EXAMPLE 19.10

A Mixture of Series and Parallel Capacitance

Find the total capacitance of the combination of capacitors shown in Figure 19.21. Assume the capacitances in Figure 19.21 are known to three decimal places ($C_1 = 1.000\ \mu\text{F}$, $C_2 = 5.000\ \mu\text{F}$, and $C_3 = 8.000\ \mu\text{F}$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}}. \tag{19.71}$$

Inverting gives

$$C_\text{S} = 0.833\ \mu\text{F}. \tag{19.72}$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

$$\begin{aligned} C_\text{tot} &= C_\text{S} + C_\text{S} \\ &= 0.833\ \mu\text{F} + 8.000\ \mu\text{F} \\ &= 8.833\ \mu\text{F}. \end{aligned} \tag{19.73}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

19.7 Energy Stored in Capacitors

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- List some uses of capacitors.
- Express in equation form the energy stored in a capacitor.
- Explain the function of a defibrillator.

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review Figure 19.22.) Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See Figure 19.22.) Capacitors are also used to supply energy for flash lamps on cameras.

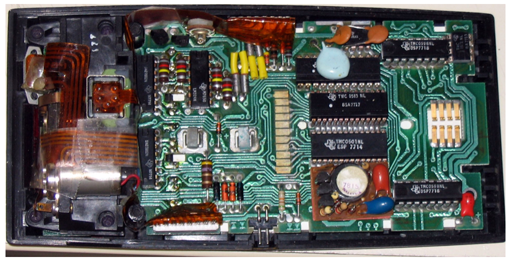

FIGURE 19.22 Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta \text{PE} = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV. But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap}, is

$$E_{\text{cap}} = \frac{QV}{2}, \qquad 19.74$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV, but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \qquad 19.75$$

where Q is the charge and V the voltage on a capacitor C. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \qquad 19.76$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places (Figure 19.23). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.

FIGURE 19.23 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies, Wikimedia Commons)

EXAMPLE 19.11

Capacitance in a Heart Defibrillator

A heart defibrillator delivers 4.00×10^2 J of energy by discharging a capacitor initially at 1.00×10^4 V. What is its capacitance?

Strategy

We are given E_{cap} and V, and we are asked to find the capacitance C. Of the three expressions in the equation for E_{cap}, the most convenient relationship is

$$E_{cap} = \frac{CV^2}{2}. \quad 19.77$$

Solution

Solving this expression for C and entering the given values yields

$$\begin{aligned} C &= \frac{2E_{cap}}{V^2} = \frac{2(4.00\times10^2 \text{ J})}{(1.00\times10^4 \text{ V})^2} = 8.00\times10^{-6} \text{ F} \\ &= 8.00 \text{ µF}. \end{aligned} \quad 19.78$$

Discussion

This is a fairly large, but manageable, capacitance at 1.00×10^4 V.