# Special Relativity



**FIGURE 28.1** Special relativity has implications and applications ranging from space travel and our understanding of time to everyday technologies such as Global Positioning Systems. In order to be effective, GPS, whether it is a part of a mobile phone or a shark tracking system, must account for relativistic principles such as time dilation. (credit: UW News/Flickr)

## CHAPTER OUTLINE

**28.1 Einstein's Postulates**
**28.2 Simultaneity And Time Dilation**
**28.3 Length Contraction**
**28.4 Relativistic Addition of Velocities**
**28.5 Relativistic Momentum**
**28.6 Relativistic Energy**

**INTRODUCTION TO SPECIAL RELATIVITY** Have you ever looked up at the night sky and dreamed of traveling to other planets in faraway star systems? Would there be other life forms? What would other worlds look like? You might imagine that such an amazing trip would be possible if we could just travel fast enough, but you will read in this chapter why this is not true. In 1905 Albert Einstein developed the theory of special relativity. This theory explains the limit on an object's speed and describes the consequences.

Relativity does not only apply to far-reaching and (as yet) unrealized activities like human interstellar travel. It affects everyday life in the form of communication, global trade, and even medicine. For example, Global Positioning Systems, which drive everything from airplane navigation to smart phone maps, rely on signals captured by multiple orbiting satellites and highly accurate measurements of time. Every signal passing between satellites, towers, and devices must be precisely measured and account for the relativistic effect of curved space and time dilation (discussed below). Variations in Earth's landscape, its non-spherical shape, and the effects of gravity must also be considered in order to obtain accurate measurements. One of the most important contributors to these systems was Gladys West, a computer scientist and mathematician working at the Naval Proving Ground, where GPS and related

technologies were advanced. West had previously developed altimeter models and managed the world's first satellite-based ocean mapping project (Seastat). She then developed and programmed the algorithms capable of calculating positions and Earth's shape to sufficient precisions to enable the existence of GPS. In these calculations, she accounted for the impacts of relativity and other complex principles related to it.

*Relativity*. The word *relativity* might conjure an image of Einstein, but the idea did not begin with him. People have been exploring relativity for many centuries. Relativity is the study of how different observers measure the same event. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. *Special relativity* deals with observers who are moving at constant velocity. *General relativity* deals with observers who are undergoing acceleration. Einstein is famous because his theories of relativity made revolutionary predictions. Most importantly, his theories have been verified to great precision in a vast range of experiments, altering forever our concept of space and time.
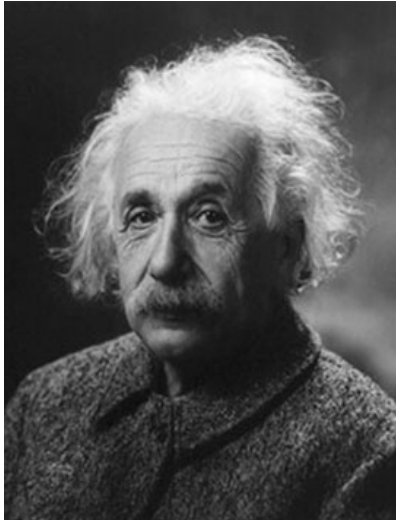


**FIGURE 28.2** Many people think that Albert Einstein (1879–1955) was the greatest physicist of the 20th century. Not only did he develop modern relativity, thus revolutionizing our concept of the universe, he also made fundamental contributions to the foundations of quantum mechanics. (credit: The Library of Congress)

It is important to note that although classical mechanics, in general, and classical relativity, in particular, are limited, they are extremely good approximations for large, slow-moving objects. Otherwise, we could not use classical physics to launch satellites or build bridges. In the classical limit (objects larger than submicroscopic and moving slower than about 1% of the speed of light), relativistic mechanics becomes the same as classical mechanics. This fact will be noted at appropriate places throughout this chapter.

## 28.1 Einstein's Postulates

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State and explain both of Einstein's postulates.
- Explain what an inertial frame of reference is.
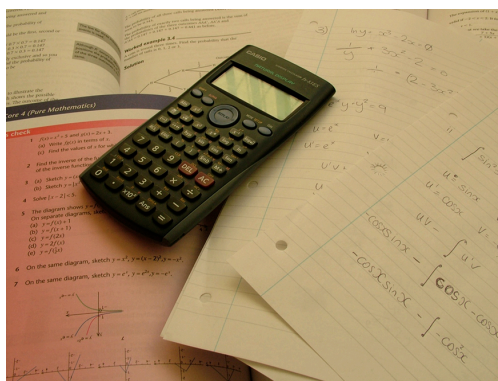- Describe one way the speed of light can be changed.

**FIGURE 28.3** Special relativity resembles trigonometry in that both are reliable because they are based on postulates that flow one from another in a logical way. (credit: Jon Oakley, Flickr)

Have you ever used the Pythagorean Theorem and gotten a wrong answer? Probably not, unless you made a mistake in either your algebra or your arithmetic. Each time you perform the same calculation, you know that the answer will be the same. Trigonometry is reliable because of the certainty that one part always flows from another in a logical way. Each part is based on a set of postulates, and you can always connect the parts by applying those postulates. Physics is the same way with the exception that *all* parts must describe nature. If we are careful to choose the correct postulates, then our theory will follow and will be verified by experiment.

Einstein essentially did the theoretical aspect of this method for **relativity**. With two deceptively simple postulates and a careful consideration of how measurements are made, he produced the theory of **special relativity.**

### Einstein's First Postulate

The first postulate upon which Einstein based the theory of special relativity relates to reference frames. All velocities are measured relative to some frame of reference. For example, a car's motion is measured relative to its starting point or the road it is moving over, a projectile's motion is measured relative to the surface it was launched from, and a planet's orbit is measured relative to the star it is orbiting around. The simplest frames of reference are those that are not accelerated and are not rotating. Newton's first law, the law of inertia, holds exactly in such a frame.

> ### Inertial Reference Frame
>
> An **inertial frame of reference** is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force.

The laws of physics seem to be simplest in inertial frames. For example, when you are in a plane flying at a constant altitude and speed, physics seems to work exactly the same as if you were standing on the surface of the Earth. However, in a plane that is taking off, matters are somewhat more complicated. In these cases, the net force on an object, $F$, is not equal to the product of mass and acceleration, $ma$. Instead, $F$ is equal to $ma$ plus a fictitious force. This situation is not as simple as in an inertial frame. Not only are laws of physics simplest in inertial frames, but they should be the same in all inertial frames, since there is no preferred frame and no absolute motion. Einstein incorporated these ideas into his **first postulate of special relativity**.

> ### First Postulate of Special Relativity
>
> The laws of physics are the same and can be stated in their simplest form in all inertial frames of reference.

As with many fundamental statements, there is more to this postulate than meets the eye. The laws of physics include only those that satisfy this postulate. We shall find that the definitions of relativistic momentum and energy must be altered to fit. Another outcome of this postulate is the famous equation $E = mc^2$.

## Einstein's Second Postulate

The second postulate upon which Einstein based his theory of special relativity deals with the speed of light. Late in the 19th century, the major tenets of classical physics were well established. Two of the most important were the laws of electricity and magnetism and Newton's laws. In particular, the laws of electricity and magnetism predict that light travels at $c = 3.00 \times 10^8$ m/s in a vacuum, but they do not specify the frame of reference in which light has this speed.

There was a contradiction between this prediction and Newton's laws, in which velocities add like simple vectors. If the latter were true, then two observers moving at different speeds would see light traveling at different speeds. Imagine what a light wave would look like to a person traveling along with it at a speed $c$. If such a motion were possible then the wave would be stationary relative to the observer. It would have electric and magnetic fields that varied in strength at various distances from the observer but were constant in time. This is not allowed by Maxwell's equations. So either Maxwell's equations are wrong, or an object with mass cannot travel at speed $c$. Einstein concluded that the latter is true. An object with mass cannot travel at speed $c$. This conclusion implies that light in a vacuum must always travel at speed $c$ relative to any observer. Maxwell's equations are correct, and Newton's addition of velocities is not correct for light.

Investigations such as Young's double slit experiment in the early-1800s had convincingly demonstrated that light is a wave. Many types of waves were known, and all travelled in some medium. Scientists therefore assumed that a medium carried light, even in a vacuum, and light travelled at a speed $c$ relative to that medium. Starting in the mid-1880s, the American physicist A. A. Michelson, later aided by E. W. Morley, made a series of direct measurements of the speed of light. The results of their measurements were startling.

> ### Michelson-Morley Experiment
>
> The **Michelson-Morley experiment** demonstrated that the speed of light in a vacuum is independent of the motion of the Earth about the Sun.

The eventual conclusion derived from this result is that light, unlike mechanical waves such as sound, does not need a medium to carry it. Furthermore, the Michelson-Morley results implied that the speed of light $c$ is independent of the motion of the source relative to the observer. That is, everyone observes light to move at speed $c$ regardless of how they move relative to the source or one another. For a number of years, many scientists tried unsuccessfully to explain these results and still retain the general applicability of Newton's laws.

It was not until 1905, when Einstein published his first paper on special relativity, that the currently accepted conclusion was reached. Based mostly on his analysis that the laws of electricity and magnetism would not allow another speed for light, and only slightly aware of the Michelson-Morley experiment, Einstein detailed his **second postulate of special relativity**.

> ### Second Postulate of Special Relativity
>
> The speed of light $c$ is a constant, independent of the relative motion of the source.

Deceptively simple and counterintuitive, this and the first postulate leave all else open for change. Some fundamental concepts do change. Among the changes are the loss of agreement on the elapsed time for an event, the variation of distance with speed, and the realization that matter and energy can be converted into one another. You will read about these concepts in the following sections.

> ### Misconception Alert: Constancy of the Speed of Light
>
> The speed of light is a constant $c = 3.00 \times 10^8$ m/s *in a vacuum*. If you remember the effect of the index of refraction from [The Law of Refraction](#), the speed of light is lower in matter.

## ⊘ CHECK YOUR UNDERSTANDING

Explain how special relativity differs from general relativity.

**Solution**

Special relativity applies only to unaccelerated motion, but general relativity applies to accelerated motion.

## 28.2 Simultaneity And Time Dilation

### LEARNING OBJECTIVES

By the end of this section, you will be able to:
- Describe simultaneity.
- Describe time dilation.
- Calculate γ.
- Compare proper time and the observer's measured time.
- Explain why the twin paradox is a false paradox.



**FIGURE 28.4** Elapsed time for a foot race is the same for all observers, but at relativistic speeds, elapsed time depends on the relative motion of the observer and the event that is observed. (credit: Jason Edward Scott Bain, Flickr)

Do time intervals depend on who observes them? Intuitively, we expect the time for a process, such as the elapsed time for a foot race, to be the same for all observers. Our experience has been that disagreements over elapsed time have to do with the accuracy of measuring time. When we carefully consider just how time is measured, however, we will find that elapsed time depends on the relative motion of an observer with respect to the process being measured.

### Simultaneity

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turning green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps. (See Figure 28.5.) Two flash lamps with observer A midway between them are on a rail car that moves to the right relative to observer B. Observer B arranges for the light flashes to be emitted just as A passes B, so that both A and B are equidistant from the lamps when the light is emitted. Observer B measures the time interval between the arrival of the light flashes. According to postulate 2, the speed of light is not affected by the motion of the lamps relative to B. Therefore, light travels equal distances to him at equal speeds. Thus observer B measures the flashes to be simultaneous.