-\frac{0.825c}{c}}} \\ &= 1.70\text{ m.}\end{aligned} \quad (28.38)$$

Discussion

Because the galaxy is moving away from the Earth, we expect the wavelengths of radiation it emits to be redshifted. The wavelength we calculated is 1.70 m, which is redshifted from the original wavelength of 0.525 m.

The relativistic Doppler shift is easy to observe. This equation has everyday applications ranging from Doppler-shifted radar velocity measurements of transportation to Doppler-radar storm monitoring. In astronomical observations, the relativistic Doppler shift provides velocity information such as the motion and distance of stars.

CHECK YOUR UNDERSTANDING

Suppose a space probe moves away from the Earth at a speed $0.350c$. It sends a radio wave message back to the Earth at a frequency of 1.50 GHz. At what frequency is the message received on the Earth?

Solution

$$f_{\text{obs}} = f_s\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} = (1.50\text{ GHz})\sqrt{\frac{1-\frac{0.350c}{c}}{1+\frac{0.350c}{c}}} = 1.04\text{ GHz} \quad (28.39)$$

28.5 Relativistic Momentum

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate relativistic momentum.
- Explain why the only mass it makes sense to talk about is rest mass.

FIGURE 28.18 Momentum is an important concept for these football players from the University of California at Berkeley and the University of California at Davis. Players with more mass often have a larger impact because their momentum is larger. For objects moving at relativistic speeds, the effect is even greater. (credit: John Martinez Pavliga)

In classical physics, momentum is a simple product of mass and velocity. However, we saw in the last section that when special relativity is taken into account, massive objects have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. All of Work, Energy, and Energy Resources is devoted to momentum, and momentum has been important for many other topics as well, particularly where collisions were involved. We will see that momentum has the same importance in modern physics. Relativistic momentum is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic Momentum

Relativistic momentum p is classical momentum multiplied by the relativistic factor γ.

$$p = \gamma mu, \tag{28.40}$$

where m is the **rest mass** of the object, u is its velocity relative to an observer, and the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}. \tag{28.41}$$

Note that we use u for velocity here to distinguish it from relative velocity v between observers. Only one observer is being considered here. With p defined in this way, total momentum p_{tot} is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum γmu becomes the classical mu at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities, but, because of the factor γ, relativistic momentum approaches infinity as u approaches c. (See Figure 28.19.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, an unreasonable value.

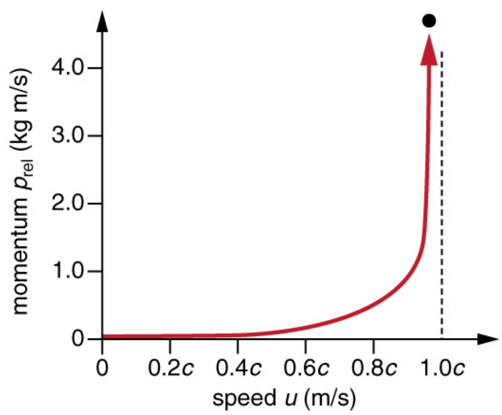

FIGURE 28.19 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Misconception Alert: Relativistic Mass and Momentum

The relativistically correct definition of momentum as $p = \gamma mu$ is sometimes taken to imply that mass varies with velocity: $m_{\text{var}} = \gamma m$, particularly in older textbooks. However, note that m is the mass of the object as measured by a person at rest relative to the object. Thus, m is defined to be the rest mass, which could be measured at rest, perhaps using gravity. When a mass is moving relative to an observer, the only way that its mass can be determined is through collisions or other means in which momentum is involved. Since the mass of a moving object cannot be determined independently of momentum, the only meaningful mass is rest mass. Thus, when we use the term mass, assume it to be identical to rest mass.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

In Relativistic Energy, the relationship of relativistic momentum to energy is explored. That subject will produce our first inkling that objects without mass may also have momentum.

CHECK YOUR UNDERSTANDING

What is the momentum of an electron traveling at a speed $0.985c$? The rest mass of the electron is 9.11×10^{-31} kg.

Solution

$$p = \gamma m u = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{(9.11 \times 10^{-31}\text{ kg})(0.985)(3.00 \times 10^{8}\text{ m/s})}{\sqrt{1-\frac{(0.985c)^2}{c^2}}} = 1.56 \times 10^{-21}\text{ kg} \cdot \text{m/s} \quad (28.42)$$

28.6 Relativistic Energy

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Compute total energy of a relativistic object.
- Compute the kinetic energy of a relativistic object.
- Describe rest energy, and explain how it can be converted to other forms.
- Explain why massive particles cannot reach C.

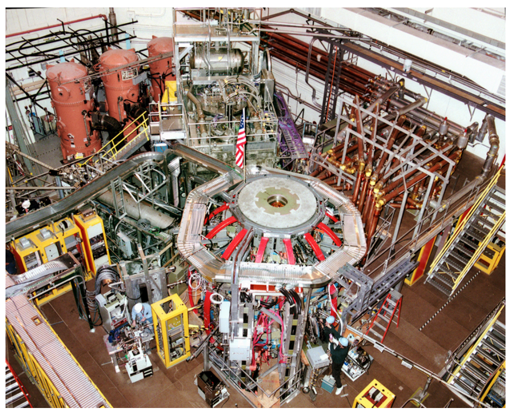

FIGURE 28.20 The National Spherical Torus Experiment (NSTX) has a fusion reactor in which hydrogen isotopes undergo fusion to produce helium. In this process, a relatively small mass of fuel is converted into a large amount of energy. (credit: Princeton Plasma Physics Laboratory)

A tokamak is a form of experimental fusion reactor, which can change mass to energy. Accomplishing this requires an understanding of relativistic energy. Nuclear reactors are proof of the conservation of relativistic energy.

Conservation of energy is one of the most important laws in physics. Not only does energy have many important forms, but each form can be converted to any other. We know that classically the total amount of energy in a system remains constant. Relativistically, energy is still conserved, provided its definition is altered to include the possibility of mass changing to energy, as in the reactions that occur within a nuclear reactor. Relativistic energy is intentionally defined so that it will be conserved in all inertial frames, just as is the case for relativistic momentum. As a consequence, we learn that several fundamental quantities are related in ways not known in classical physics. All of these relationships are verified by experiment and have fundamental consequences. The altered definition of energy contains some of the most fundamental and spectacular new insights into nature found in recent history.

Total Energy and Rest Energy

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor.

Total Energy

Total energy E is defined to be