

not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

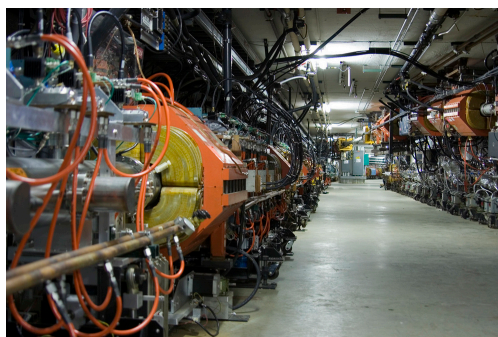


FIGURE 22.24 The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcgrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [Figure 22.25](#).) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.

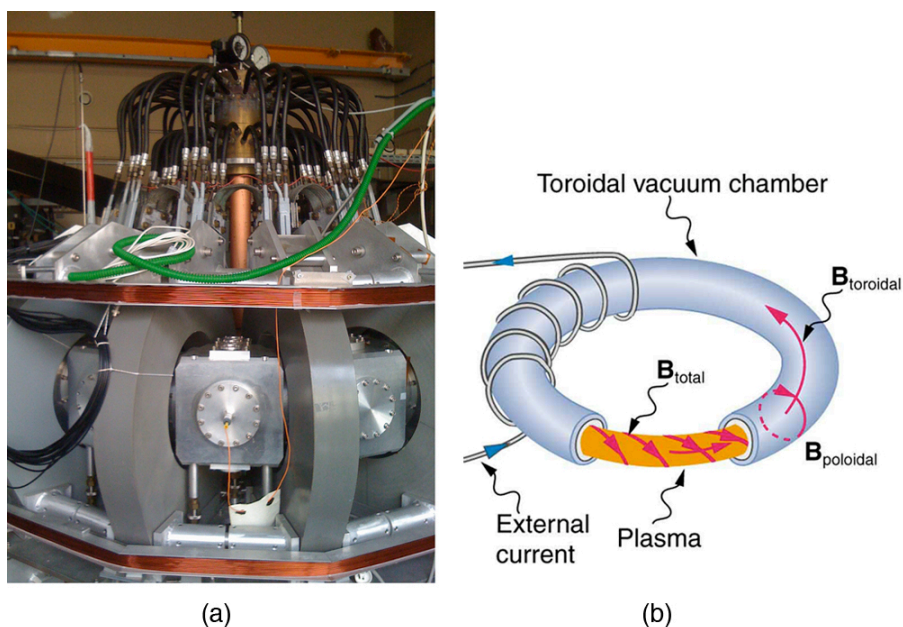


FIGURE 22.25 Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See [More Applications of Magnetism](#).) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

22.6 The Hall Effect

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

Figure 22.26 shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ϵ , known as the **Hall emf**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect**, after Edwin Hall, the American physicist who discovered it in 1879.

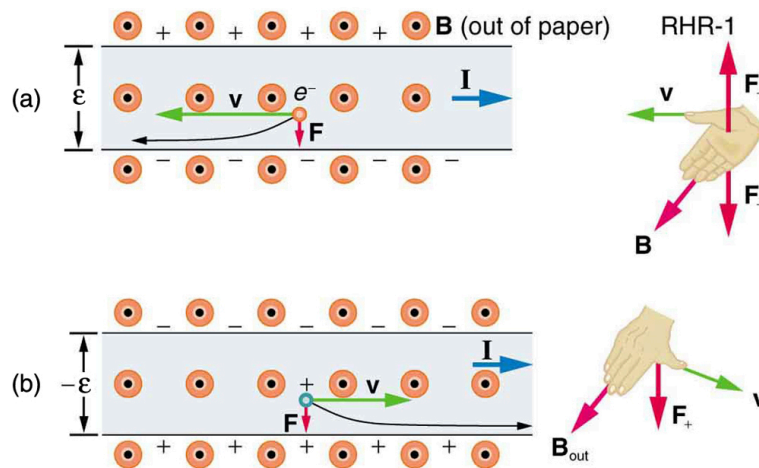


FIGURE 22.26 The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ϵ , the Hall emf, across the conductor. (b) Positive charges moving to the right (conventional current also to the right) are moved to the side, producing a Hall emf of the opposite sign, $-\epsilon$. Thus, if the direction of the field and current are known, the sign of the charge carriers can be determined from the Hall effect.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in Figure 22.26(b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ϵ , across a conductor. Consider the balance of forces on a moving charge in a situation where B , v , and l are mutually perpendicular, such as shown in Figure 22.27. Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = qvB$, and the electric force, $F_e = qE$, eventually grows to equal it. That is,

$$qE = qvB \quad 22.10$$

or

$$E = vB. \quad 22.11$$

Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \epsilon/l$, where l is the width of the conductor and ϵ is the Hall emf. Entering this into the last expression gives

$$\frac{\epsilon}{l} = vB. \quad 22.12$$

Solving this for the Hall emf yields

$$\epsilon = Blv \text{ (} B, v, \text{ and } l, \text{ mutually perpendicular),} \quad 22.13$$

where ε is the Hall effect voltage across a conductor of width l through which charges move at a speed v .

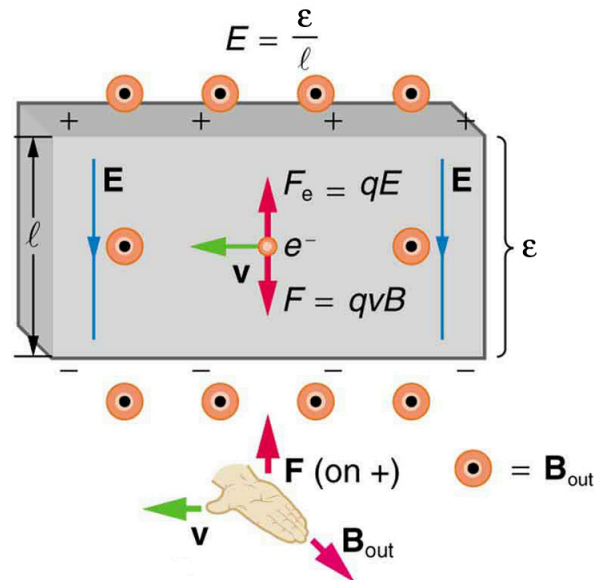


FIGURE 22.27 The Hall emf ε produces an electric force that balances the magnetic force on the moving charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B . Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See [Figure 22.28](#).) A magnetic field applied perpendicular to the flow direction produces a Hall emf ε as shown. Note that the sign of ε depends not on the sign of the charges, but only on the directions of B and v . The magnitude of the Hall emf is $\varepsilon = Blv$, where l is the pipe diameter, so that the average velocity v can be determined from ε providing the other factors are known.

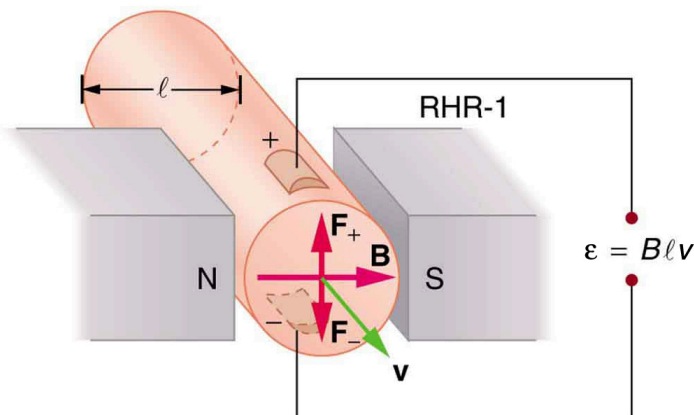


FIGURE 22.28 The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ε is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v .



EXAMPLE 22.3

Calculating the Hall emf: Hall Effect for Blood Flow

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in [Figure 22.28](#). What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B , v , and l are mutually perpendicular, the equation $\varepsilon = Blv$ can be used to find ε .

Solution

Entering the given values for B , v , and l gives

$$\begin{aligned}\epsilon &= Blv = (0.100 \text{ T})(4.00 \times 10^{-3} \text{ m})(0.200 \text{ m/s}) \\ &= 80.0 \mu\text{V}\end{aligned}\quad 22.14$$

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ϵ is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

22.7 Magnetic Force on a Current-Carrying Conductor

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.

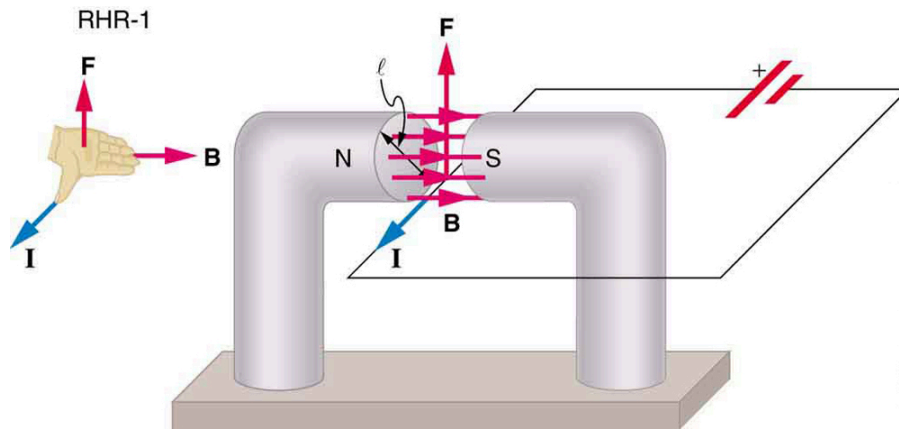


FIGURE 22.29 The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_d is given by $F = qv_d B \sin \theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_d B \sin \theta)(N)$, where N is the number of charge carriers in the section of wire of length l . Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (qv_d B \sin \theta)(nAl)$. Gathering terms,

$$F = (nqAv_d)lB \sin \theta. \quad 22.15$$

Because $nqAv_d = I$ (see [Current](#)),

$$F = IlB \sin \theta \quad 22.16$$

is the equation for *magnetic force on a length l of wire carrying a current I in a uniform magnetic field B* , as shown in [Figure 22.30](#). If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB \sin \theta$. The direction of this force is given by RHR-1, with the thumb in the direction of