

works even better if you use a laser pencil.)

25.3 The Law of Refraction

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Determine the index of refraction, given the speed of light in a medium.

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [Figure 25.10](#).) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Refraction

The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

Speed of Light

The speed of light c not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. As the accuracy of the measurements of the speed of light were improved, c was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in [Special Relativity](#). It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.

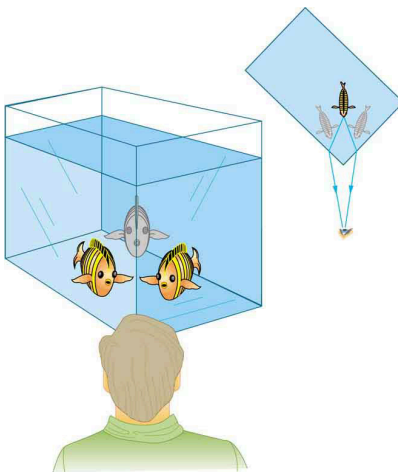


FIGURE 25.10 Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter's moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be 2.26×10^8 m/s (only 25% different from today's accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [Figure 25.11](#). Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer's eye.

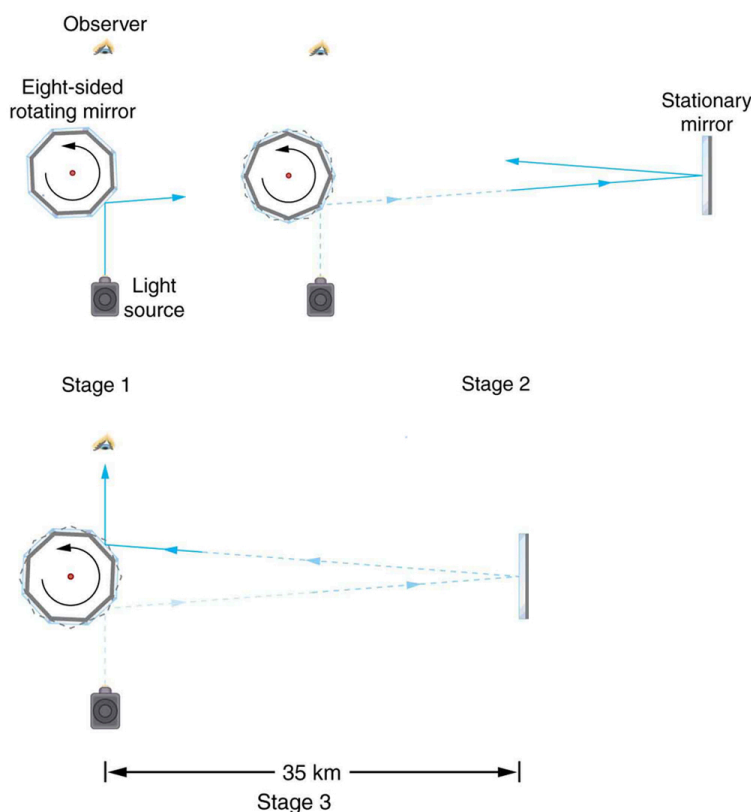


FIGURE 25.11 A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson's calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}, \quad 25.1$$

where the approximate value of 3.00×10^8 m/s is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

$$n = \frac{c}{v}, \quad 25.2$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Value of the Speed of Light

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s} \quad 25.3$$

Index of Refraction

$$n = \frac{c}{v} \quad 25.4$$

That is, $n \geq 1$. [Table 25.1](#) gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n = 1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Medium	n
Gases at 0°C, 1 atm	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at 20°C	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at 20°C	
Diamond	2.419

TABLE 25.1 Index of Refraction in Various Media

Medium	n
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 20°C	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

TABLE 25.1 Index of Refraction in Various Media



EXAMPLE 25.1

Speed of Light in Matter

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Strategy

The speed of light in a material, v , can be calculated from the index of refraction n of the material using the equation $n = c/v$.

Solution

The equation for index of refraction states that $n = c/v$. Rearranging this to determine v gives

$$v = \frac{c}{n}. \quad 25.5$$

The index of refraction for zircon is given as 1.923 in [Table 25.1](#), and c is given in the equation for speed of light. Entering these values in the last expression gives

$$\begin{aligned} v &= \frac{3.00 \times 10^8 \text{ m/s}}{1.923} \\ &= 1.56 \times 10^8 \text{ m/s}. \end{aligned} \quad 25.6$$

Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [Table 25.1](#) that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

Figure 25.12 shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. (Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in Figure 25.12, medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in Figure 25.12(a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in Figure 25.12(b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.

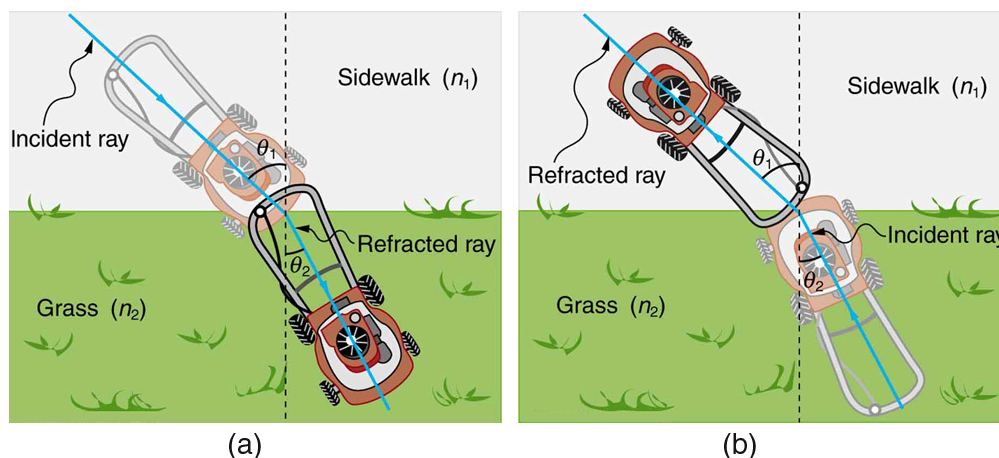


FIGURE 25.12 The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the **law of refraction**, or “Snell’s Law,” which is stated in equation form as

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \quad 25.7$$

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and θ_2 are the angles between the rays and the perpendicular in medium 1 and 2, as shown in Figure 25.12. The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell’s law after the Dutch mathematician Willebrord Snell (1591–1626). While the law has been named after Snell, the Arabian physicist, Ibn Sahl, found the law of refraction in 984 and used it in his work *On Burning Mirrors and Lenses*. Snell’s experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium. Snell was not aware that the speed of light varied in different media, but through experiments he was able to determine indices of refraction from the way light rays changed direction.

The Law of Refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad 25.8$$

Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.



EXAMPLE 25.2

Determine the Index of Refraction from Refraction Data

Find the index of refraction for medium 2 in [Figure 25.12\(a\)](#), assuming medium 1 is air and given the incident angle is 30.0° and the angle of refraction is 22.0° .

Strategy

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus $n_1 = 1.00$ here. From the given information, $\theta_1 = 30.0^\circ$ and $\theta_2 = 22.0^\circ$. With this information, the only unknown in Snell's law is n_2 , so that it can be used to find this unknown.

Solution

Snell's law is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \quad 25.9$$

Rearranging to isolate n_2 gives

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2}. \quad 25.10$$

Entering known values,

$$\begin{aligned} n_2 &= 1.00 \frac{\sin 30.0^\circ}{\sin 22.0^\circ} = \frac{0.500}{0.375} \\ &= 1.33. \end{aligned} \quad 25.11$$

Discussion

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.



EXAMPLE 25.3

A Larger Change in Direction

Suppose that in a situation like that in [Example 25.2](#), light goes from air to diamond and that the incident angle is 30.0° . Calculate the angle of refraction θ_2 in the diamond.

Strategy

Again the index of refraction for air is taken to be $n_1 = 1.00$, and we are given $\theta_1 = 30.0^\circ$. We can look up the index of refraction for diamond in [Table 25.1](#), finding $n_2 = 2.419$. The only unknown in Snell's law is θ_2 , which we wish to determine.

Solution

Solving Snell's law for $\sin \theta_2$ yields

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1. \quad 25.12$$

Entering known values,

$$\sin \theta_2 = \frac{1.00}{2.419} \sin 30.0^\circ = (0.413)(0.500) = 0.207. \quad 25.13$$

The angle is thus

$$\theta_2 = \sin^{-1} 0.207 = 11.9^\circ. \quad 25.14$$

Discussion

For the same 30° angle of incidence, the angle of refraction in diamond is significantly smaller than in water (11.9° rather than 22° —see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

25.4 Total Internal Reflection

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the phenomenon of total internal reflection.
- Describe the workings and uses of fiber optics.
- Analyze the reason for the sparkle of diamonds.

A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce *total reflection* using an aspect of *refraction*.

Consider what happens when a ray of light strikes the surface between two materials, such as is shown in [Figure 25.13\(a\)](#). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since $n_1 > n_2$, the angle of refraction is greater than the angle of incidence—that is, $\theta_2 > \theta_1$.) Now imagine what happens as the incident angle is increased. This causes θ_2 to increase also. The largest the angle of refraction θ_2 can be is 90° , as shown in [Figure 25.13\(b\)](#). The **critical angle** θ_c for a combination of materials is defined to be the incident angle θ_1 that produces an angle of refraction of 90° . That is, θ_c is the incident angle for which $\theta_2 = 90^\circ$. If the incident angle θ_1 is greater than the critical angle, as shown in [Figure 25.13\(c\)](#), then all of the light is reflected back into medium 1, a condition called **total internal reflection**.

Critical Angle

The incident angle θ_1 that produces an angle of refraction of 90° is called the critical angle, θ_c .