$$E = -\frac{\Delta V}{\Delta s}, \tag{19.36}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

19.3 Electrical Potential Due to a Point Charge

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential V of a point charge* is

$$V = \frac{kQ}{r} \text{ (Point Charge)}, \tag{19.37}$$

where k is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kQ}{r} \text{ (Point Charge)}. \tag{19.38}$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2}. \tag{19.39}$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

EXAMPLE 19.6

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a -3.00 nC static charge?

Strategy

As we have discussed in Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$\begin{aligned} V &= k\frac{Q}{r} \\ &= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{-3.00\times10^{-9} \text{ C}}{5.00\times10^{-2} \text{ m}}\right) \\ &= -539 \text{ V}. \end{aligned} \quad \textbf{19.40}$$

Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

EXAMPLE 19.7

What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See Figure 19.7.) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

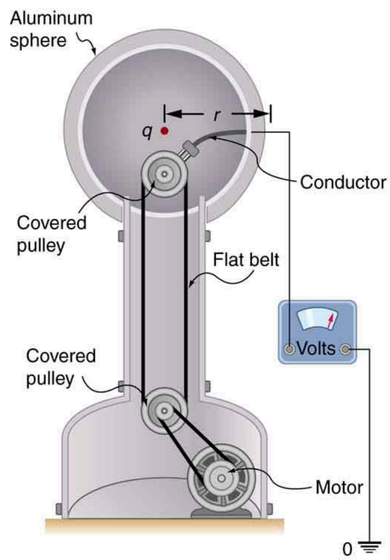

FIGURE 19.7 The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kQ}{r}. \quad \textbf{19.41}$$

Solution

Solving for Q and entering known values gives

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125\ \text{m})\left(100\times10^3\ \text{V}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} \\ &= 1.39\times10^{-6}\ \text{C} = 1.39\ \mu\text{C}. \end{aligned} \quad (19.42)$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in Electric Potential Energy: Potential Difference, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $\text{PE}_g = mgh$.

19.4 Equipotential Lines

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider Figure 19.8, which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius *r* surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance *r* from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure 19.8. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

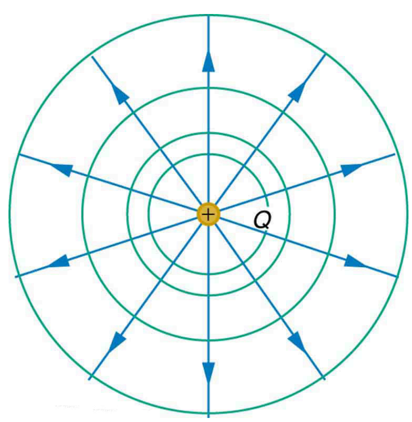

FIGURE 19.8 An isolated point charge Q with its electric field lines in blue and equipotential lines in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is