18.4 Electric Field: Concept of a Field Revisited

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $F = k |q_1q_2|/r^2$, its magnitude is given by the equation $F = k |qQ|/r^2$, for a **point charge** (a particle having a charge *Q*) acting on a **test charge** *q* at a distance *r* (see Figure 18.18). Both the magnitude and direction of the Coulomb force field depend on *Q* and the test charge *q*.



FIGURE 18.18 The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1 , and so the force F_2 acting on it is attractive and stronger than F_1 . The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$
 18.11

where **F** is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that **E** is in the same direction as **F**. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is

 $F = k | qQ | /r^2$. Thus the magnitude of the electric field, *E*, for a point charge is

$$E = \left|\frac{F}{q}\right| = k \left|\frac{qQ}{qr^2}\right| = k \frac{|Q|}{r^2}.$$
18.12

Since the test charge cancels, we see that

$$E = k \frac{|Q|}{r^2}.$$
18.13

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

EXAMPLE 18.2

Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9}$ C and $r = 5.00 \times 10^{-3}$ m. Entering those values into the above equation gives

$$E = k \frac{Q}{r^2}$$

= (8.99 × 10⁹ N · m²/C²)× $\frac{(2.00 \times 10^{-9} \text{ C})}{(5.00 \times 10^{-3} \text{ m})^2}$
= 7.19 × 10⁵ N/C.
18.14

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

EXAMPLE 18.3

Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250 \ \mu$ C?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250 \,\mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5 \,\text{N/C}$ is thus,

$$F = -qE$$

= (0.250 × 10⁻⁶ C)(7.20 × 10⁵ N/C) 18.15
= 0.180 N.

Because *q* is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

PHET EXPLORATIONS

Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

Click to view content (https://openstax.org/l/02electric_field_dreams).

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18.5 Electric Field Lines: Multiple Charges

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 18.19 shows two pictorial representations of the same electric field created by a positive point charge Q. Figure 18.19 (b) shows the standard representation using continuous lines. Figure 18.19 (a) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.



FIGURE 18.19 Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 18.20.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2 . This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.