$$v = \sqrt{\frac{2qV}{m}}. \tag{19.19}$$

Entering values for q, V, and m gives

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\text{ C}\right)(-100\text{ J/C})}{9.11\times10^{-31}\text{ kg}}} \\ &= 5.93\times10^{6}\text{ m/s}. \end{aligned} \tag{19.20}$$

Discussion

Note that both the charge and the initial voltage are negative, as in Figure 19.4. From the discussions in Electric Charge and Electric Field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

19.2 Electric Potential in a Uniform Electric Field

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See Figure 19.5.) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist’s point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in Electric Potential Energy: Potential Difference, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

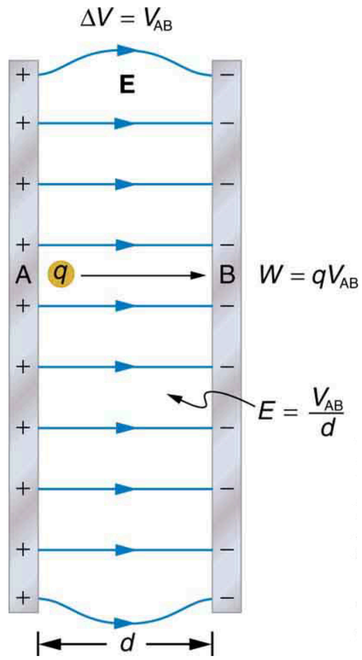

FIGURE 19.5 The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{\text{AB}}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}$. See the text for details.)

The work done by the electric field in Figure 19.5 to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta \text{PE} = -q\Delta V. \tag{19.21}$$

The potential difference between points A and B is

$$-\Delta V = -(V_{\text{B}} - V_{\text{A}}) = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}. \tag{19.22}$$

Entering this into the expression for work yields

$$W = qV_{\text{AB}}. \tag{19.23}$$

Work is $W = Fd \cos \theta$; here $\cos \theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

$$qEd = qV_{\text{AB}}. \tag{19.24}$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left.\begin{aligned} V_{\text{AB}} &= Ed \\ E &= \frac{V_{\text{AB}}}{d} \end{aligned}\right\} (\text{uniform } E\text{ - field only}), \tag{19.25}$$

where d is the distance from A to B, or the distance between the plates in Figure 19.5. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1 \text{ N/C} = 1 \text{ V/m}. \tag{19.26}$$

Voltage between Points A and B

$$\left.\begin{aligned} V_{\text{AB}} &= Ed \\ E &= \frac{V_{\text{AB}}}{d} \end{aligned}\right\} (\text{uniform } E\text{ - field only}), \tag{19.27}$$

where d is the distance from A to B, or the distance between the plates.

EXAMPLE 19.4

What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about $3.0\times10^{6}\ \text{V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{\text{AB}} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$\text{V}_{\text{AB}} = Ed. \tag{19.28}$$

Entering the given values for E and d gives

$$V_{\text{AB}} = (3.0\times10^{6}\ \text{V/m})(0.025\ \text{m}) = 7.5\times10^{4}\ \text{V} \tag{19.29}$$

or

$$V_{\text{AB}} = 75\ \text{kV}. \tag{19.30}$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.

FIGURE 19.6 A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

EXAMPLE 19.5

Field and Force inside an Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{\text{AB}}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q\ \mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q\ E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{\text{AB}}}{d}. \quad \text{19.31}$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\ \text{kV}}{0.0400\ \text{m}} = 6.25 \times 10^5\ \text{V/m}. \quad \text{19.32}$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE. \quad \text{19.33}$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^5\ \text{V/m}) = 0.313\ \text{N}. \quad \text{19.34}$$

Discussion

Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \quad \text{19.35}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \quad 19.36$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

19.3 Electrical Potential Due to a Point Charge

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential V of a point charge* is

$$V = \frac{kQ}{r} \text{ (Point Charge)}, \quad 19.37$$

where k is a constant equal to $9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kQ}{r} \text{ (Point Charge)}. \quad 19.38$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2}. \quad 19.39$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

EXAMPLE 19.6

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb $(\mu\mathrm{C})$ range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a $-3.00\ \mathrm{nC}$ static charge?

Strategy

As we have discussed in Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and