separated by $1/10,000$ of a centimeter. Once the angles are found, the distances along the screen can be found using simple trigonometry.

Solution for (a)

The distance between slits is $d = (1\text{ cm})/10{,}000 = 1.00\times10^{-4}\text{ cm}$ or $1.00\times10^{-6}\text{ m}$. Let us call the two angles θ_{V} for violet (380 nm) and θ_{R} for red (760 nm). Solving the equation $d\sin\theta_{\text{V}} = m\lambda$ for $\sin\theta_{\text{V}}$,

$$\sin\theta_{\text{V}} = \frac{m\lambda_{\text{V}}}{d}, \tag{27.13}$$

where $m = 1$ for first order and $\lambda_{\text{V}} = 380\text{ nm} = 3.80\times10^{-7}\text{ m}$. Substituting these values gives

$$\sin\theta_{\text{V}} = \frac{3.80\times10^{-7}\text{ m}}{1.00\times10^{-6}\text{ m}} = 0.380. \tag{27.14}$$

Thus the angle θ_{V} is

$$\theta_{\text{V}} = \sin^{-1} 0.380 = 22.33^{\circ}. \tag{27.15}$$

Similarly,

$$\sin\theta_{\text{R}} = \frac{7.60\times10^{-7}\text{ m}}{1.00\times10^{-6}\text{ m}}. \tag{27.16}$$

Thus the angle θ_{R} is

$$\theta_{\text{R}} = \sin^{-1} 0.760 = 49.46^{\circ}. \tag{27.17}$$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

Solution for (b)

The distances on the screen are labeled y_{V} and y_{R} in Figure 27.20. Noting that $\tan\theta = y/x$, we can solve for y_{V} and y_{R}. That is,

$$y_{\text{V}} = x\tan\theta_{\text{V}} = (2.00\text{ m})(\tan 22.33^{\circ}) = 0.815\text{ m} \tag{27.18}$$

and

$$y_{\text{R}} = x\tan\theta_{\text{R}} = (2.00\text{ m})(\tan 49.46^{\circ}) = 2.338\text{ m}. \tag{27.19}$$

The distance between them is therefore

$$y_{\text{R}} - y_{\text{V}} = 1.52\text{ m}. \tag{27.20}$$

Discussion

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

27.5 Single Slit Diffraction

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the single slit diffraction pattern.

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. Figure 27.21 shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.

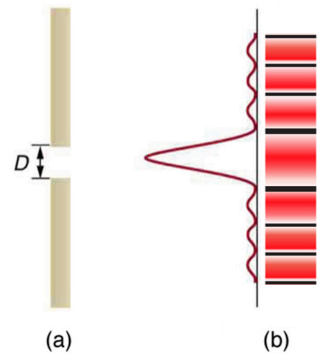

FIGURE 27.21 (a) Single slit diffraction pattern. Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side.

The analysis of single slit diffraction is illustrated in Figure 27.22. Here we consider light coming from different parts of the *same* slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in Figure 27.22(a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle θ relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In Figure 27.22(b), the ray from the bottom travels a distance of one wavelength λ farther than the ray from the top. Thus a ray from the center travels a distance $\lambda/2$ farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a minimum in intensity will occur at this angle. There will be another minimum at the same angle to the right of the incident direction of the light.

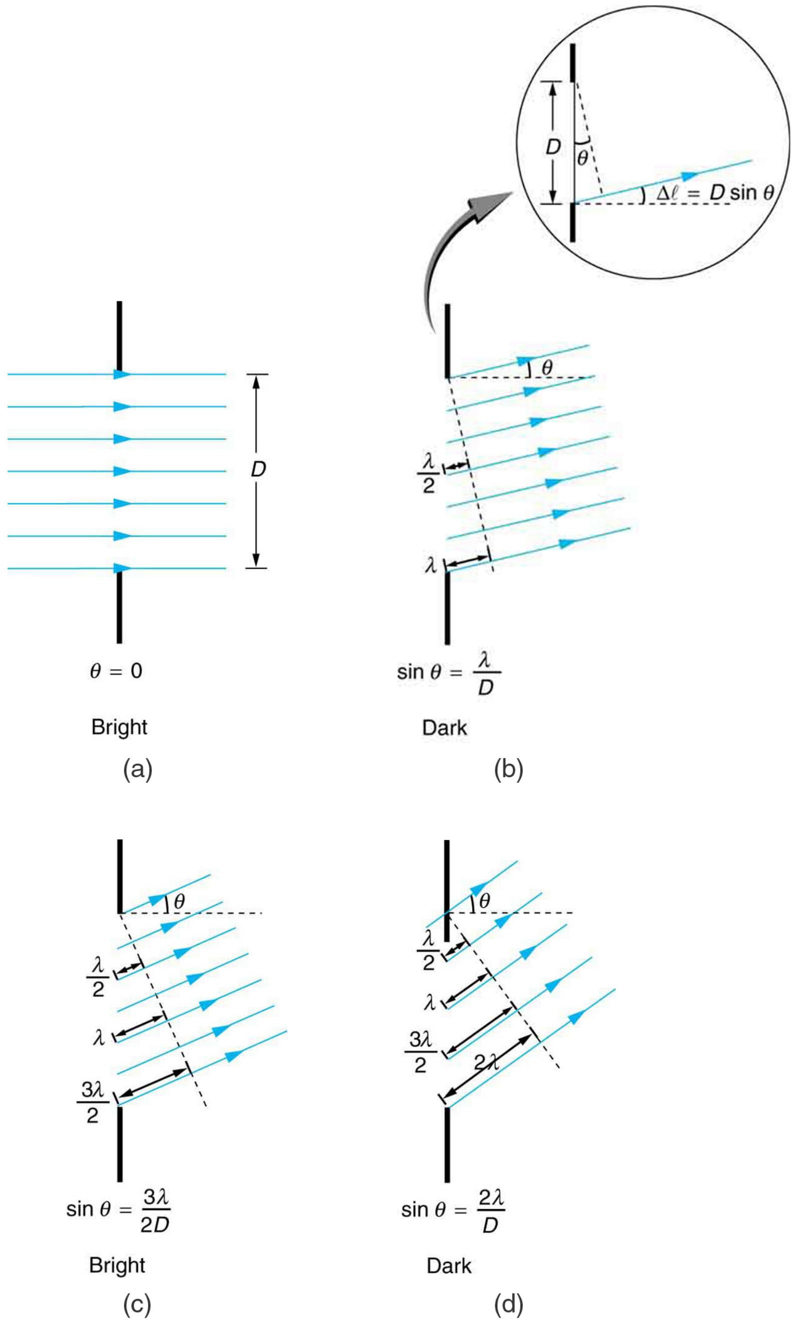

FIGURE 27.22 Light passing through a single slit is diffracted in all directions and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be $D \sin \theta$.

At the larger angle shown in Figure 27.22(c), the path lengths differ by $3\lambda/2$ for rays from the top and bottom of the slit. One ray travels a distance λ different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, will also add constructively. Most rays from the slit will have another to interfere with constructively, and a maximum in intensity will occur at this angle. However, all rays do not interfere constructively for this situation, and so the maximum is not as intense as the central maximum. Finally, in Figure 27.22(d), the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is $D \sin \theta$, and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.

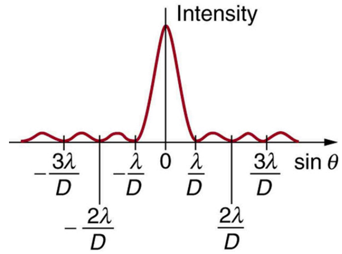

FIGURE 27.23 A graph of single slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact the central maximum is six times higher than shown here.

Thus, to obtain **destructive interference for a single slit**,

$$D \sin\theta = m\lambda, \text{ for } m = 1, -1, 2, -2, 3, \ \ldots \ \text{(destructive)}, \qquad 27.21$$

where D is the slit width, λ is the light's wavelength, θ is the angle relative to the original direction of the light, and m is the order of the minimum. Figure 27.23 shows a graph of intensity for single slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This is consistent with the illustration in Figure 27.21(b).

EXAMPLE 27.4

Calculating Single Slit Diffraction

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of 45.0° relative to the incident direction of the light. (a) What is the width of the slit? (b) At what angle is the first minimum produced?

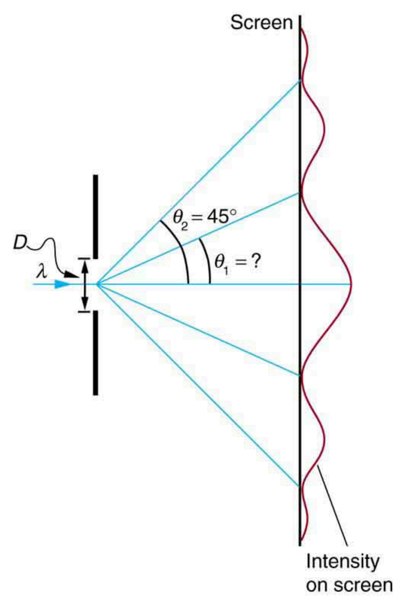

FIGURE 27.24 A graph of the single slit diffraction pattern is analyzed in this example.

Strategy

From the given information, and assuming the screen is far away from the slit, we can use the equation $D \sin\theta = m\lambda$ first to find D, and again to find the angle for the first minimum θ_1.

Solution for (a)

We are given that $\lambda = 550 \text{ nm}$, $m = 2$, and $\theta_2 = 45.0^\circ$. Solving the equation $D \ \sin\theta = m\lambda$ for D and substituting known values gives

$$\begin{aligned} D &= \frac{m\lambda}{\sin\theta_2} = \frac{2(550\text{ nm})}{\sin 45.0^\circ} \\ &= \frac{1100\times10^{-9}}{0.707} \\ &= 1.56\times10^{-6}. \end{aligned} \quad (27.22)$$

Solution for (b)

Solving the equation $D\ \sin\theta = m\lambda$ for $\sin\theta_1$ and substituting the known values gives

$$\sin\theta_1 = \frac{m\lambda}{D} = \frac{1\left(550\times10^{-9}\text{ m}\right)}{1.56\times10^{-6}\text{ m}}. \quad (27.23)$$

Thus the angle θ_1 is

$$\theta_1 = \sin^{-1} 0.354 = 20.7^\circ. \quad (27.24)$$

Discussion

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single slit diffraction pattern. We also see that the central maximum extends 20.7° on either side of the original beam, for a width of about 41°. The angle between the first and second minima is only about $24^\circ\ (45.0^\circ - 20.7^\circ)$. Thus the second maximum is only about half as wide as the central maximum.

27.6 Limits of Resolution: The Rayleigh Criterion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the Rayleigh criterion.

Light diffracts as it moves through space, bending around obstacles, interfering constructively and destructively. While this can be used as a spectroscopic tool—a diffraction grating disperses light according to wavelength, for example, and is used to produce spectra—diffraction also limits the detail we can obtain in images. Figure 27.25(a) shows the effect of passing light through a small circular aperture. Instead of a bright spot with sharp edges, a spot with a fuzzy edge surrounded by circles of light is obtained. This pattern is caused by diffraction similar to that produced by a single slit. Light from different parts of the circular aperture interferes constructively and destructively. The effect is most noticeable when the aperture is small, but the effect is there for large apertures, too.

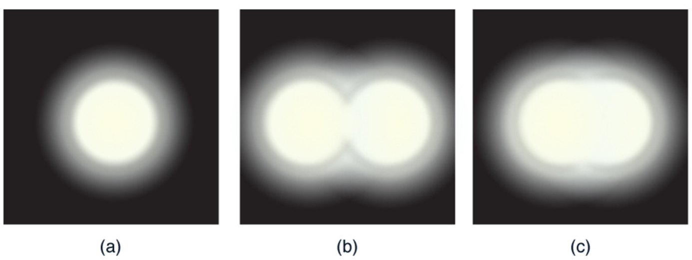

FIGURE 27.25 (a) Monochromatic light passed through a small circular aperture produces this diffraction pattern. (b) Two point light sources that are close to one another produce overlapping images because of diffraction. (c) If they are closer together, they cannot be resolved or distinguished.

How does diffraction affect the detail that can be observed when light passes through an aperture? Figure 27.25(b) shows the diffraction pattern produced by two point light sources that are close to one another. The pattern is similar to that for a single point source, and it is just barely possible to tell that there are two light sources rather than one. If they were closer together, as in Figure 27.25(c), we could not distinguish them, thus limiting the detail or resolution we can obtain. This limit is an inescapable consequence of the wave nature of light.

There are many situations in which diffraction limits the resolution. The acuity of our vision is limited because light